Question 1

The “no-slip” condition means that a fluid “sticks” to a solid surface. This is true for both fixed and moving surfaces. Let two layers of fluid be dragged along by the motion of an upper plate as shown in Figure 1. The bottom plate is stationary. The top fluid puts a shear stress on the upper plate, and the lower fluid puts a shear stress on the bottom plate. Determine the ratio of these two shear stresses.

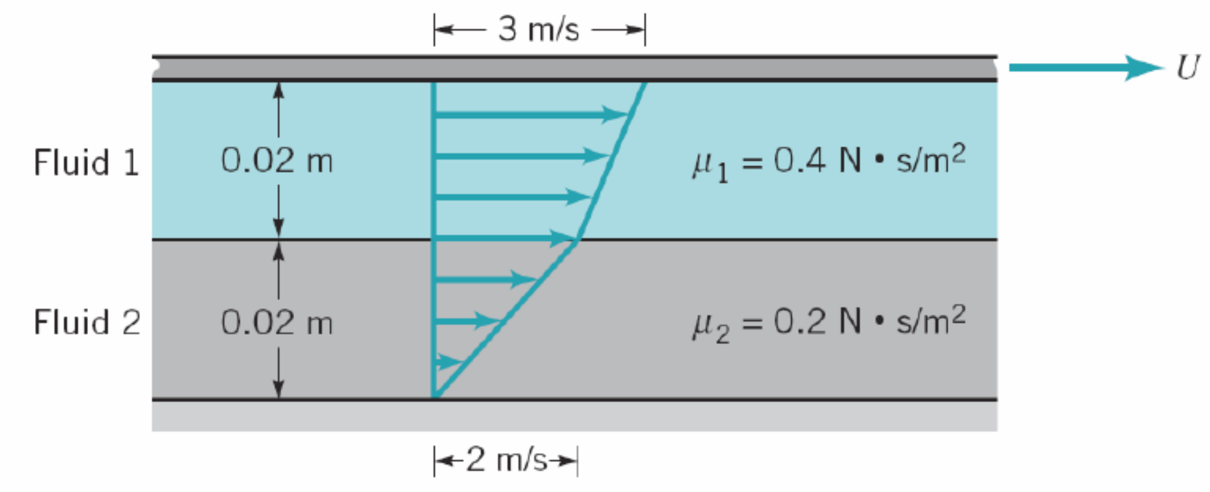


Figure 1

$$Shear\ stress, \tau = \mu \frac{du}{dy}$$

$$\tau_1 = \mu_1 \frac{du}{dy} = 0.4 \left(\frac{3-2}{0.04-0.02} \right) = 20\ (\mathrm{N/m^2})$$

$$\tau_2 = \mu_2 \frac{du}{dy} = 0.2 \left(\frac{2-0}{0.02-0} \right) = 20\ (\mathrm{N/m^2})$$

$$ratio\ \ \frac{\tau_1}{\tau_2} = \frac{20\ (\mathrm{N/m^2})}{20\ (\mathrm{N/m^2})} = 1$$

Question 2

A 25-mm-diameter shaft is pulled through a cylindrical bearing as shown in Figure 2. The lubricant that fills the 0.3-mm gap between the shaft and bearing is an oil having a kinematic viscosity of 8.0 × 10−4 m2/s and a specific gravity of 0.91. Determine the force P required to pull the shaft at a velocity of 3 m/s. Assume the velocity distribution in the gap is linear.

