

# Fluid Properties

# 1.1 Introduction

Understanding fluid mechanics is needed for:

- Biomechanics - To understand the flow of blood and cerebral fluid.
- Meteorology and Ocean engineering - To understand the motion of air movements and ocean currents.
- Chemical engineering - To design different kinds of chemical-processing equipment.
- Aeronautical engineering - To maximize lift, minimize drag on aircraft, and to design fan-jet engines.
- Mechanical engineering - To design pumps, turbines, internal combustion engines, etc.

# 1.2 Dimensions, Units, and Physical Quantities

- **Fundamental Dimensions** - Nine quantities that can express all other quantities.
  1. Length
  2. Mass
  3. Time
  4. Temperature
  5. Amount of a substance
  6. Electric current
  7. Luminous intensity
  8. Plane angle
  9. Solid angle

## 1.2 Dimensions, Units, and Physical Quantities

**Example:**

$$F = ma$$

$$[F] = [m][a]$$

$$F = M \frac{L}{T^2}$$

- There are two primary systems of units:
  - English units
  - Système International units (SI)

## 1.2 Dimensions, Units, and Physical Quantities

**Table 1.1** Fundamental Dimensions and Their Units

<i>Quantity</i>	<i>Dimensions</i>	<i>SI units</i>		<i>English units</i>	
Length $\ell$	$L$	meter	m	foot	ft
Mass $m$	$M$	kilogram	kg	slug	slug
Time $t$	$T$	second	s	second	s
Electric current $i$		ampere	A	ampere	A
Temperature $T$	$\Theta$	kelvin	K	Rankine	$^{\circ}\text{R}$
Amount of substance	$M$	kg-mole	kmol	lb-mole	lbmol
Luminous intensity		candela	cd	candela	cd
Plane angle		radian	rad	radian	rad
Solid angle		steradian	sr	steradian	sr

- **Derived Quantities** - Combinations of fundamental quantities to form different parameters.

# 1.2 Dimensions, Units, and Physical Quantities

**Table 1.2** Derived Units

<i>Quantity</i>	<i>Dimensions</i>	<i>SI units</i>	<i>English units</i>
Area $A$	$L^2$	$m^2$	$ft^2$
Volume $\mathcal{V}$	$L^3$	$m^3$	$ft^3$
Velocity $V$	$L/T$	L (liter)	
Acceleration $a$	$L/T^2$	m/s	ft/s
Angular velocity $\omega$	$T^{-1}$	$m/s^2$	$ft/s^2$
Force $F$	$ML/T^2$	rad/s	rad/s
Density $\rho$	$M/L^3$	$kg \cdot m/s^2$	slug-ft/s <sup>2</sup>
Specific weight $\gamma$	$M/L^2T^2$	N (newton)	lb (pound)
Frequency $f$	$T^{-1}$	$kg/m^3$	slug/ft <sup>3</sup>
Pressure $p$	$M/LT^2$	N/m <sup>3</sup>	lb/ft <sup>3</sup>
Stress $\tau$	$M/LT^2$	hertz (cycles/s)	s <sup>-1</sup> (hertz)
Surface tension $\sigma$	$M/T^2$	N/m <sup>2</sup>	lb/ft <sup>2</sup>
Work $W$	$ML^2/T^2$	Pa (pascal)	(psf)
Energy $E$	$ML^2/T^2$	N/m <sup>2</sup>	lb/ft <sup>2</sup>
Heart rate $\dot{Q}$	$ML^2/T^3$	Pa (pascal)	(psf)
Torque $T$	$ML^2/T^2$	N/m	lb/ft
Power $P$	$ML^2/T^3$	N · m	ft-lb
Viscosity $\mu$	$M/LT$	J (joule)	
Kinematic viscosity $\nu$	$L^2/T$	N · m	ft-lb
Mass flux $\dot{m}$	$M/T$	J (joule)	
Flow rate $Q$	$L^3/T$	J/s	Btu/s
Specific heat $c$	$L^2/T^2\Theta$	N · m	ft-lb
Conductivity $K$	$ML/T^3\Theta$	J/s	ft-lb/s
		W (watt)	
		$N \cdot s/m^2$	lb-s/ft <sup>2</sup>
		$m^2/s$	ft <sup>2</sup> /s
		kg/s	slug/s
		$m^3/s$	ft <sup>3</sup> /s
		J/kg · K	Btu/slug-°R
		W/m · K	lb/s-°R

## 1.2 Dimensions, Units, and Physical Quantities

**Table 1.3** SI Prefixes

<i>Multiplication factor</i>	<i>Prefix</i>	<i>Symbol</i>
$10^{12}$	tera	T
$10^9$	giga	G
$10^6$	mega	M
$10^3$	kilo	k
$10^{-2}$	centi <sup>a</sup>	c
$10^{-3}$	milli	m
$10^{-6}$	micro	$\mu$
$10^{-9}$	nano	n
$10^{-12}$	pico	p

<sup>a</sup>Permissible if used alone as cm, cm<sup>2</sup>, or cm<sup>3</sup>.

