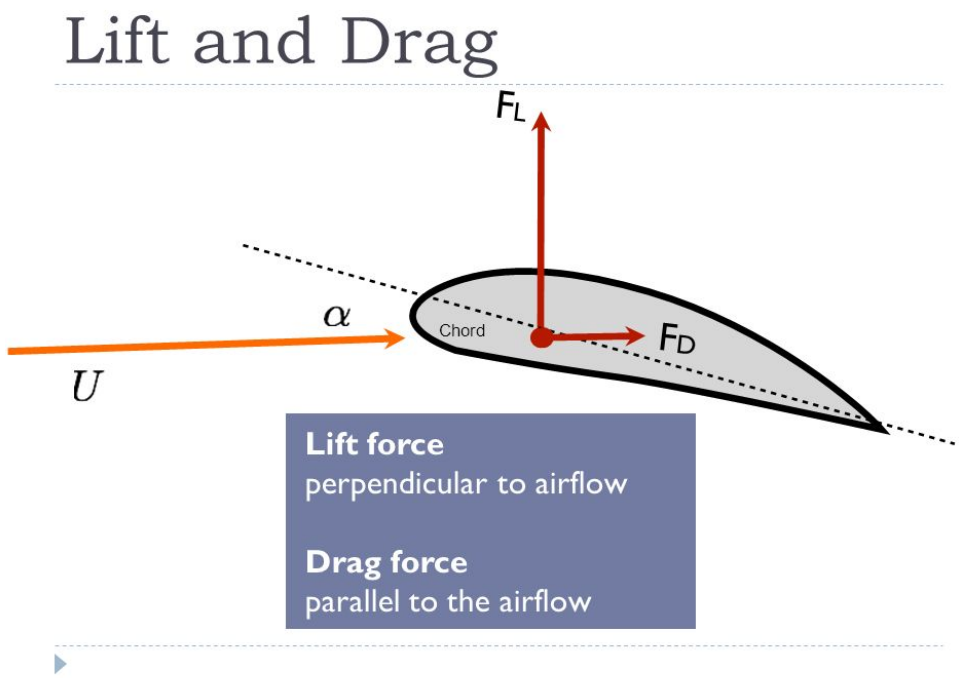
CHAPTER 4
LAMINAR BOUNDARY LAYER

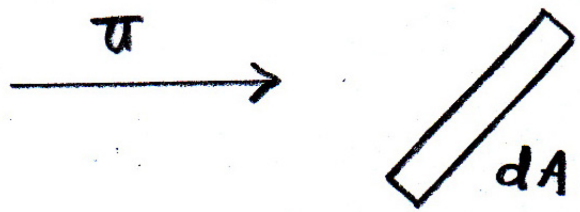
LIFT AND DRAG FORCE



Drag force $= \int dF_X$

Lift force $= \int dF_Y$

Assume there is an inclined plate of dA and a fluid flow passes it with velocity U



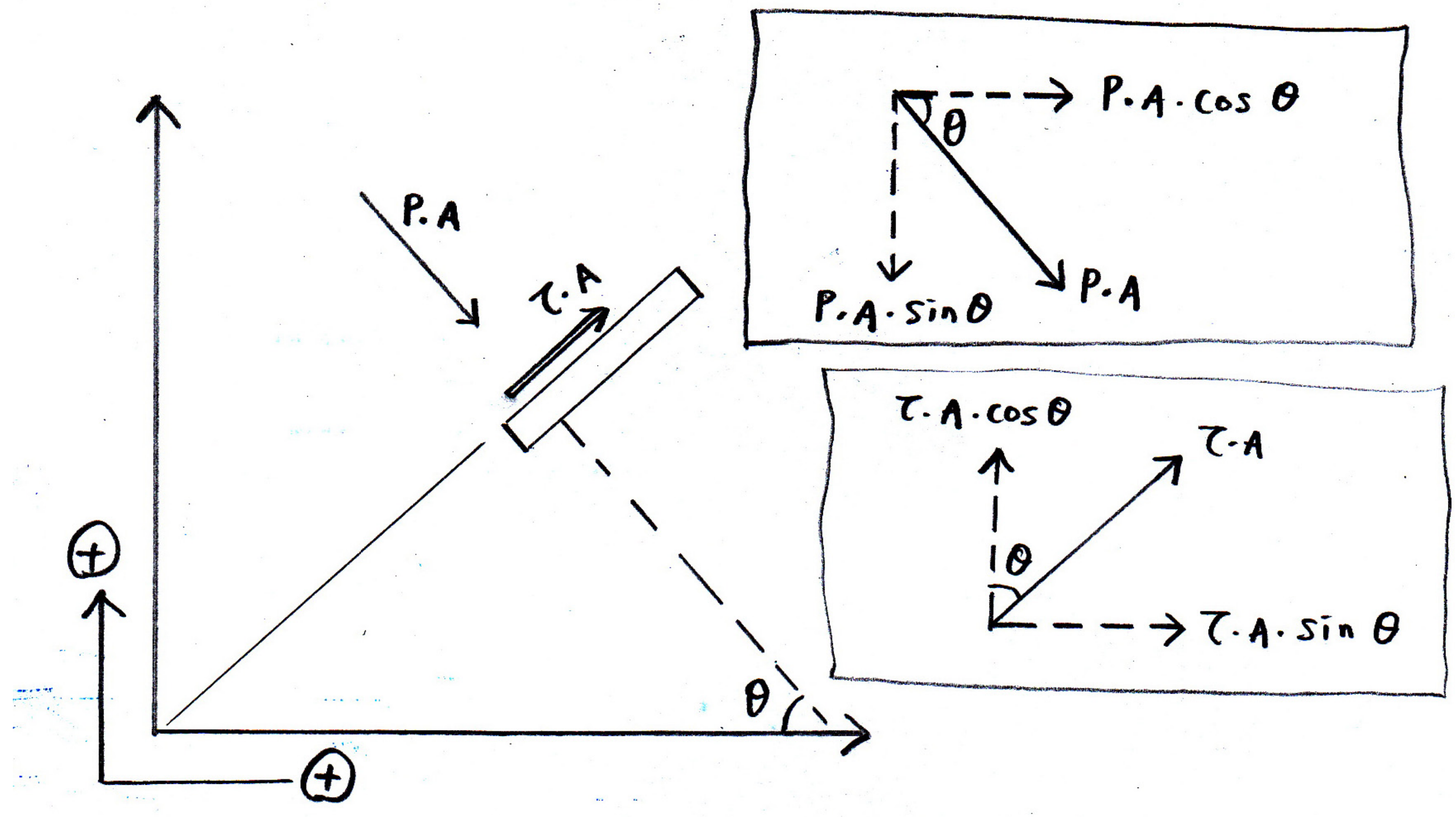
(1)
Due to the U flow velocity, there is a change in the pressure acting on the plane surface. It can be described according to Bernoulli's law.
There is a pressure, then the force could be produced.

