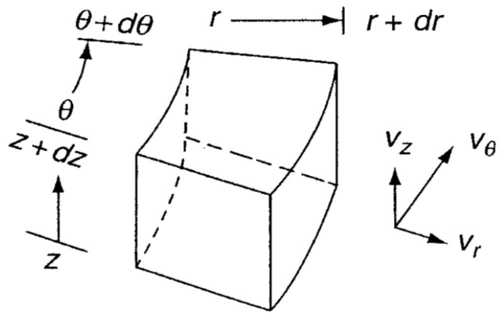
PROBLEMS FOR CHAPTER 2

2-1

By consideration of the cylindrical elemental control volume as shown below, use the conservation of mass to derive the continuity equation in cylindrical coordinates.



2-2

Simplify the equation of continuity in cylindrical coordinates (r, θ, z) to the case of steady compressible flow in polar coordinates $\left(\frac{\partial}{\partial z} = 0\right)$ and derive a stream function for this case.

2-3

Simplify the equation of continuity in cylindrical coordinates to the case of steady compressible flow in axisymmetric coordinates $\left(\frac{\partial}{\partial \theta} = 0\right)$ and derive a stream function for this case.

2-4

For steady incompressible flow with negligible viscosity, show that the Navier-Stokes relation (Eq.2-30) reduces to the condition that $\frac{P}{\rho} + \frac{V^2}{2} + gh$ is constant along a streamline of the flow, where h denotes the height of the fluid particle above a horizontal datum. The is the weaker form of the so-called Bernoulli relation.

2-11

The differential equation for irrotational plane compressible gas flow is:

$$\frac{\partial^2 \phi}{\partial t^2} + \frac{\partial}{\partial t}(u^2 + v^2) + (u^2 - a^2)\frac{\partial^2 \phi}{\partial x^2} + (v^2 - a^2)\frac{\partial^2 \phi}{\partial y^2} + 2uv\frac{\partial^2 \phi}{\partial x \partial y} = 0$$

where ϕ is the velocity potential and a the (variable) speed of sound in the gas. In the spirit of Sec.2-9-2, nondimensionalize this equation and define any parameters which appear.

2-13

The equation of motion for free convection near a hot vertical plate for incompressible flow with constant properties are;

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$
$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = g\beta(T - T_1) + v\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right)$$
$$\rho c_p\left(u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y}\right) = k\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2}\right)$$

Introduce the dimensionless variables

$u^* = \frac{uL}{v}$ $v^* = \frac{vL}{v}$ $x^* = \frac{x}{L}$ $y^* = \frac{y}{L}$ $T^* = \frac{T-T_1}{T_0-T_1}$

where L is the length of the plate, v is kinematic viscosity. Use these variable to nondimensionalize the free convection equations and define any parameters which arise.

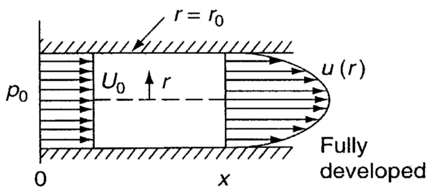
2-14

Laminar flow in the entrance to a pipe, as shown below, the entrance flow is uniform, $u = U_0$ and the flow downstream is parabolic in profile,

$$u(r) = C(r_0^2 - r^2)$$

Using the integral relation, sec.2-13, show that the viscous drag exerted on the pipe walls between 0 and x is given by:

$$Drag = \pi r_0^2\left(p_0 - p_x - \frac{1}{3}\rho U_0^2\right)$$



2-18

Flow through a well-designed contraction or nozzle is nearly frictionless. Suppose that water at 20°C flows through a horizontal nozzle at a weight flow of 50N/s. If entrance and exit diameters are 8cm and 3cm, respectively, and the exit pressure is 1 atm, estimate the entrance pressure from Bernoulli's equation.