## 1.2 Dimensions, Units, and Physical Quantities

A mass of 100 kg is acted on by a 400-N force acting vertically upward and a 600-N force acting upward at a 45° angle. Calculate the vertical component of the acceleration. The local acceleration of gravity is 9.81 m/s<sup>2</sup>. (The rollers are frictionless.)

### Solution

The first step in solving a problem involving forces is to draw a free-body diagram with all forces acting on it, as shown in Figure E1.1.

Next, apply Newton's second law (Eq. 1.2.4). It relates the net force acting on a mass to the acceleration and is expressed as

$$\Sigma F_y = ma_y$$

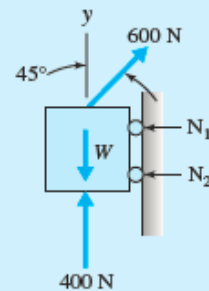


Figure E1.1

Using the appropriate components in the positive  $y$ -direction, with  $W = mg$ , we have

$$400 + 600 \sin 45^\circ - 100 \times 9.81 = 100a_y$$

$$a_y = \underline{-1.57 \text{ m/s}^2}$$

The negative sign indicates that the acceleration is in the negative  $y$ -direction, i.e., down.

*Note:* We have used only three significant digits in the answer since the information given in the problem is assumed known to three significant digits.



# 1.3 Continuum View of Gases and Liquids

- Substances can be both liquids or gases.
  - **Liquids** - Matter in which molecules are relatively free to change their positions with respect to each other. The molecules are restricted by cohesive forces so as to maintain a relatively fixed volume.
  - **Gas** – Matter in which molecules are unrestricted by cohesive forces. Gas has neither definite shape nor volume.



Figure 1.1 Normal and tangential components of a force.

- A force  $\Delta F$  that acts on an area  $\Delta A$  can be broken into tangential ( $F_t$ ) and normal ( $F_n$ ) components.
- **Stress** - Force divided by the area upon which it acts.

## 1.3 Continuum View of Gases and Liquids

- **Stress Vector** - The force vector divided by the area.
- **Normal Stress** - Normal component of force divided by the area.
- **Shear Stress ( $\tau$ )** - Tangential force divided by the area. (defined as shown below)

$$\tau = \lim_{\Delta A \rightarrow 0} \frac{\Delta F_t}{\Delta A}$$

# 1.3 Continuum View of Gases and Liquids

- **Microscopic Behavior of Fluids**
- Molecules are not stationary, but move about with high velocities.
- The molecules collide with each other and strike the walls of a container in which they are confined.
  - Gives rise to the pressure exerted by the gas.
- If volume increases (at a constant temperature):
  - Number of collisions (per unit area) decreases
  - Hence pressure decreases.
- If temperature increases:
  - Velocity of molecules increases
  - Hence pressure increases.

# 1.3 Continuum View of Gases and Liquids

- Assume that fluids act as a continuum:
  - A continuous distribution of a liquid or gas throughout a region of interest.

$$\rho = \lim_{\Delta V \rightarrow 0} \frac{\Delta m}{\Delta V}$$

- Density is used to find out if the continuum assumption is appropriate.
  - $\Delta m$ : Incremental mass                       $\Delta V$ : Incremental volume
- Standard Atmospheric Conditions
  - Pressure: 101.3 kPa
  - Temperature: 15°C
  - Density of air: 1.23 kg/m<sup>3</sup>
  - Density of water: 1000 kg/m<sup>3</sup>

## 1.3 Continuum View of Gases and Liquids

- The continuum model can be checked for accuracy by comparing the characteristic length,  $l$ , with the mean free path.
- **Mean Free Path,  $\lambda$ : Average distance a molecule travels before it collides with another molecule.**
  - **If  $l \gg \lambda$  : The continuum model is acceptable.**

$$\lambda = 0.225 \frac{m}{\rho d^2}$$

## 1.4 Pressure and Temperature Scales

- The pressure,  $p$ , can be defined as:

$$p = \lim_{\Delta A \rightarrow 0} \frac{\Delta F_n}{\Delta A}$$

$\Delta F_n$ : Incremental normal compressive force

$\Delta A$ : Incremental area

Units: N/m<sup>2</sup>

- Absolute Pressure**: Zero is reached for an ideal vacuum.
- Gage Pressure**: Pressure relative to the local atmospheric pressure.