$$F = P \cdot A$$

(2)
The airflow also causes friction due to fluid viscosity, and this also causes the resulting force.

$$F = \tau \cdot A$$

Thus, there are two (2) resulting forces and it can be summarized as follows:



$$\text{Drag Force, } F_D = \int d \cdot F_x = \int P \cdot cos\theta . dA + \int \tau \cdot sin\theta . dA$$

$$\text{Lift Force, } F_L = \int d \cdot F_y = \int P \cdot sin\theta . dA + \int \tau \cdot cos\theta . dA$$

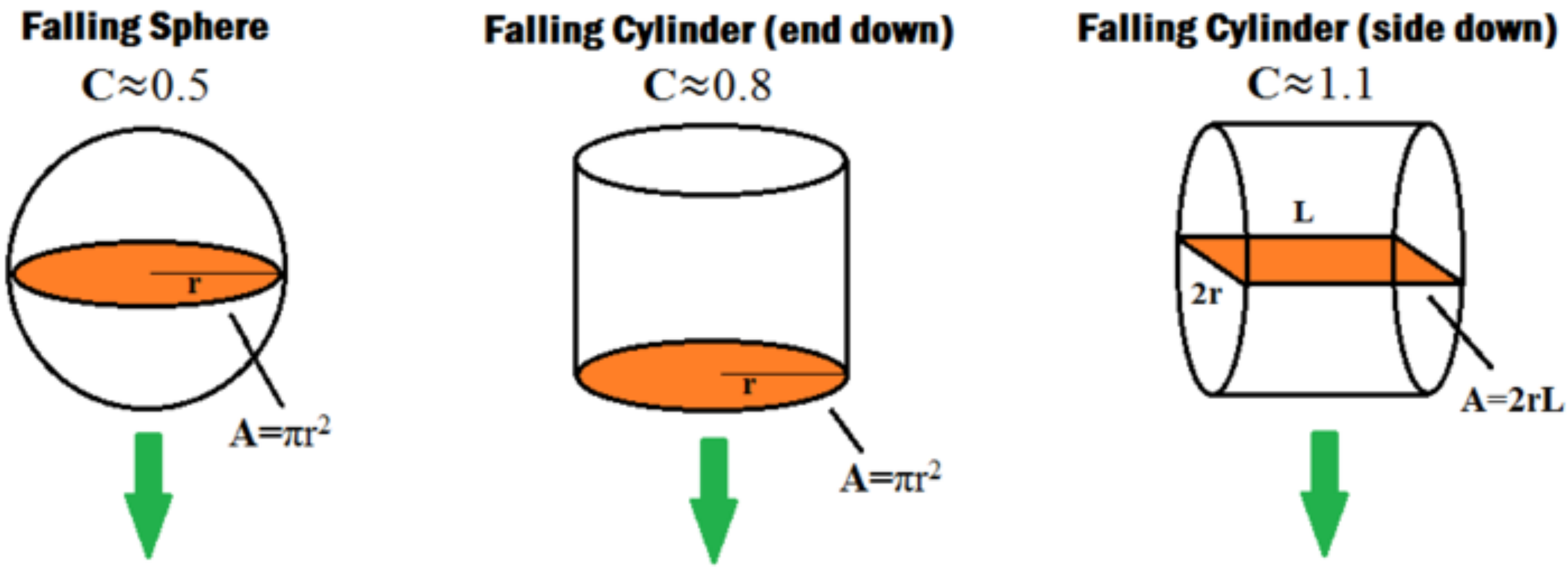
The drag force and lift force calculations can be done using the following basic formula:

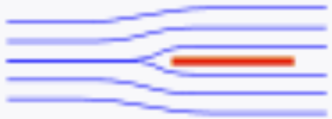
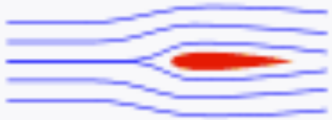
$$F_D = C_D \cdot \frac{1}{2} \cdot \rho A U^2$$

$$F_L = C_L \cdot \frac{1}{2} \cdot \rho A U^2$$

where;
$F_D =$ Drag Force
$F_L =$ Lift Force
$C_D =$ Coefficient of Drag
$C_L =$ Coefficient of Lift
$\rho =$ Density of fluid
$A =$ Area
$U =$ Velocity of fluid

The value of the area, A , will change according to the conditions and definitions.


Falling Sphere
C≈0.5
r
A=πr²
Falling Cylinder (end down)
C≈0.8
r
A=πr²
Falling Cylinder (side down)
C≈1.1
L
2r
A=2rL

Shape and flow	Form Drag	Skin friction
	0%	100%
	~10%	~90%
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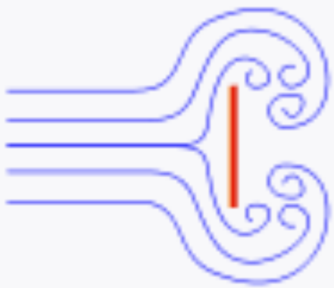
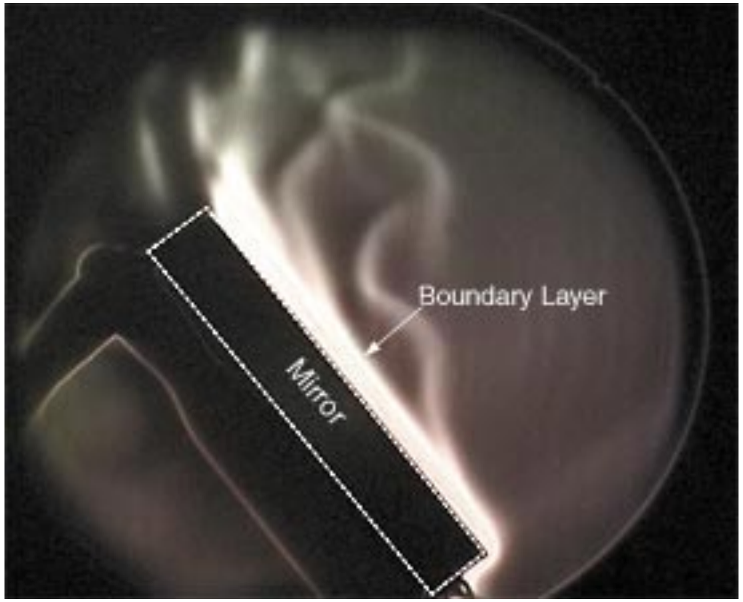
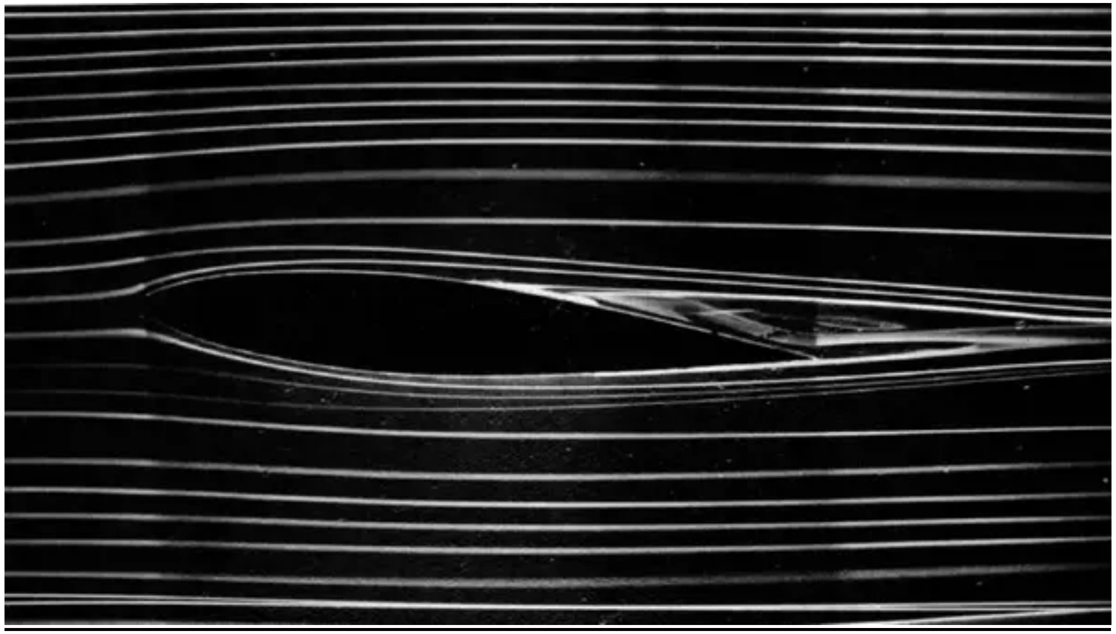
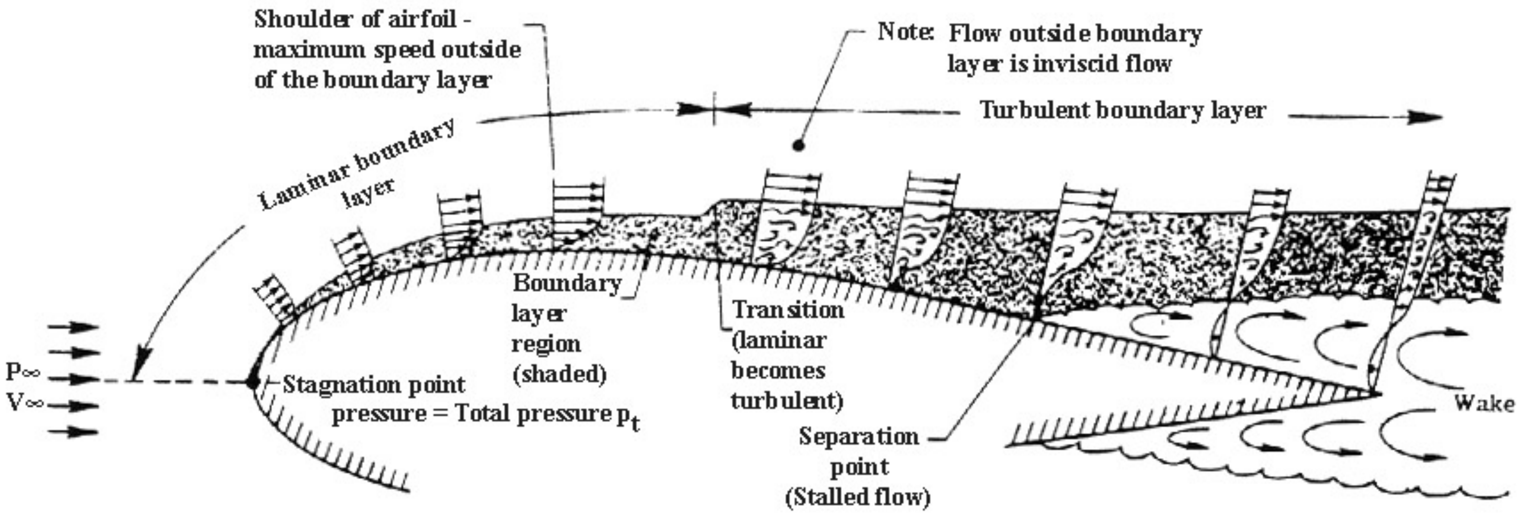
	100%	0%