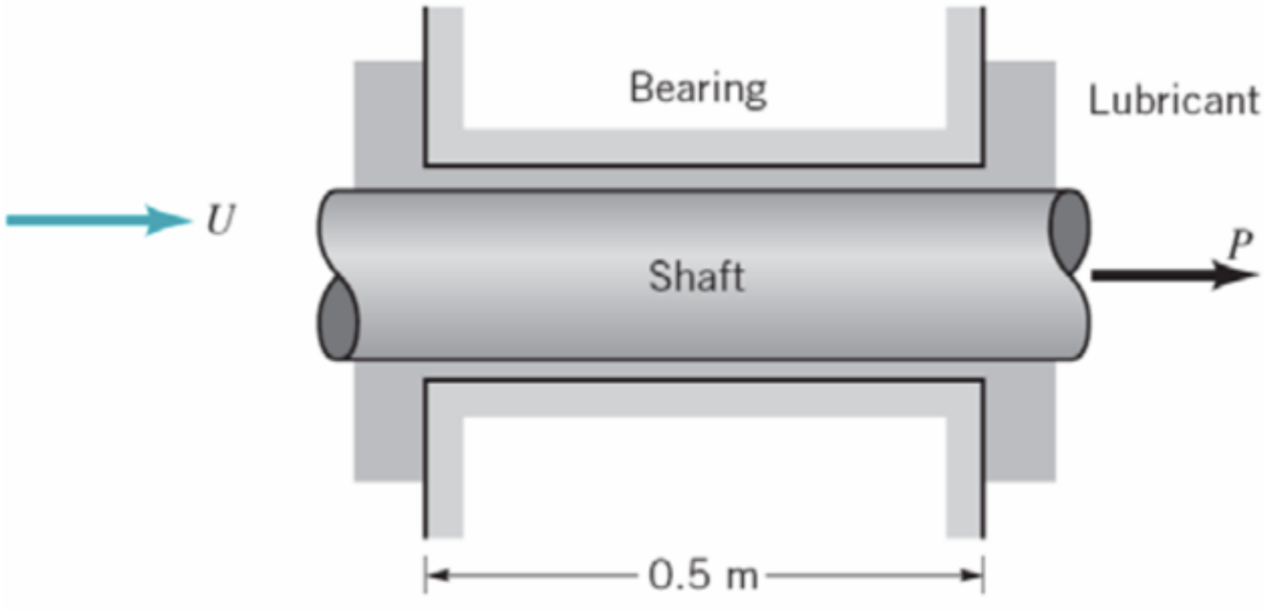


Figure 2

$$Shear\ stress, \tau = \mu \frac{du}{dy} \qquad \text{……………………......}(1)$$

$$Force, F = \tau A \qquad \text{……………………......}(2)$$

Using equation (1) and (2)

From figure

SG = 0.9

$\nu = 8\times 10^{-4} \quad m^2/s$

$$F = \mu \frac{du}{dy} \cdot A$$

$$F = \rho\nu \frac{du}{dy} \cdot A$$

$$= \rho\nu \frac{du}{dy} \cdot 2\pi r L$$

$$= (910)(8 \times 10^{-4}) \left(\frac{3}{0.0003}\right) \cdot 2\pi \left(\frac{0.025}{2}\right) (0.5)$$

$$= 286 \quad (\mathrm{N})$$

Question 3

A layer of water flows down an inclined fixed surface with the velocity profile shown in Figure 3. Determine the magnitude and direction of the shearing stress that the water exerts on the fixed surface for U = 2 m/s and h = 0.1 m.

