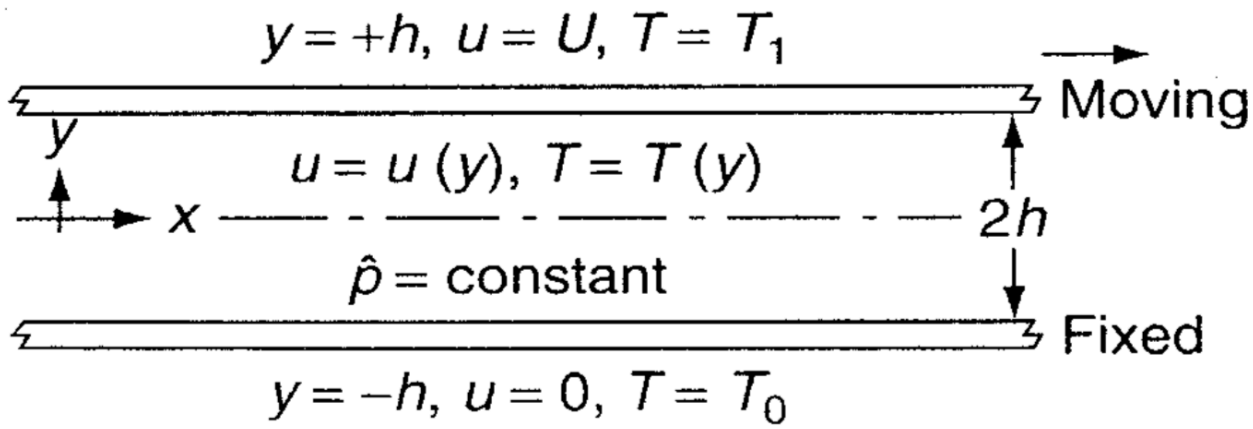
PROBLEMS FOR CHAPTER 3

3-1

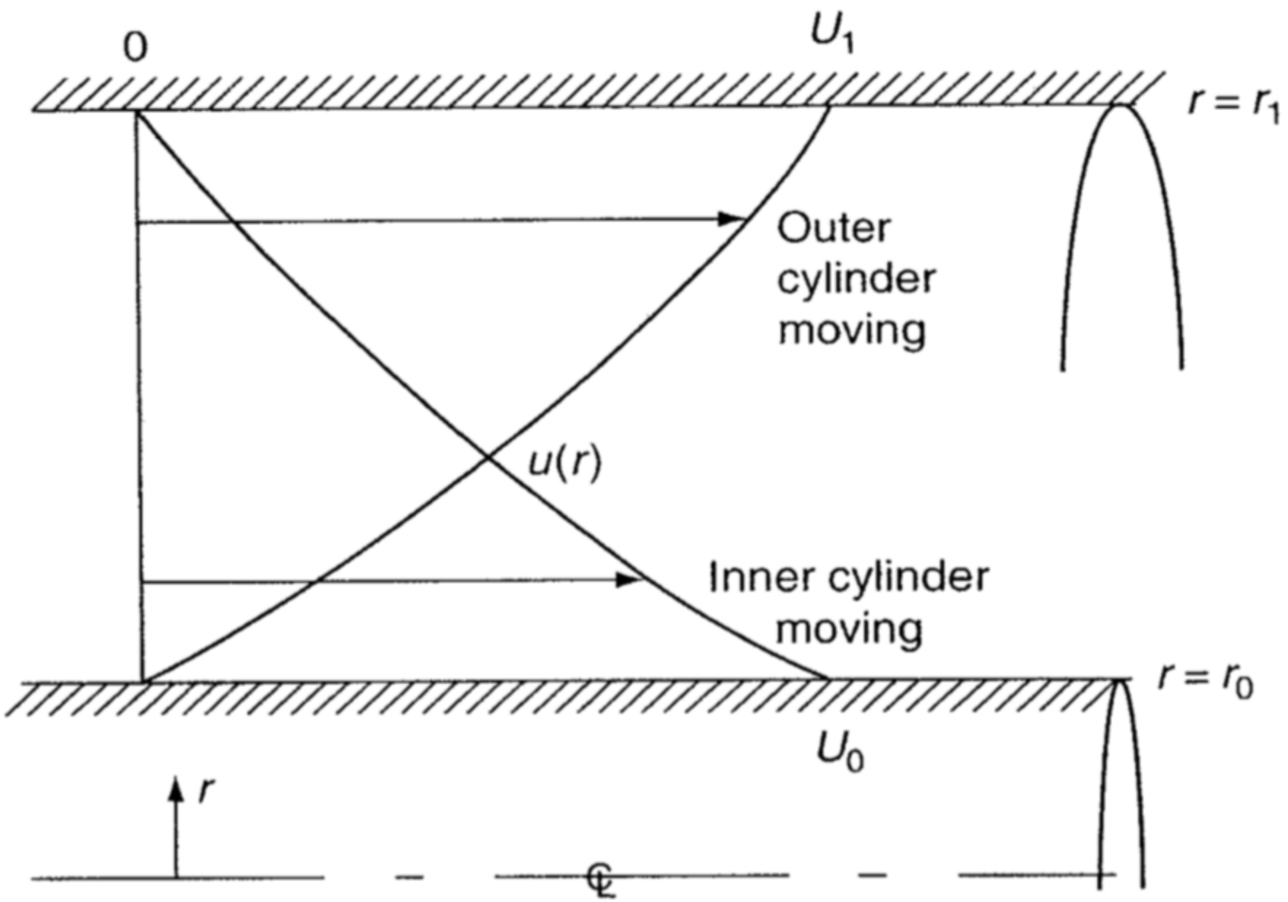
Reconsider the problem of Couette flow between parallel plates (Fig.3-1), for a power-law non-Newtonian fluid, $\tau_{xy} = K\left(\frac{du}{dy}\right)^n$, where $n \neq 1$. Assuming constant pressure and temperature, solve for the velocity distribution $u(y)$ between the plates of (a) $n < 1$ and (b) $n > 1$, and compare with the Newtonian solution (Eq.3-6).