WHAT IS BOUNDARY LAYER

Boundary layer, in fluid mechanics, thin layer of a flowing gas or liquid in contact with a surface such as that of an airplane wing or of the inside of a pipe. The fluid in the boundary layer is subjected to shearing forces. A range of velocities exists across the boundary layer from maximum to zero, provided the fluid is in contact with the surface. Boundary layers are thinner at the leading edge of an aircraft wing and thicker toward the trailing edge. The flow in such boundary layers is generally laminar at the leading or upstream portion and turbulent in the trailing or downstream portion.







Shoulder of airfoil - maximum speed outside of the boundary layer
Note: Flow outside boundary layer is inviscid flow
Turbulent boundary layer
Laminar boundary layer
Boundary layer region (shaded)
Transition (laminar becomes turbulent)
P∞
V∞
Stagnation point pressure = Total pressure p_t
Separation point (Stalled flow)
Wake

BOUNDARY LAYER THICKNESS, δ

Definition:

A distance from the surface where the local velocity equals 99% of the free stream velocity, U.

$$\delta = y_{(u=0.99U)}$$

$\delta =$ boundary layer thickness
$y =$ distance in y-direction
$u =$ local velocity
$U =$ free stream velocity

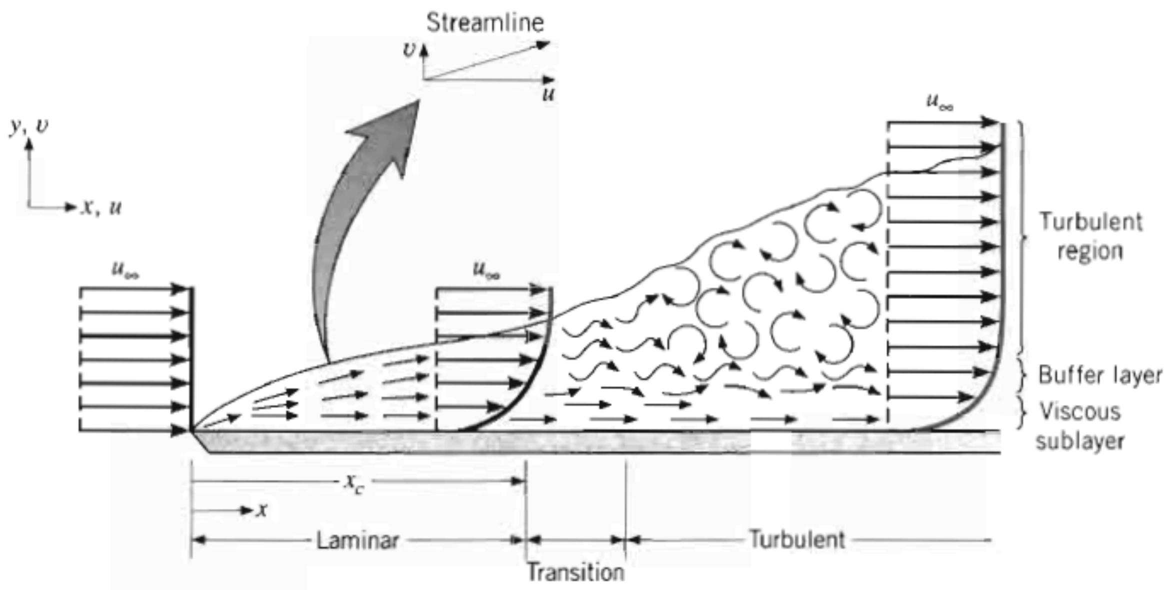


FIGURE 6.6 Velocity boundary layer development on a flat plate.

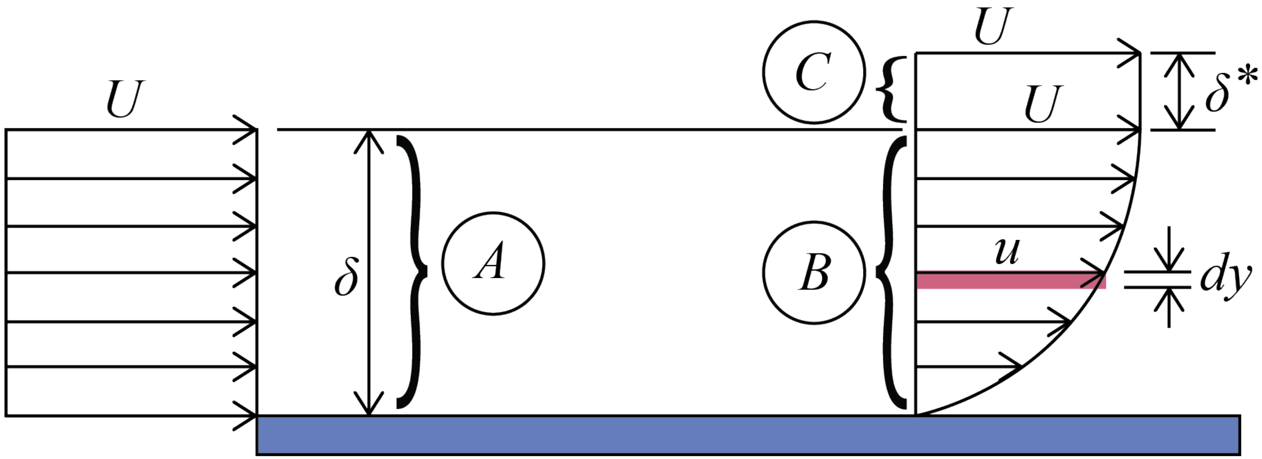
DISPLACEMENT THICKNESS, δ^*

Definition:

The distance the surface would have to move in the y-direction to reduce the flow passing by a volume equivalent to the real effect of the boundary layer.

It is written as delta-star, δ^*

Calculation of the displacement thickness is done by using the idea of conservation of mass.



$$\text{Density}, \rho = \frac{\text{Mass}}{\text{Volume}} = \frac{m}{V}$$

$$\begin{aligned} m &= \rho V \\ &= \rho \times (\text{length} \times \text{height} \times \text{width}) \\ &= \rho \times (U \times \delta \times 1) \\ &= \rho U \delta \end{aligned}$$

Conservation of mass

$$
\begin{aligned}
m_A &= m_B + m_C \\
\rho U \delta &= \int_0^{\delta} \rho u \cdot dy + \rho U \delta^* \\
\rho U \delta^* &= \rho U \delta - \int_0^{\delta} \rho u \cdot dy = \int_0^{\delta} (\rho U - \rho u)\, dy \\
\delta^* &= \int_0^{\delta} \left[\frac{(\rho U - \rho u)}{\rho U}\right] dy = \int_0^{\delta} \left(1 - \frac{\rho u}{\rho U}\right) dy \\
&= \int_0^{\delta} \left(1 - \frac{u}{U}\right) dy
\end{aligned}
$$

Therefore, the formula for determining the displacement thickness is:

$$\delta^* = \int_0^\delta \left(1 - \frac{u}{U}\right) dy$$

δ^* = displacement thickness
δ = boundary layer thickness
u = local velocity
U = free stream velocity