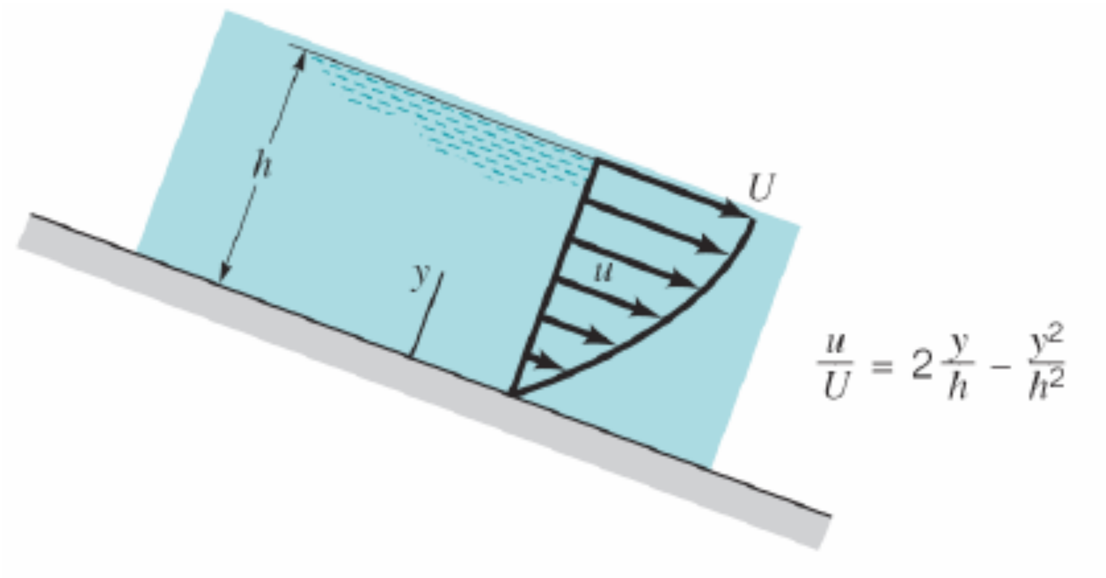


Figure 3

$$\tau = \mu \frac{d\mu}{dy}$$

$$\frac{u}{U} = 2\frac{y}{h} - \frac{y^2}{h^2}$$

$$u = 2U\frac{y}{h} - \frac{y^2}{h^2}U$$

$$\frac{du}{dy} = \frac{2U}{h} - \frac{2yU}{h^2}$$

Thus, at fixed surface, y=0

$$\left(\frac{du}{dy}\right)_{y=0} = \frac{2U}{h} - \frac{2(0)}{h^2} = \frac{2U}{h}$$

So that,

$$\tau = \mu \frac{du}{dy}$$

$$= (1.12 \times 10^{-3}) \left(\frac{2U}{h}\right)$$

$$= (1.12 \times 10^{-3}) \left(\frac{(2)(2)}{0.1}\right)$$

$$= 0.0448 \text{ N/m}^2$$

Question 4

The viscosity of liquids can be measured through the use of a rotating cylinder viscometer of the type illustrated in Figure 4. In this device the outer cylinder is fixed and the inner cylinder is rotated with an angular velocity, ω. The torque T required to develop ω is measured and the viscosity is calculated from these two measurements. Develop an equation to determine the torque? Neglect end effects and assume the velocity distribution in the gap is linear.