$$P_{\text{absolute}} = P_{\text{atmospheric}} + P_{\text{gage}}$$

## 1.4 Pressure and Temperature Scales

- Temperature scales (Celsius and Fahrenheit scales)

$$K = ^\circ C + 273.15 \quad \text{Celsius to Kelvin}$$

$$^\circ R = ^\circ F + 459.67 \quad \text{Fahrenheit to Rankine}$$

	$^\circ C$	K
Steam point	$100^\circ$	373
Ice point	$0^\circ$	273
Special point	$-18^\circ$	255
Absolute zero temperature	$-273^\circ$	$0^\circ$

**Figure 1.5** Temperatures of special points.

## 1.4 Pressure and Temperature Scales

### Example 1.2

A pressure gage attached to a rigid tank measures a vacuum of 42 kPa inside the tank shown in Figure E1.2, which is situated at a site in Colorado where the elevation is 2000 m. Determine the absolute pressure inside the tank.

#### Solution

To determine the absolute pressure, the atmospheric pressure must be known. If the elevation were not given, we would assume a standard atmospheric pressure of 100 kPa. However, with the elevation given, the atmospheric pressure is found from Table B.3 in Appendix B to be 79.5 kPa. Thus

$$p = -42 + 79.5 = \underline{37.5 \text{ kPa abs}}$$

*Note:* A vacuum is always a negative gage pressure.



Figure E1.2



# 1.5 Fluid Properties

## 1.5.1 Density and Specific Weight

- **Specific Weight,  $\gamma$** : Weight per unit volume

- Units: N/m<sup>3</sup>

$$\gamma = \frac{W}{V} = \frac{mg}{V} = \rho g$$

g: Local gravity

- **Specific Gravity,  $S$** : Ratio of density of a substance to the density of water at 4°C.

- Units: Dimensionless

$$S = \frac{\rho}{\rho_{\text{water}}} = \frac{\gamma}{\gamma_{\text{water}}}$$

# 1.5 Fluid Properties

## 1.5.1 Density and Specific Weight

**Table 1.4** Density, Specific Weight, and Specific Gravity of Air and Water at Standard Conditions

	<i>Density <math>\rho</math></i>	<i>Specific weight <math>\gamma</math></i>	<i>Specific gravity <math>S</math></i>
	kg/m <sup>3</sup>	N/m <sup>3</sup>	
Air	1.23	12.1	0.00123
Water	1000	9810	1

# 1.5 Fluid Properties

## 1.5.2 Viscosity

- **Viscosity,  $\mu$** : Measure of the resistance of a fluid to gradual deformations by shear stress.
  - Accounts for energy losses in the transport of fluids in ducts or pipes
  - Plays a role in the generation of turbulence

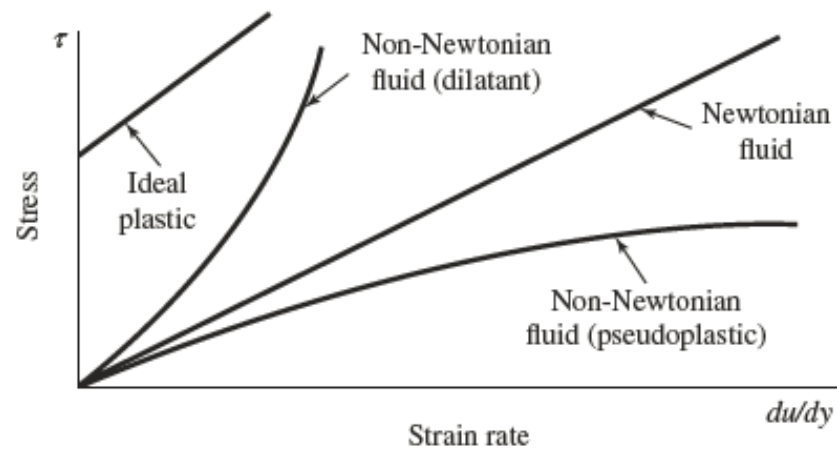
$$\tau = \mu \frac{du}{dy}$$

- Units:  $\text{N}\cdot\text{s}/\text{m}^2$
- The differential,  $\frac{du}{dy}$ , is a velocity gradient (**strain rate**).
  - This is the rate at which a fluid element deforms.