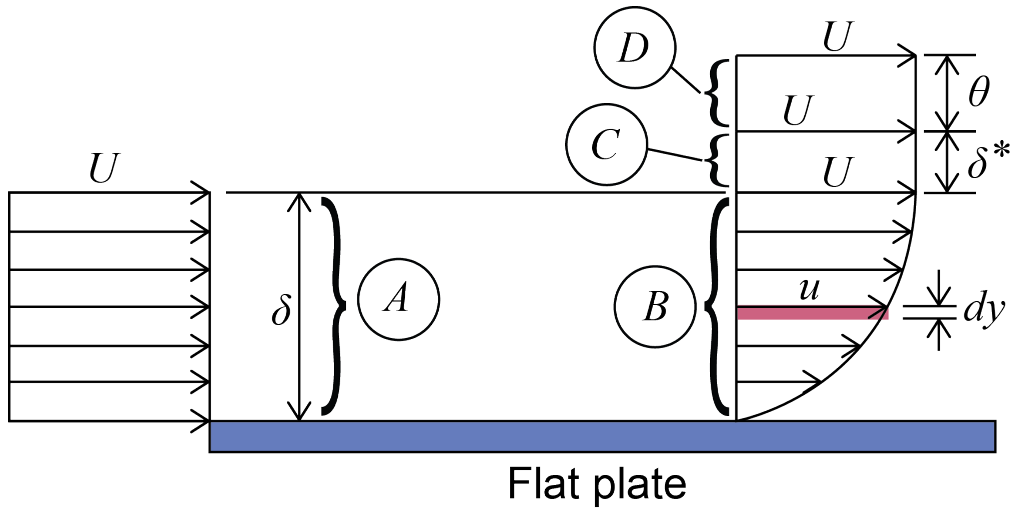
MOMENTUM THICKNESS, θ

Definition:

The distance by which a surface would have to be moved parallel to itself towards the reference plane in an inviscid fluid stream of velocity, u to give the same total momentum as exists between the surface and the reference plane in a real fluid.

It is written as theta, θ

Calculation of the momentum thickness is done by using the idea of conservation of momentum.



$$\sum mV_{in} = \sum mV_{out}$$

	Mass	Velocity
IN	$\rho U\delta$	U
OUT	$\int_0^\delta \rho u \cdot dy$	u
OUT	$\rho U\delta^*$	U
OUT	$\rho U\theta$	U

$$\sum mV_{in} = \sum mV_{out}$$

$$\rho U\delta \cdot U = \int_0^\delta \rho u \cdot u \cdot dy + \rho U\delta^* \cdot U + \rho U\theta \cdot U$$

$$\rho U^2\delta = \int_0^\delta \rho u^2 \cdot dy + \rho U^2\delta^* + \rho U^2\theta$$

It is known that: $$\delta^* = \int_0^\delta \left(1 - \frac{u}{U}\right) dy$$

$$\rho U^2\delta = \int_0^\delta \rho u^2 \cdot dy + \rho U^2 \int_0^\delta \left(1 - \frac{u}{U}\right) dy + \rho U^2\theta$$

$$= \int_0^\delta \rho u^2 \cdot dy + \rho U^2\delta - \int_0^\delta \rho U u \cdot dy + \rho U^2\theta$$

$$\rho U^2\theta = \rho U^2\delta - \int_0^\delta \rho u^2 \cdot dy - \rho U^2\delta + \int_0^\delta \rho U u \cdot dy$$

$$\rho U^2\theta = \int_0^\delta \rho U u \cdot dy - \int_0^\delta \rho u^2 \cdot dy$$

$$\theta = \int_0^\delta \frac{\rho U u}{\rho U^2} \cdot dy - \int_0^\delta \frac{\rho u^2}{\rho U^2} \cdot dy$$

$$= \int_0^\delta \frac{u}{U} \cdot dy - \int_0^\delta \frac{u^2}{U^2} \cdot dy$$

$$= \int_0^\delta \frac{u}{U}\left(1 - \frac{u}{U}\right) dy$$

Therefore, the formula for determining the momentum thickness is:

$$\theta = \int_0^{\delta} \frac{u}{U}\left(1 - \frac{u}{U}\right) dy$$

$\theta =$ momentum thickness
$\delta =$ boundary layer thickness
$u =$ local velocity
$U =$ free stream velocity

EXAMPLE

Determine the $\frac{\delta^*}{\delta}$ and $\frac{\theta}{\delta}$ for the velocity distribution of $\frac{u}{U} = \frac{y}{\delta}$

$$\delta^* = \int_0^{\delta} \left(1 - \frac{u}{U}\right) dy = \int_0^{\delta} \left(1 - \frac{y}{\delta}\right) dy = \frac{1}{2}\delta$$