$$u = \frac{U}{2}\left(1 + \frac{y}{h}\right) \qquad \text{(Eq.3-6)}$$



3-2

Consider the axial Couette flow (Fig. 3-3) with both cylinders moving. Find the velocity distribution $u(r)$ and plot it for (a) $U_1 = U_0$, (b) $U_1 = -U_0$, (c) $U_1 = 2U_0$.



3-5

A circular cylinder or radius R is rotating at steady angular rate ω in an infinite fluid of constant ρ and μ. Assuming purely circular stream lines, find the velocity and pressure distribution in the fluid and compare with the flow field of an inviscid "potential" vortex.

3-6

Assuming that the velocity distribution between rotating concentric cylinders is known from Eq.3-22, find the pressure distribution $p(r)$ if the pressure is p_0 at the inner cylinder.

3-8

Air at 20°C and 1 atm is driven between two parallel plates 1 cm apart by an imposed pressure gradient $(\frac{dp}{dx})$ and by the upper plate moving at 20 cm/s.

Find (a) The volume flow rate per unit width if $\frac{dp}{dx} = -0.3\ Pa/m$

(b) The value of $(\frac{dp}{dx})$ which causes the shear stress at the lower plate to be zero.

3-9

Derive the solution $u(y,z)$ for flow through an elliptical duct (Fig.3-9), by solving Eq.3-30. Begin with a guessed quadratic solution, $u = A + By^2 + Cz^2$, and work your way through to the exact solution.