# 1.5 Fluid Properties

## 1.5.2 Viscosity - Newtonian fluid

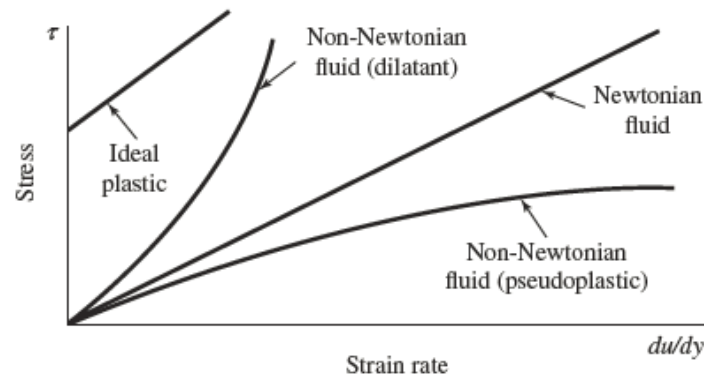
- **Newtonian fluid:** A fluid in which the shear stress is directly proportional to the velocity gradient.
  - E.g., Air, water, and oil



**Figure 1.8** Newtonian and non-Newtonian fluids.

# 1.5 Fluid Properties

## 1.5.2 Viscosity - Non-Newtonian fluid



**Figure 1.8** Newtonian and non-Newtonian fluids.

- **Dilatants** - Non-Newtonian fluids which become more resistant to motion as the strain rate increases.
  - E.g., Quicksand, slurries
- **Pseudoplastic** - A fluid which becomes less resistant to motion with increased strain rate.
  - E.g., Paint
- **Bingham fluids** - Require a minimum shear stress to cause motion.
  - E.g., Clay suspensions, toothpaste

# 1.5 Fluid Properties

## 1.5.2 Viscosity - Non-Newtonian fluid

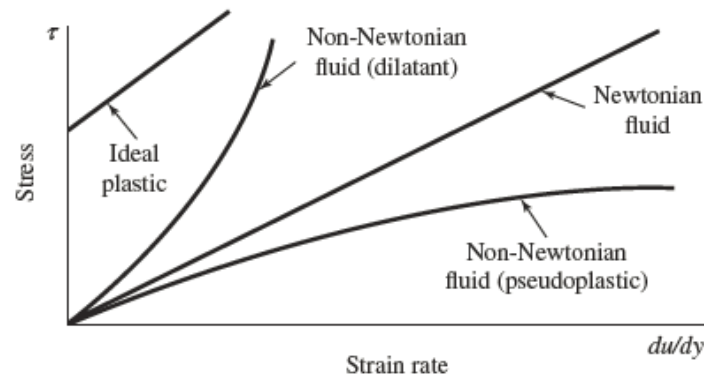


Figure 1.8 Newtonian and non-Newtonian fluids.

- **No-slip Condition:** Causes fluid to adhere to the surface (due to viscosity)
- In equations, viscosity is often divided by density (kinematic viscosity).
- Units:  $\text{m}^2/\text{s}$

$$\nu = \frac{\mu}{\rho}$$

# 1.5 Fluid Properties

## 1.5.3 Compressibility

- Can be described using the **Bulk modulus of elasticity, B**.
  - This is the ratio of the change in pressure to relative change in density.
  - Same units as pressure.
- For gases:
  - Significant changes in density (~4%) - Compressible.
  - Small density changes (under 3%) - Incompressible.

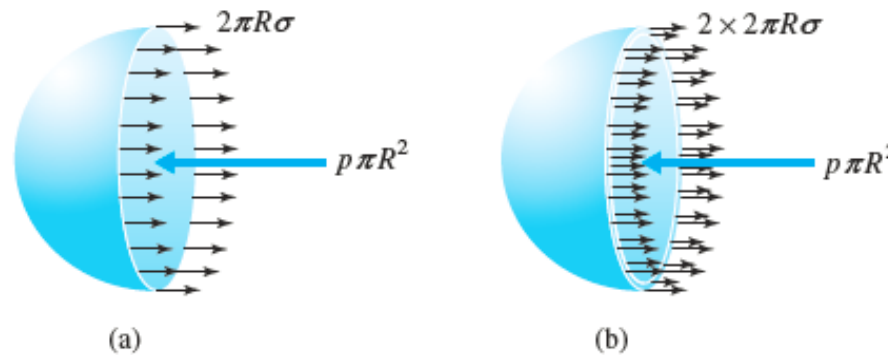
$$c = \sqrt{\left. \frac{\partial p}{\partial \rho} \right|_T} \cong \sqrt{\left. \frac{\Delta p}{\Delta \rho} \right|_T} = \sqrt{\frac{B}{\rho}}$$

- The speed of sound in a liquid can be found using the Bulk modulus of elasticity and density, as shown above.