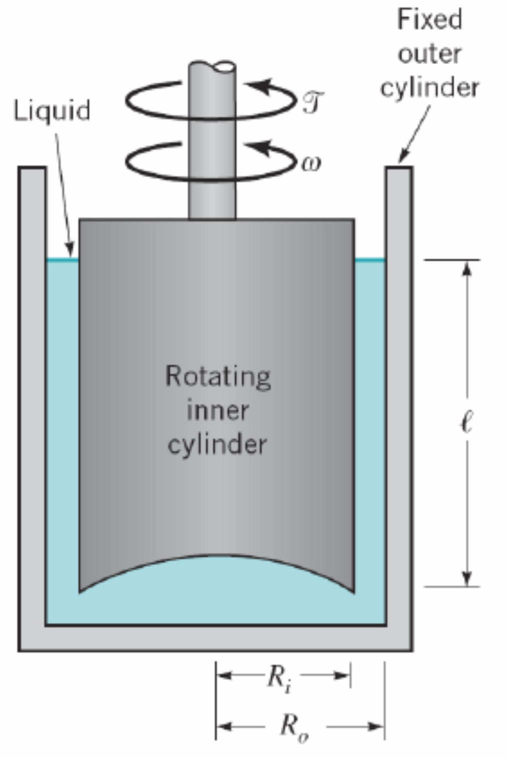


Figure 4

$$Force, F = \tau A$$

$$Shear\ stress, \tau = \mu \frac{du}{dy}$$

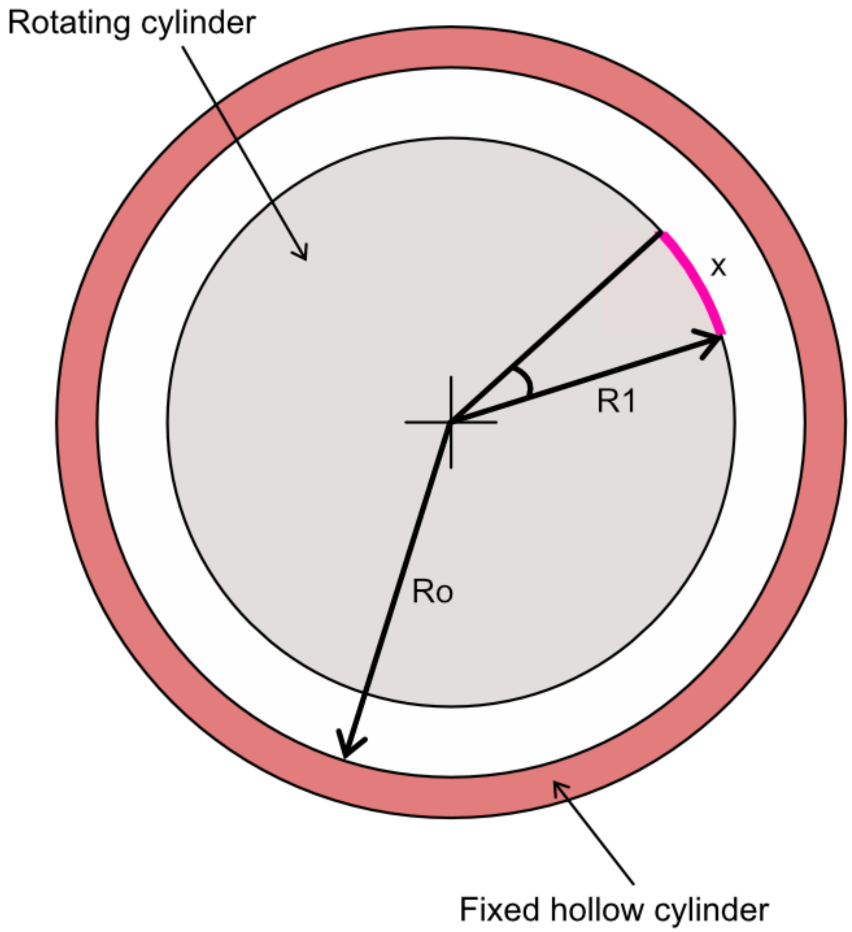
$$Torque, q = Fr$$

For small area in rotating cylinder

$$dq = dF \cdot r = \tau \cdot dA \cdot r = \tau \cdot r \cdot dA$$

$$dA = xL = R_1 L\, d\theta$$

$$dq = \tau \cdot r \cdot R_1 L\, d\theta$$



$$r = R_1$$

$$dq = \tau(R_1)^2 L\, d\theta$$

$$\int dq = \tau(R_1)^2 L \int_0^{2\pi} 1 \cdot d\theta$$

$$q = \tau(R_1)^2 L 2\boldsymbol{\pi}$$

To find value of shear stress, τ

$$\tau = \mu \frac{du}{dy}$$

We know that, $u = R_1\omega \quad y = R_0 - R_1$

So that,

$$\tau = \mu \frac{R_1\omega}{R_0 - R_1}$$

We found that torque,

$$q = \mu \frac{R_1\omega}{R_0 - R_1}(R_1)^2 L 2\pi$$

$$q = \frac{\mu(R_1)^3 \omega L 2\pi}{R_0 - R_1}$$

Question 4

(Alternative solution)

$$Torque, q = Fr$$

$$F = \tau A$$

$$\tau = \mu \frac{d\mu}{dy}$$

$$q = \tau A r$$

$$= \mu \frac{d\mu}{dy} A r$$

$$= \mu \frac{R_1 \omega}{R_0 - R_1} 2\boldsymbol{\pi} R_1 L R_1$$

$$q = \frac{\mu (R_1)^3 \omega L 2\pi}{R_0 - R_1}$$

Question 5

Determine the equation of Torque for rotating cone, as shown in Figure 5. Ignore the value in British unit.