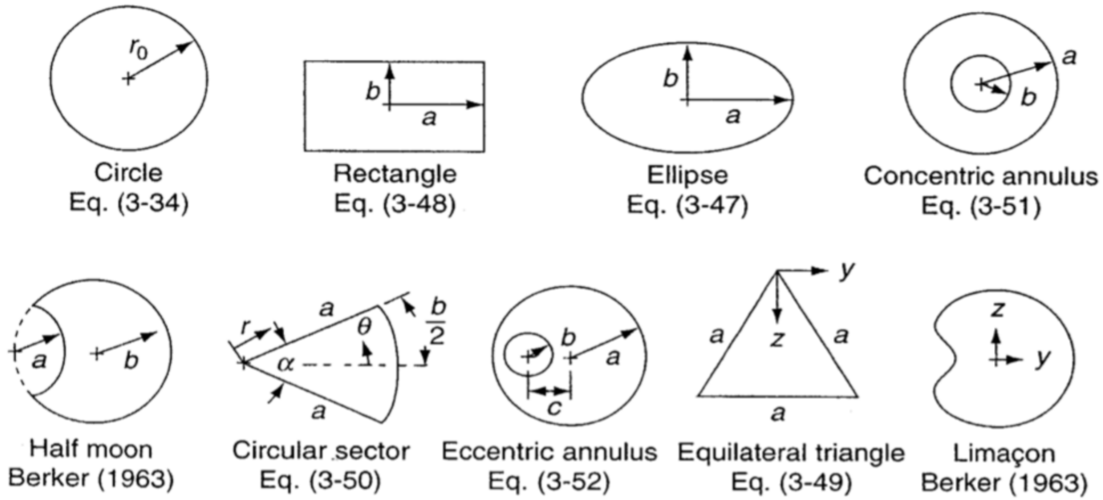


FIGURE 3-9

Some cross sections for which fully developed flow solutions are known; for still more, consult Berker (1963, pp. 67ff.) or Shah and London (1978).

Elliptical section: $y^2/a^2 + z^2/b^2 \leq 1$:

$$
\begin{aligned}
u(y, z) &= \frac{1}{2\mu}\left(-\frac{d\hat{p}}{dx}\right)\frac{a^2b^2}{a^2 + b^2}\left(1 - \frac{y^2}{a^2} - \frac{z^2}{b^2}\right) \\
Q &= \frac{\pi}{4\mu}\left(-\frac{d\hat{p}}{dx}\right)\frac{a^3b^3}{a^2 + b^2}
\end{aligned} \qquad (3\text{-}47)
$$

3-12

It is desired to measure the viscosity of light lubricating oils $(\mu \approx 0.02\ to\ 0.1\ Pa.s)$ by passing approximately 1 m^3/h of fluid through an annulus of length 30 cm with inner and outer radii of 9 mm and 10 mm, respectively. Estimate the expected pressure drop through the device and an appropriate instrument for the pressure measurement.

3-13

Lubricating oil at 20°C is to be cooled by flowing at an average velocity of 2m/s through a 3cm diameter pipe whose walls are at 10°C.

Estimate: (a) The heat loss (in W/m^2) at x=10 cm

(b) The mean oil temperature at the pipe exit, L=2m.

3-15

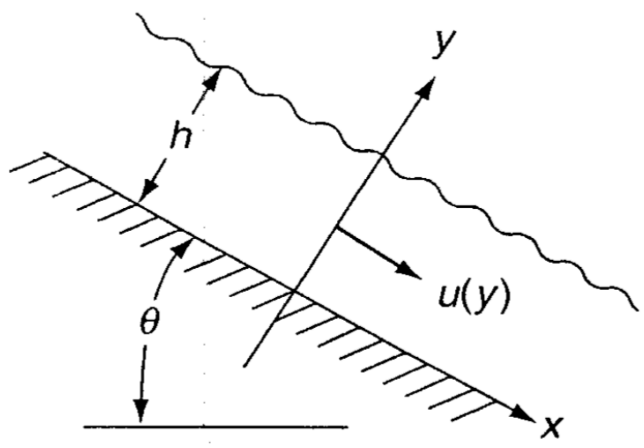
Consider a wide liquid film of constant thickness, h, flowing steadily due to gravity down an inclined plane at angle θ, as shown in Fig.P3-15. The atmosphere exerts constant pressure and negligible shear on the free surface. Show that the velocity distribution is given by:

$$u = \frac{\rho g \sin\theta}{2\mu} y(2h - y)$$

and that the volume flow rate per unit width is:

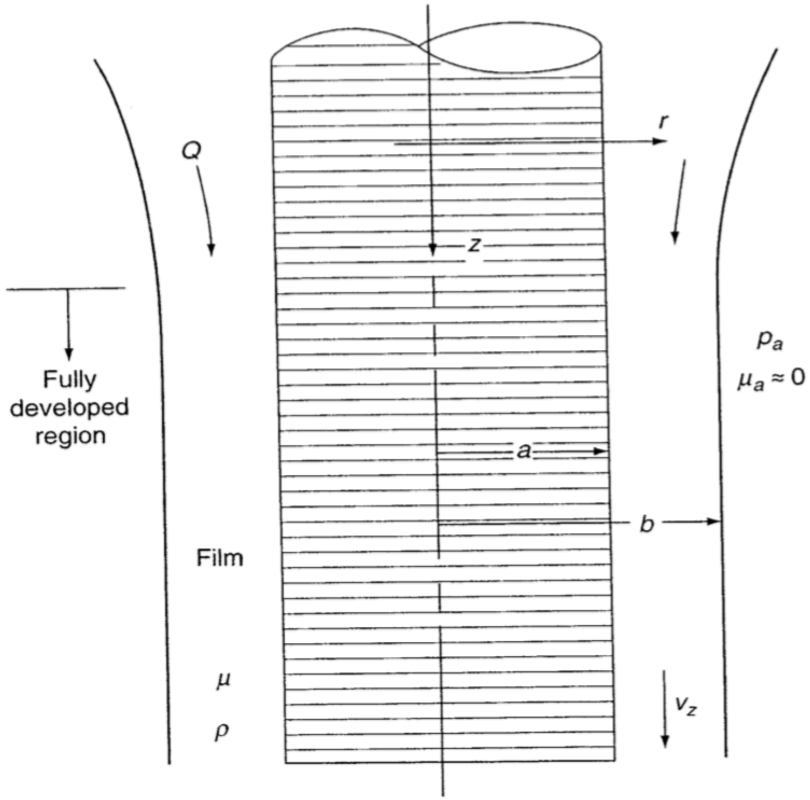
$$Q = \frac{\rho g h^3 \sin\theta}{3\mu}$$

Compare this result with flow between parallel plates, Eq.3-44 and Eq.3-45.



3-16

Consider a film of liquid draining a volume flow rate Q down outside of a vertical rod of radius *a*, as shown in Figure P3-16. Some distance down the rod, a fully developed region is reached where fluid shear balances gravity and the thin film thickness remains constant. Assuming incompressible laminar flow and negligible shear interaction with the atmosphere, find an expression for $v_z(r)$ and a relation between Q and film radius, *b*.



3-20

Air at 20°C and 1 atm is at rest between two fixed parallel plates 2 cm apart. At time, t=0, the lower plate suddenly begins to move tangentially at 30cm/s. Compute the air velocity in the center between plates after 2 seconds. When will the center velocity reach 14 cm/s.

3-27

The practical difficulty with the Ekman spiral solution Eq.3-144, is that it assumes laminar flow whereas the real ocean is turbulent. One approximate remedy is to replace kinematic viscosity everywhere by a constant turbulent or "eddy viscosity" correlated with wind shear and penetration depth using a suggestion by Clauser (1956):

$$V_{turbulent} = 0.04D\left(\frac{\tau_0}{\rho}\right)^{\frac{1}{2}}$$

Repeat our text example, $V_{wind} = 6$m/s over a 20°C air-water interface of 41°N latitude. Compute penetration depth D, and surface velocity V_0.

Ekman solution for surface velocity and penetration depth is:

$$V_0 = \frac{\tau_0/\rho}{\sqrt{2\omega\upsilon\sin\phi}} \qquad and \qquad D\pi\sqrt{\frac{\upsilon}{\omega\sin\phi}}$$

3-34

A sphere of specific gravity 7.8 is dropped into oil of specific gravity 0.88 and viscosity $\mu = 0.15 \ Pa.s$. Estimate the terminal of the sphere if its diameter is (a) 0.1 mm (b) 1mm (c) 10mm. Which of these is a creeping motion?

3-54

Glycerin, with a density of 1260 kg/m^3 and viscosity of 1.5 kg/m.s, flows at 1 m/s past a small smooth sphere.

(a) What sphere diameter will cause the Reynolds number to be exactly unity?

(b) For the sphere size in (a), according to Stokes-flow theory, what is the gage pressure (relative to freestream pressure) at the front stagnation point and at the rear stagnation point.

(c) What is the surface shear stress at these two points?