# 1.5 Fluid Properties

## 1.5.4 Surface Tension

- Results from the attractive forces between molecules.
- Hence seen only in liquids at an interface (liquid-gas).
- Forces between molecules in a liquid bulk are equal in all directions.
  - No net force is exerted on them.
- At an interface, the molecules exert a force that has a resultant force.
  - Holds a drop of water on a rod and limits its size.



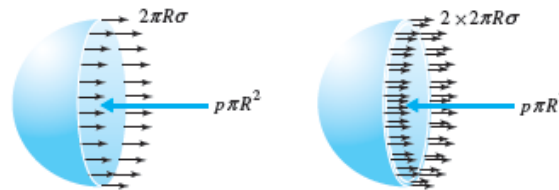
**Figure 1.9** Internal forces in (a) a droplet and (b) a bubble.



# 1.5 Fluid Properties

## 1.5.4 Surface Tension

- Unit: Force per unit length, N/m
- Force results from the length (of fluid in contact with a solid) multiplied by the surface tension.
  - A droplet has one surface.
  - A bubble is a thin film of liquid with an inside and an outside surface.



$$p\pi R^2 = 2\pi R\sigma$$

$$\therefore p = \frac{2\sigma}{R}$$

Pressure in the droplet balances the surface tension around the circumference.

$$2 \times (2\pi R\sigma)$$

$$\therefore p = \frac{4\sigma}{R}$$

Pressure in the bubble is balanced by the surface tension forces on the two circumferences.

# 1.5 Fluid Properties

## 1.5.4 Surface Tension

- As seen before, the internal pressure in a bubble is twice as large as that in a droplet of a similar size.

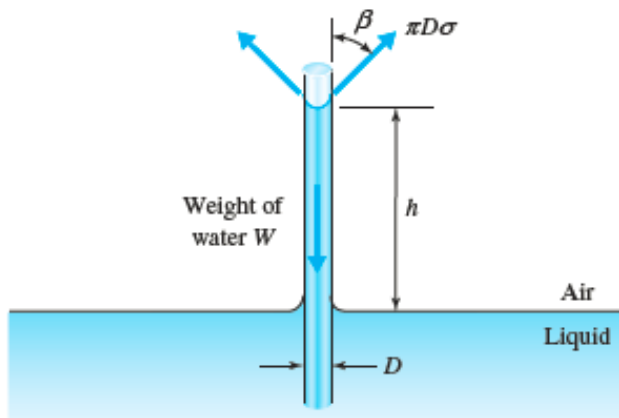


Figure 1.10 Rise in a capillary tube.

- Liquid rises in a glass capillary tube due to surface tension.
- The liquid makes a contact angle  $\beta$  with a glass tube.
  - For most liquids (and water) this is zero.
  - Mercury has an angle greater than  $90^\circ$ .

# 1.5 Fluid Properties

## 1.5.4 Surface Tension

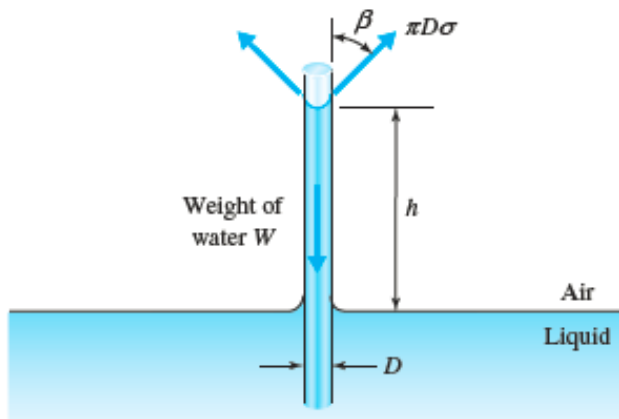


Figure 1.10 Rise in a capillary tube.

$$\sigma \pi D \cos \beta = \gamma \frac{\pi D^2}{4} h$$

$$h = \frac{4\sigma \cos \beta}{\gamma D}$$

- h: Capillary rise
- D: Diameter
- p: Density
- $\sigma$ : Surface tension
  
- 'h' can be determined by equating the vertical component of the surface tension force to the weight of the liquid column.

# 1.5 Fluid Properties

## 1.5.4 Surface Tension

A 2-mm-diameter clean glass tube is inserted in water at 15°C (Figure E1.4). Determine the height that the water will climb the tube. The water makes a contact angle of 0° with the clean glass.