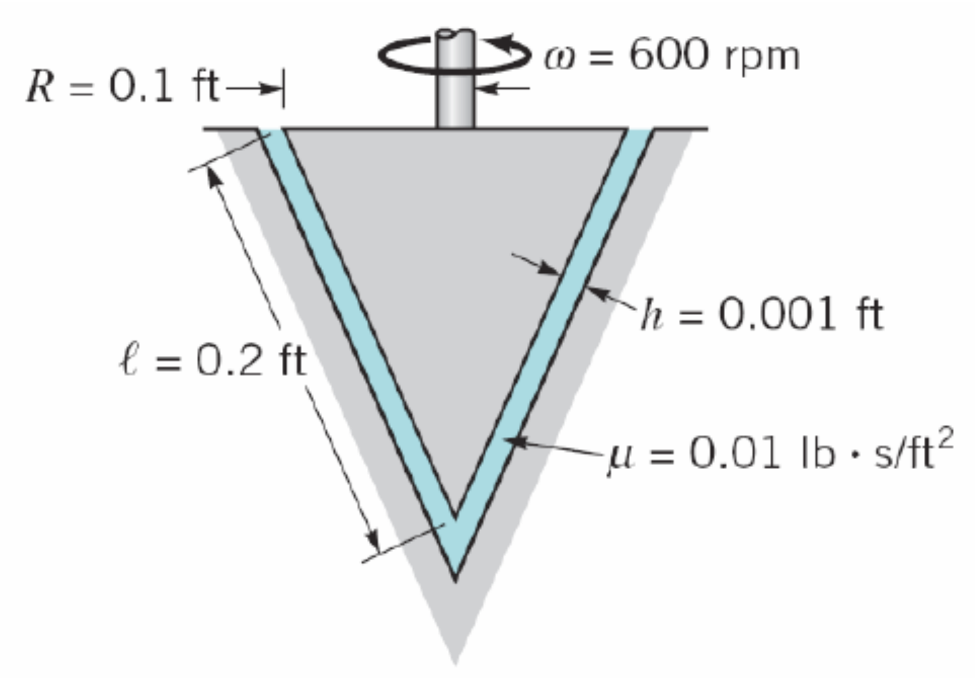
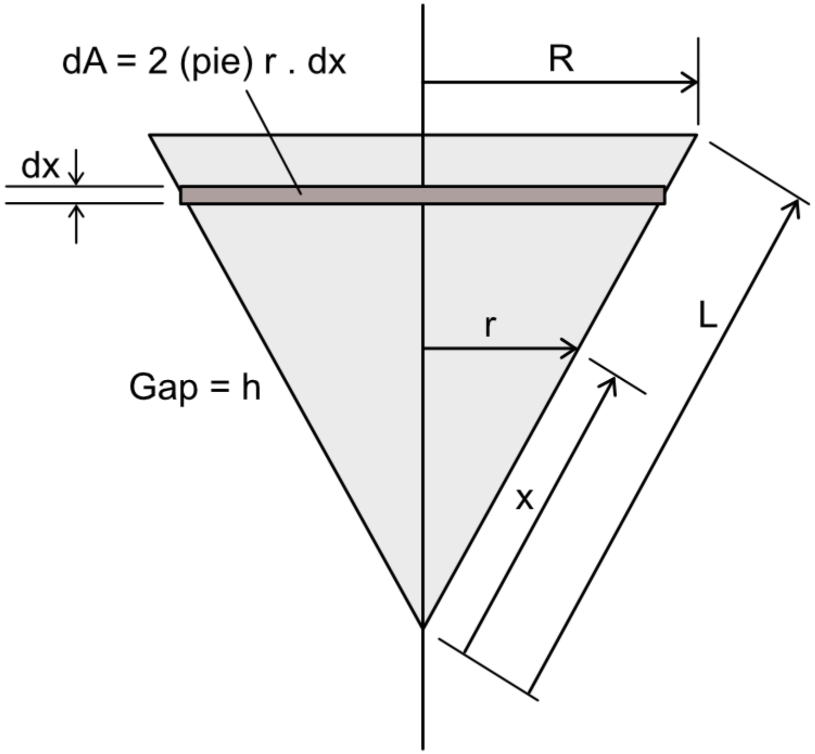


Figure 5



Basic equation

$$Torque, q = Fr$$

$$Force, F = \tau A$$

$$Shear\ stress, \tau = \mu \frac{du}{dy}$$

$$\frac{R}{L} = \frac{r}{x}$$

$$x = \frac{L}{R} r$$

$$dx = \frac{L}{R} dr$$

$$dq = dF \cdot r = \tau \cdot dA \cdot r = \tau \cdot r \cdot dA = \tau \cdot r \cdot 2\pi r \cdot dx$$

