IDEAL FLOW

COMBINATION OF BASIC FLOW PATTERNS

- DOUBLET + LINEAR
  = FLOW AROUND CYLINDER

- DOUBLET + VORTEX + LINEAR
  = FLOW AROUND ROTATING CYLINDER

DOUBLET + LINEAR = FLOW AROUND CYLINDER

STREAM FUNCTION COMBINATION

$$\psi_{combination} = \psi_{doublet} + \psi_{linear}$$

$$= \frac{\mu}{2\pi r} \cdot \sin\theta - Ur \cdot \sin\theta$$

This can describe a uniform flow through a cylinder. In real life, a cylinder is a solid body, and has a radius value. Therefore, the value of $\psi_{combination}$ need to be modified.

Substitute the value of $(r, \theta) = (r, 0)$ in the $\psi_{combination}$.

$$\psi_{combination} = 0$$

$$0 = \frac{\mu}{2\pi r} \cdot \sin\theta - Ur \cdot \sin\theta$$

If $r = A$

$$0 = \frac{\mu}{2\pi A} \cdot \sin\theta - UA \cdot \sin\theta$$

$$= \frac{\mu}{2\pi A} \cdot \sin\theta - UA \cdot \sin\theta$$

$$= \sin\theta \left(\frac{\mu}{2\pi A} - UA \right)$$

$$0 = \frac{\mu}{2\pi A} - UA$$

Strength of doublet can be determined as ;

$$0 = \frac{\mu}{2\pi A} - UA$$

$$\mu = 2\pi U A^2$$

Substitute value of μ in the $\psi_{combination}$

$$0 = \frac{\mu}{2\pi r} \cdot \sin\theta - Ur \cdot \sin\theta$$

$$= \frac{2\pi U A^2}{2\pi r} \cdot \sin\theta - Ur \cdot \sin\theta$$

$$= \frac{UA^2}{r} \cdot \sin\theta - Ur \cdot \sin\theta$$

$$= -Ur \cdot \sin\theta + \frac{UA^2}{r} \cdot \sin\theta$$

$$= -Ur \cdot \sin\theta \left(1 - \frac{A^2}{r^2}\right)$$

Stream function of flow around cylinder can be shown as :

$$\psi_{cyl} = -Ur \cdot \sin\theta \left(1 - \frac{A^2}{r^2}\right)$$

Velocity potential of flow around cylinder can be shown as :

$$\phi_{cyl} = -Ur \cdot \cos\theta \left(1 - \frac{A^2}{r^2}\right)$$

VELOCITY COMPONENT

$$u_r = \frac{d\psi}{rd\theta} = \frac{d}{rd\theta}\left(-Ur \cdot \sin\theta\left(1 - \frac{A^2}{r^2}\right)\right)$$

$$u_r = -U \cdot \cos\theta\left(1 - \frac{A^2}{r^2}\right)$$

If $r = A$, we will get $u_r = 0$

$$u_\theta = -\frac{d\psi}{dr} = \frac{d}{dr}\left(-Ur \cdot \sin\theta\left(1 - \frac{A^2}{r^2}\right)\right)$$

$$u_\theta = U \cdot \sin\theta\left(1 + \frac{A^2}{r^2}\right)$$

If $\quad r = A\ ,\ u_\theta = 2U \sin\theta$

Maximum velocity occurs at $\theta = 90^\circ$
and the $\quad u_\theta = 2U$

If. $\quad r = A\ ,\ u_\theta = 0$

$$\sin\theta = 0$$

$$\theta = 0 \quad \text{dan} \quad \pi$$

PRESSURE ON CYLINDER SURFACE

Pressure on the cylinder surface can be written as :

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} = \frac{P_2}{\rho g} + \frac{V_2^2}{2g}$$

$$\frac{P_2 - P_1}{\rho g} = \frac{V_1^2}{2g} - \frac{V_2^2}{2g}$$

$$V_1 = U$$
$$V_2 = 2U \sin\theta$$

$$P_2 - P_1 = \frac{1}{2}\rho U^2 - \frac{1}{2}\rho U^2(4\sin^2\theta)$$

Pressure coefficient, C_p can be written as :

$$C_p = \frac{P_2 - P_1}{\frac{1}{2}\rho U^2} = 1 - 4\sin^2\theta$$

Pressure at point 2:

$$P_2 = P_1 + \frac{1}{2}\rho U^2(1 - 4\sin^2\theta)$$

Drag force on the cylinder surface:

$$\begin{aligned} F_D &= \int dF \cdot \cos\theta \\ &= \int_0^{2\pi} P_2 \cos\theta \cdot A \cdot d\theta \\ &= \int_0^{2\pi} \left(P_1 + \frac{1}{2}\rho U^2(1 - 4\sin^2\theta)\right)\cos\theta \cdot A \\ &\quad \cdot d\theta \\ &= 0 \end{aligned}$$