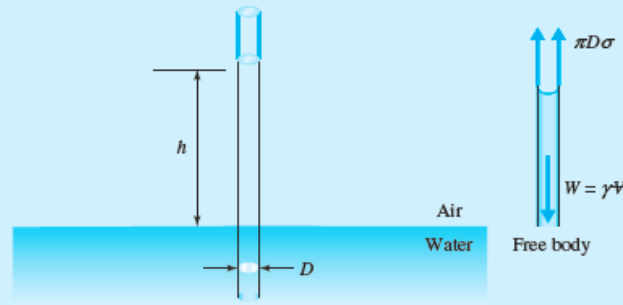


Figure E1.4

### Solution

A free-body diagram of the water shows that the upward surface-tension force is equal and opposite to the weight. Writing the surface-tension force as surface tension times distance, we have

$$\sigma\pi D = \gamma \frac{\pi D^2}{4} h$$

or

$$h = \frac{4\sigma}{\gamma D} = \frac{4 \times 0.0741 \text{ N/m}}{9810 \text{ N/m}^3 \times 0.002 \text{ m}} = 0.01512 \text{ m} \quad \text{or} \quad \underline{15.12 \text{ mm}}$$

The numerical values for  $\sigma$  and  $\rho$  were obtained from Table B.1 in Appendix B. *Note:* If temperature is not given, the nominal value used for the specific weight of water is  $\gamma = \rho g = 9810 \text{ N/m}^3$ .

# 1.5 Fluid Properties

## 1.5.5 Vapor Pressure

- A certain fraction of a liquid will vaporize when a small quantity is placed in a closed container.
  - Will end when equilibrium between the liquid and gaseous states is reached.
- **Vapor Pressure:** Pressure resulting from molecules in a gaseous state.
  - E.g., Water at 15°C has a vapor pressure of 1.70 kPa absolute.
  - Depends on temperature (increases when temperature increases).
- **Boiling** occurs where vapor pressure equals atmospheric pressure.
- **Cavitation** is when bubbles form in a liquid when the local pressure falls below the vapor pressure of the liquid.
  - This is very damaging as these bubbles collapse in high-pressure regions.
  - Leads to pressure spikes (can damage ship's propellers, etc.).

# 1.5 Fluid Properties

## 1.5.5 Vapor Pressure

Calculate the vacuum necessary to cause cavitation in a water flow at a temperature of 80°C in Colorado where the elevation is 2500 m.

### **Solution**

The vapor pressure of water at 80°C is given in Table B.1. It is 47.3 kPa absolute. The atmospheric pressure is found by interpolation using Table B.3 to be  $79.48 - (79.48 - 61.64)500/2000 \cong 75.0$ . The required pressure is then

$$p = 47.3 - 75.0 = -27.7 \text{ kPa} \quad \text{or} \quad \underline{27.7 \text{ kPa vacuum}}$$

## 1.6 Conservation Laws

- **SYSTEM:** Fixed quantity of matter upon which attention is focused.
- **NEWTON'S SECOND LAW:** The sum of all external forces acting on a system is equal to the time rate of change of linear momentum of the system.

## 1.6 Conservation Laws

- **CONSERVATION OF MASS:** Matter is indestructible.
- **CONSERVATION OF MOMENTUM:** [From Newton's Second Law] The momentum of a system remains constant if no external forces act on it.
- **CONSERVATION OF ENERGY:** The total energy of an isolated system remains constant.



# 1.7 Thermodynamic Properties and Relationships

- **Extensive Property** - Property that depends on the system's mass.
  - E.g., Momentum, Energy
- **Intensive Property** - Property that is independent of the system's mass.
  - E.g., Temperature, Pressure

## 1.7.1 Properties of an Ideal Gas

$$p = \rho RT$$

- Behavior of gases for most engineering applications can be described by the ideal gas law.
- For air, with temperatures more than  $-50^{\circ}\text{C}$  and pressures not extremely high, the ideal gas law approximates the behavior of air to a good degree.

# 1.7 Thermodynamic Properties and Relationships

## 1.7.1 Properties of an Ideal Gas

$$p = \rho RT$$

- p: Absolute pressure
- $\rho$ : Density
- T: Absolute temperature
- R: Gas constant

$$R = \frac{R_u}{M}$$

The gas constant is found using the universal gas constant and the molar mass.

$$R_u = 8.314 \text{ kJ/kmol} \cdot \text{K}$$

# 1.7 Thermodynamic Properties and Relationships

## 1.7.1 Properties of an Ideal Gas

$$pV = mRT$$

A tank with a volume of  $0.2 \text{ m}^3$  contains  $0.5 \text{ kg}$  of nitrogen. The temperature is  $20^\circ\text{C}$ . What is the pressure?