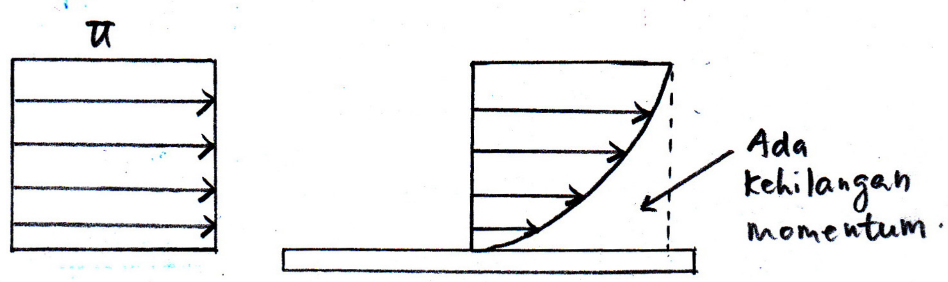
$$\frac{\delta^*}{\delta} = \frac{1}{2}$$

$$\theta = \int_0^{\delta} \frac{u}{U}\left(1 - \frac{u}{U}\right) dy = \int_0^{\delta} \frac{y}{\delta}\left(1 - \frac{y}{\delta}\right) dy$$

$$\frac{\theta}{\delta} = \frac{1}{6}$$

VON-KARMAN EQUATION

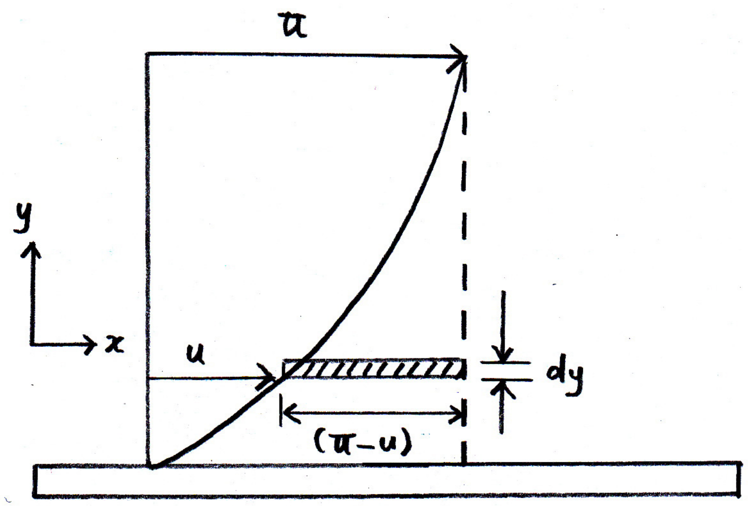
It was introduces by T. Von Karman (1881 – 1963) by using the idea of conservation of momentum.



$$\text{Momentum in} = \text{Momentum out}$$

Momentum can be calculated as:

$$\text{Momentum} = m \times V$$
$$m = \text{mass} = \rho \times (\forall)$$
$$V = \text{Velocity}$$
$$\forall = \text{Volume}$$



mass of the shaded area is:

$$m = \rho(U-u)B \cdot dy$$

Momentum deficit, d , of the shaded area is:

$$\begin{aligned} d &= mV \\ &= \rho(U-u)B \cdot dy \cdot u \end{aligned}$$

Total momentum deficit, D is:

$$\begin{aligned} D &= \rho B \int_0^{\delta} u(U-u)\cdot dy = \rho B U^2 \int_0^{\delta} \frac{u}{U}\left(\frac{U}{U}-\frac{u}{U}\right)\cdot dy \\ &= \rho B U^2 \int_0^{\delta} \frac{u}{U}\left(1-\frac{u}{U}\right)\cdot dy \\ &= \rho B U^2 \theta \end{aligned} \qquad \text{Eq.(1)}$$

Differentiate to dx

$$\frac{dD}{dx} = \rho B U^2 \frac{d\theta}{dx} \qquad \text{Eq.(2)}$$

Momentum losses also can be count as a friction:

$$\begin{aligned} dD &= \tau \cdot dA \\ &= \tau \cdot B \cdot dx \\ \frac{dD}{dx} &= \tau \cdot B \end{aligned} \qquad \text{Eq.(3)}$$

From Eq.(2) and Eq.(3)

$$\frac{dD}{dx} = \rho B U^2 \frac{d\theta}{dx} = \tau \cdot B$$

This equation can be simplified as :

$$\tau = \rho U^2 \frac{d\theta}{dx}$$

This is known as Von Karman equation.

Value of shear stress τ can be found by using the Newton's law of viscosity:

$$\tau = \mu \left(\frac{du}{dy}\right)_{y=0}$$

The condition $y = 0$ was used here because the maximum shear stress occur at $y = 0$.

EXAMPLE:

Determine the boundary layer thickness and drag coefficient in laminar boundary layer that have polynomial velocity profile.

Polynomial velocity profile:

$$\frac{u}{U} = 2\left(\frac{y}{\delta}\right) - \left(\frac{y}{\delta}\right)^2$$

$$\tau = \rho U^2 \frac{d\theta}{dx}$$

$$\tau = \mu \left(\frac{du}{dy}\right)_{y=0}$$

$$\theta = \int_0^\delta \frac{u}{U}\left(1 - \frac{u}{U}\right)^2 dy = \frac{2}{15}\delta$$

By using the Von Karman equation;

$$\tau = \rho U^2 \frac{d\theta}{dx} = \rho U^2 \frac{d}{dx}(\theta) = \rho U^2 \frac{d}{dx}\left(\frac{2}{15}\delta\right) \qquad \text{Eq.(1)}$$