Lift force on the cylinder surface:

$$
\begin{aligned}
F_L &= -\int dF \cdot \sin\theta \\
&= \int_0^{2\pi} P_2 \sin\theta \cdot A \cdot d\theta \\
&= \int_0^{2\pi} \left(P_1 + \frac{1}{2}\rho U^2 (1 - 4\sin^2\theta)\right) \sin\theta \cdot A \cdot d\theta \\
&= 0
\end{aligned}
$$

DOUBLET + VORTEX + LINEAR = FLOW AROUND ROTATING CYLINDER

STREAM FUNCTION COMBINATION

$$\psi_{combination} = \psi_{cyl} + \psi_{vortex}$$

$$= -Ur \cdot \sin\theta \left(1 - \frac{A^2}{r^2}\right) - \frac{\Gamma}{2\pi} \cdot \ln\left(\frac{r}{A}\right)$$

If the cylinder does not rotate, the value of the vortex, Γ, becomes zero.

If the cylinder rotates, the vortex, Γ, has a value. This will cause the top of the cylinder to have high velocity and low pressure. The bottom of the cylinder will have low velocity and high pressure.

This pressure difference will result in a LIFT FORCE.
It is known as MAGNUS EFFECT.

(H.G., Magnus, a German physicist, 1802-1870)

VELOCITY COMPONENT

$$u_r = \frac{d\psi}{rd\theta} = \frac{d}{rd\theta}\left(-Ur \cdot \sin\theta\left(1 - \frac{A^2}{r^2}\right) - \frac{\Gamma}{2\pi} \cdot \ln\left(\frac{r}{A}\right)\right)$$

$$u_r = -U \cdot \cos\theta\left(1 - \frac{A^2}{r^2}\right)$$

If $r = A$, $u_r = 0$

$$u_\theta = -\frac{d\psi}{dr} = \frac{d}{dr}\left(-Ur \cdot \sin\theta \left(1 - \frac{A^2}{r^2}\right) - \frac{\Gamma}{2\pi} \cdot \ln\left(\frac{r}{A}\right)\right)$$

$$u_\theta = U \cdot \sin\theta \left(1 + \frac{A^2}{r^2}\right) + \frac{\Gamma}{2\pi r}$$

STAGNATION POINT

At the stagnation point, velocity is zero.

$$u_r = -U \cdot \cos\theta \left(1 - \frac{A^2}{r^2}\right) = 0$$

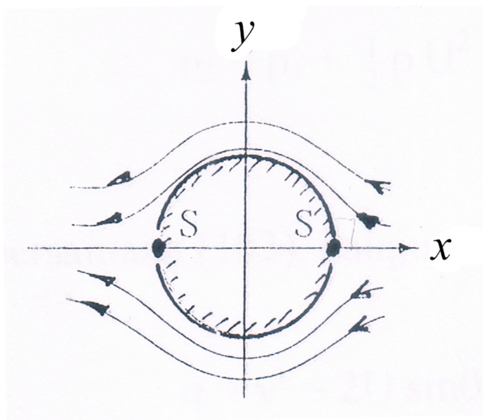
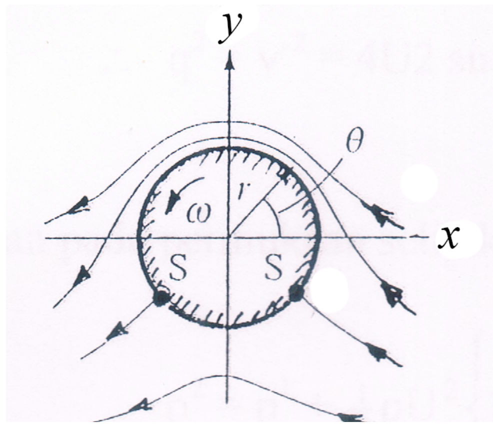
$$r = A$$