### Solution

Assume this is an ideal gas. Apply Eq. 1.7.1 ( $R$  can be found in Table B.4). Solving the equation,  $p = \rho RT$ , we obtain, using  $\rho = m/V$ ,

$$p = \frac{0.5 \text{ kg}}{0.2 \text{ m}^3} \times 0.2968 \frac{\text{kJ}}{\text{kg} \cdot \text{K}} (20 + 273) \text{K} = \underline{218 \text{ kPa abs}}$$

*Note:* The resulting units are  $\text{kJ/m}^3 = \text{kN} \cdot \text{m/m}^3 = \text{kN/m}^2 = \text{kPa}$ . The ideal gas law requires that pressure and temperature be in absolute units.

# 1.7 Thermodynamic Properties and Relationships

## 1.7.2 First Law of Thermodynamics

- States that when a system changes from State 1 to State 2, its energy changes from  $E_1$  to  $E_2$ .
  - This energy change is **heat transfer** or **work**.
  - Heat Transfer **to** the system and work done **by** the system are **positive**.

$$Q_{1-2} - W_{1-2} = E_2 - E_1$$

- $Q_{1-2}$ : Amount of heat transfer to the system.
- $W_{1-2}$ : Amount of work done by the system.

# 1.7 Thermodynamic Properties and Relationships

## 1.7.2 First Law of Thermodynamics

$$E = m \left( \frac{V^2}{2} + gz + \tilde{u} \right)$$

- Energy (E) for the total energy consists of kinetic energy ( $\frac{1}{2}mV^2$ ), potential energy ( $mgz$ ), and internal energy ( $m\tilde{u}$ ).
  - $\tilde{u}$ : Internal energy per unit mass
- Work results from a force moving through a distance.
- If the force is because of pressure:

$$\begin{aligned} W_{1-2} &= \int_{l_1}^{l_2} F dl \\ &= \int_{l_1}^{l_2} pA dl = \int_{V_1}^{V_2} p dV \end{aligned}$$

# 1.7 Thermodynamic Properties and Relationships

A cart with a mass of 29.2 kg is pushed up a ramp with an initial force of 445 N (Figure E1.7). The force decreases according to

$$F = 72.95(6.1 - l) \text{ N}$$

If the cart starts from rest at  $l = 0$ , determine its velocity after it has traveled 6.1 m up the ramp. Neglect friction.

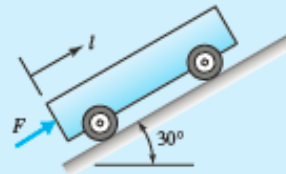


Figure E1.7

## Solution

The energy equation (Eq. 1.7.6) allows us to relate the quantities of interest. Since there is no heat transfer, we have

$$-W_{1-2} = E_2 - E_1$$

Recognizing that the force is doing work on the system, the work is negative. Hence the energy equation becomes, using  $W = \int F dl$ ,

$$-\left[ -\int_0^{6.1} 72.95(6.1 - l) dl \right] = m \left( \frac{V_2^2}{2} + gz_2 \right) - m \left( \frac{V_1^2}{2} + gz_1 \right)$$

Taking the datum as  $z_1 = 0$ , we have  $z_2 = 6.1 \sin 30^\circ = 3.05$  m. Thus

$$72.95 \times 6.1^2 - 72.95 \times \frac{6.1^2}{2} = 29.2 \left( \frac{V_2^2}{2} + 9.81 \times 3.05 \right)$$
$$\therefore V_2 = \underline{5.76 \text{ m}}$$

*Note:* We have assumed no internal energy change and no heat transfer.

# 1.7 Thermodynamic Properties and Relationships

## 1.7.3 Other Thermodynamic Quantities

- Enthalpy (H), created to help with thermodynamic calculations.

$$H = m\tilde{u} + pV$$

- Constant-pressure specific heat  $C_p$  and constant-volume specific heat  $C_v$  are used to calculate enthalpy and internal energy changes.

$$\Delta h = \int c_p dT \quad dh = c_p dT$$

$$\Delta \tilde{u} = \int c_v dT \quad d\tilde{u} = c_v dT$$

# 1.7 Thermodynamic Properties and Relationships

## 1.7.3 Other Thermodynamic Quantities

- **Ratio of specific heats (k):** The ratio of specific heats.

$$k = \frac{c_p}{c_v}$$

- A process in which pressure, temperature, and other properties are constant at any instant throughout the system is called a quasi-equilibrium process.
  - E.g., Compression/expansion in the cylinder of an internal combustion engine.
- **If no heat is transferred: Process is an isentropic process.**



# 1.7 Thermodynamic Properties and Relationships

## 1.7.3 Other Thermodynamic Quantities

- For an isentropic process:

$$\frac{p_1}{p_2} = \left(\frac{\rho_1}{\rho_2}\right)^k \quad \frac{T_1}{T_2} = \left(\frac{p_1}{p_2}\right)^{(k-1)/k} \quad \frac{T_1}{T_2} = \left(\frac{\rho_1}{\rho_2}\right)^{k-1}$$