$$dq = \mu \frac{du}{dy} \cdot r \cdot 2\pi r \cdot dx$$

$$= \mu \frac{r\omega}{h} \cdot r \cdot 2\pi r \cdot dx$$

$$= \frac{\mu\omega}{h} 2\pi r^3 dx$$

$$(Replace\ dx = \frac{L}{R} dr)$$

$$dq = \frac{\mu\omega}{h} 2\pi r^3 \frac{L}{R} dr$$

$$(Note: r \neq R)$$

$$dq = \frac{\mu\omega}{h} 2\pi \frac{L}{R} r^3 dr$$

Integrating both side

$$\int dq = \frac{\mu\omega}{h} 2\pi \frac{L}{R} \int_0^R r^3 dr$$

$$q = \frac{\mu\omega}{h} 2\pi \frac{L}{R} \frac{R^4}{4} + C$$

(At R=0, q=0, C=0)

$$q = \frac{\mu\omega}{h} 2\pi \frac{L}{R} \frac{R^4}{4}$$

$$= \frac{\mu\omega}{h} 2\pi L \frac{R^3}{4}$$

$$q = \frac{\mu\omega\pi L R^3}{2h}$$

Question 6

Determine the equation of Torque for rotating disk, as shown in Figure 6. Ignore the value in British unit.

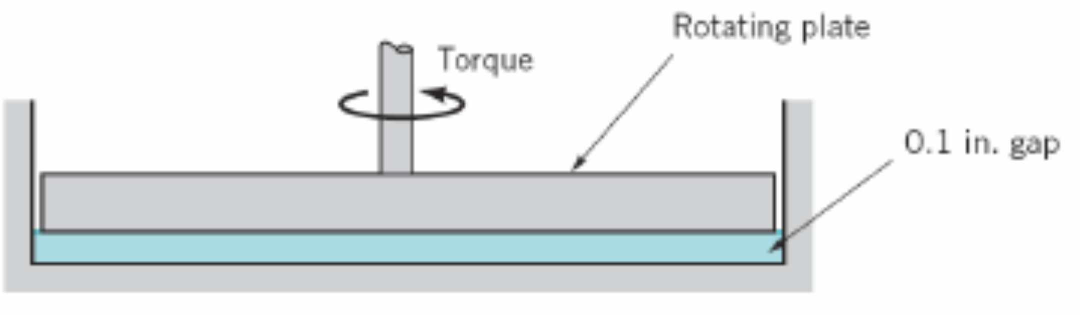
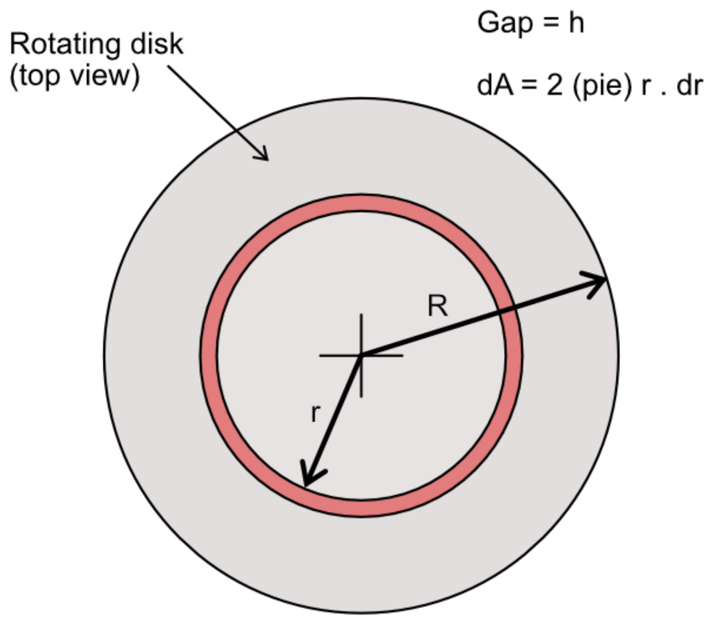


Figure 6



$$q = Fr$$

$$F = \tau A \qquad A = \pi r^2$$

$$\tau = \mu \frac{du}{dy} \qquad \frac{dA}{dr} = 2\pi r$$

$$dq = \mu \frac{du}{dy} dAr \qquad dA = 2\pi r dr$$

$$= \mu \frac{r\omega}{h} r 2\pi r dr$$

$$q = \frac{\mu\omega}{h} 2\pi \frac{r^4}{4} dr$$