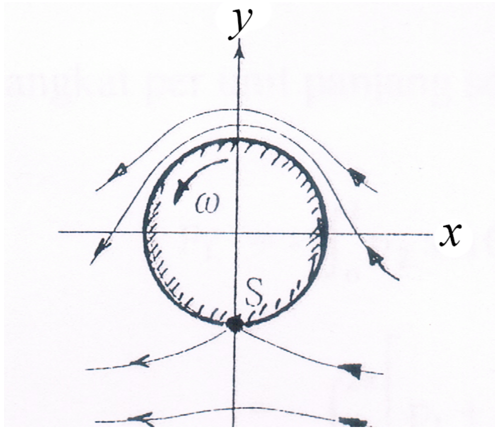
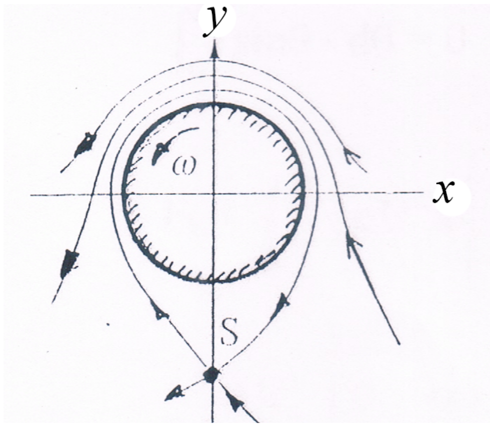
$$u_\theta = U \cdot \sin\theta \left(1 + \frac{A^2}{r^2}\right) + \frac{\Gamma}{2\pi r} = 0$$

$$\sin\theta = \frac{-\Gamma}{4\pi U A}$$

This means that the position of θ is influenced by the strength of the vortex, Γ (gamma).

There are four (4) main possibilities :

$\Gamma = 0$ $\sin\theta = 0$ $\theta = 0 \text{ dan } \pi$	
$\Gamma < 4\pi UA$ $\sin\theta < -1.0$ $\pi < \theta < 2\pi$	

$\Gamma = 4\pi UA$ $\sin\theta = -1.0$ $\theta = \frac{3}{2}\pi$	
$\Gamma > 4\pi UA$ $\sin\theta > -1.0$ $\theta =$ impossible	

LIFT FORCE

Pressure on cylinder surface;

$$P_1 + \frac{1}{2}\rho U^2 = P_1 + \frac{1}{2}\rho q^2$$

$U =$ free stream velocity
$q =$ resultant velocity

$$P_2 = P_1 + \frac{1}{2}\rho U^2 \left(1 - \frac{q^2}{U^2}\right)$$

$$q = u_\theta = 2U \sin\theta + \frac{\Gamma}{2\pi A}$$

$$q^2 = 4U^2 \sin^2\theta + \frac{2\mathrm{U}\Gamma}{\pi A}\sin\theta + \left(\frac{\Gamma}{2\pi A}\right)^2$$

$$P_2 = P_1 + \frac{1}{2}\rho U^2 \left[1 - 4\sin^2\theta - \frac{2\Gamma}{\pi U A}\sin\theta - \left(\frac{\Gamma}{2\pi U A}\right)^2\right]$$

Lift force per unit width:

$$F_L = -\int_0^{2\pi} P_2 \sin\theta \cdot A \cdot d\theta$$

$$= -\int_0^{2\pi} \left[P_1 + \frac{1}{2}\rho U^2 \left(1 - 4\sin^2\theta - \frac{2\Gamma}{\pi U A}\sin\theta - \left(\frac{\Gamma}{2\pi U A}\right)^2\right)\right] \sin\theta \cdot A \cdot d\theta$$

Known that :

$$\int_0^{2\pi} \sin\theta \, d\theta = \int_0^{2\pi} \sin^3\theta \, d\theta = 0$$

$$F_L = -\int_0^{2\pi} \frac{1}{2}\rho U^2 A \left(\frac{-2\Gamma \cdot \sin^2\theta}{\pi U A}\right) d\theta$$

$$= \frac{\rho U \Gamma}{\pi}\left[\frac{\theta}{2} - \frac{\sin 2\theta}{4}\right]_0^{2\pi}$$

$$F_L = \rho U \Gamma$$

ρ = density of fluid
U = free stream velocity
Γ = strength of vortex

It is called the KUTTA-JOUKOWSKI THEOREM