- For a small pressure wave in a gas (at low frequency), the wave speed in an isentropic process is:

$$c = \sqrt{\left.\frac{dp}{d\rho}\right|_s} = \sqrt{kRT}$$

# 1.7 Thermodynamic Properties and Relationships

## 1.7.3 Other Thermodynamic Quantities

- For a small pressure wave in a gas (at a relatively high frequency):
  - Entropy is not constant.

$$c = \sqrt{\left. \frac{dp}{d\rho} \right|_T} = \sqrt{RT}$$

# 1.7 Thermodynamic Properties and Relationships

A cylinder fitted with a piston has an initial volume of  $0.5 \text{ m}^3$ . It contains  $2.0 \text{ kg}$  of air at  $400 \text{ kPa}$  absolute. Heat is transferred to the air while the pressure remains constant until the temperature is  $300^\circ\text{C}$ . Calculate the heat transfer and the work done. Assume constant specific heats.

### Solution

Using the first law (Eq. 1.7.6), and the definition of enthalpy, we see that, with no kinetic or internal energy changes, there results

$$\begin{aligned} Q_{1-2} &= p_2 V_2 - p_1 V_1 + m\tilde{u}_2 - m\tilde{u}_1 \\ &= m\tilde{u}_2 + p_2 V_2 - (m\tilde{u}_1 + p_1 V_1) \\ &= H_2 - H_1 = m(h_2 - h_1) = mc_p(T_2 - T_1) \end{aligned}$$

where Eq. 1.7.12 is used assuming  $c_p$  to be constant. The initial temperature is

$$T_1 = \frac{p_1 V_1}{mR} = \frac{400 \text{ kN/m}^2 \times 0.5 \text{ m}^3}{2.0 \text{ kg} \times 0.287 \text{ kJ/kg}\cdot\text{K}} = 348.4 \text{ K}$$

(Use  $\text{kJ} = \text{kN}\cdot\text{m}$  to check the units.) Thus the heat transfer is ( $c_p$  is found in Table B.4)

$$Q_{1-2} = 2.0 \times 1.0[(300 + 273) - 348.4] = \underline{449 \text{ kJ}}$$

The final volume is found using the ideal gas law:

$$V_2 = \frac{mRT_2}{p_2} = \frac{2 \text{ kg} \times (0.287 \text{ kJ/kg}\cdot\text{K}) \times 573 \text{ K}}{400 \text{ kN/m}^2} = 0.822 \text{ m}^3$$

The work done for the constant-pressure process is, using Eq. 1.7.9 with  $p = \text{const}$ ,

$$\begin{aligned} W_{1-2} &= p(V_2 - V_1) \\ &= 400 \text{ kN/m}^2(0.822 - 0.5) \text{ m}^3 = 129 \text{ kN}\cdot\text{m} \text{ or } \underline{129 \text{ kJ}} \end{aligned}$$

# 1.7 Thermodynamic Properties and Relationships

The temperature on a cold winter day in the mountains of Wyoming is  $-30^{\circ}\text{C}$  at an elevation of 4 km. Calculate the density of the air assuming the same pressure as in the local atmosphere; also find the speed of sound.

### Solution

From Table B.3 we find the atmospheric pressure at an elevation of 4 km to be 61.64 kPa. The absolute temperature is found to be

$$T = -30 + 273 = 243 \text{ K}$$

Using the ideal gas law, the mass density is calculated as

$$\begin{aligned}\rho &= \frac{p}{RT} \\ &= \frac{61.64}{0.287 \times 243} = \underline{0.884 \text{ kg/m}^3}\end{aligned}$$

The speed of sound, using Eq. 1.7.17, is determined to be

$$\begin{aligned}c &= \sqrt{kRT} \\ &= \sqrt{1.4 \times 287 \times 243} = \underline{312 \text{ m/s}}\end{aligned}$$

*Note:* The gas constant in the foregoing equations has units of  $\text{J/kg} \cdot \text{K}$  so that the appropriate units result.

## 1.8 Summary

- To relate units, Newton's second law is used:
  - $N = \text{kg} \cdot \text{m}/\text{s}^2$
- When making calculations, the answer should have the same number of significant digits as the **least** accurate number used in the calculations.
- Pressure is expressed as gage pressure unless stated otherwise.
- The density, or specific weight, of a fluid can be found if the specific gravity is given:

$$\rho_x = S_x \rho_{\text{water}} \quad \gamma_x = S_x \gamma_{\text{water}}$$

- The shear stress due to viscous effects in a simple flow is:

$$\tau = \mu \frac{du}{dy}$$