Shear stress can be found from Newton equation;

$$\tau = \mu \left(\frac{du}{dy}\right)_{y=0}$$

$$\frac{u}{U} = 2\left(\frac{y}{\delta}\right) - \left(\frac{y}{\delta}\right)^2$$

$$\frac{du}{dy} = \frac{2U}{\delta} - \frac{2Uy}{\delta^2}$$

$$\left(\frac{du}{dy}\right)_{y=0} = \frac{2U}{\delta} - \frac{2U(0)}{\delta^2} = \frac{2U}{\delta}$$

$$\tau = \mu \left(\frac{du}{dy}\right)_{y=0} = \mu \cdot \frac{2U}{\delta} \qquad \text{Eq.(2)}$$

From Eq.(1) and Eq.(2)

$$\rho U^2 \frac{d}{dx}\left(\frac{2}{15}\delta\right) = \mu \cdot \frac{2U}{\delta}$$

$$\frac{2}{15}\rho U^2 \frac{d\delta}{dx} = \mu \cdot \frac{2U}{\delta}$$

$$\delta \cdot d\delta = \frac{15}{2} \cdot \frac{1}{\rho U^2} \cdot \mu \cdot \frac{2U}{\delta} \cdot dx$$

$$= \frac{15\mu}{\rho U} \cdot dx$$

Integrating both sides;

$$\delta \cdot d\delta = \frac{15\mu}{\rho U} \cdot dx$$

$$\int \delta \cdot d\delta = \int \frac{15\mu}{\rho U} \cdot dx$$

$$\frac{\delta^2}{2} = \frac{15\mu}{\rho U} \cdot x + C$$

$$x = 0, \qquad \delta = 0, \qquad \Rightarrow \quad C = 0$$

$$\delta^2 = \frac{(2)15\mu x}{\rho U} = \frac{30\mu x}{\rho U}$$

$$= \frac{30\mu x}{\rho U} \cdot \frac{x}{x} = 30x^2 \cdot \frac{\mu}{\rho U x} = \frac{30x^2}{Re}$$

$$\delta = \sqrt{\frac{30x^2}{Re}}$$

$$\delta = \frac{5.48x}{\sqrt{Re}}$$

Shear stress, τ ;

$$\tau = \mu \frac{2U}{\delta}$$

Substitute $\delta = \frac{5.48x}{\sqrt{Re}}$

$$\tau = 0.365 \cdot \frac{\mu U}{x} \cdot \sqrt{Re}$$

Local drag coefficient, C_d

$$\tau = 0.365 \cdot \frac{\mu U}{x} \cdot \sqrt{Re}$$

$$\tau = C_d \cdot \frac{1}{2} \cdot \rho U^2$$

$$C_d = \frac{0.73}{\sqrt{Re}}$$

Drag force, F_D

$$
\begin{aligned}
F_D &= \int_0^L \tau \cdot B \cdot dx \\
&= \int_0^L 0.365 \cdot \frac{\mu U}{x} \cdot \sqrt{Re} \cdot B \cdot dx \\
&= \int_0^L 0.365 \cdot \frac{\mu U}{x} \cdot \sqrt{\frac{\rho U x}{\mu}} \cdot B \cdot dx \\
&= \int_0^L 0.365 \cdot U\sqrt{\rho U \mu} \cdot x^{-\frac{1}{2}} \cdot B \cdot dx \\
&= \left[0.73 \cdot U\sqrt{\rho U \mu} \cdot x^{\frac{1}{2}} \cdot B\right]_0^L \\
&= 0.73 \cdot U\sqrt{\rho U \mu} \cdot L^{\frac{1}{2}} \cdot B \\
&= 0.73 \cdot U\sqrt{\rho U \mu} \cdot \sqrt{L} \cdot B
\end{aligned}
$$

Drag coefficient, C_D

$$F_D = C_D \frac{1}{2} \rho A U^2$$

$$C_D = \frac{2 \cdot F_D}{\rho A U^2}$$

$$A = L \times B$$

$$C_D = \frac{2\left(0.73 \cdot U\sqrt{\rho U \mu} \cdot \sqrt{L} \cdot B\right)}{\rho A U^2}$$

$$= \frac{2\left(0.73 \cdot U\sqrt{\rho U \mu} \cdot \sqrt{L} \cdot B\right)}{\rho (B \cdot L) U^2}$$

$$= \frac{1.46 \cdot \sqrt{\mu}}{\sqrt{\rho U L}}$$

$$C_D = \frac{1.46}{\sqrt{Re}}$$

The value of drag coefficient, C_D for various $\frac{u}{U}$ are as follows:

Velocity	δ	C_d	C_D
$\frac{u}{U} = \frac{y}{\delta}$	$\frac{3.46x}{\sqrt{Re}}$	$\frac{0.578}{\sqrt{Re}}$	$\frac{1.153}{\sqrt{Re}}$
$\frac{u}{U} = 2\left(\frac{y}{\delta}\right) - 2\left(\frac{y}{\delta}\right)^2$	$\frac{5.48x}{\sqrt{Re}}$	$\frac{0.730}{\sqrt{Re}}$	$\frac{1.46}{\sqrt{Re}}$
$\frac{u}{U} = \frac{3}{2}\left(\frac{y}{\delta}\right) - \frac{1}{2}\left(\frac{y}{\delta}\right)^3$	$\frac{4.64x}{\sqrt{Re}}$	$\frac{0.646}{\sqrt{Re}}$	$\frac{1.292}{\sqrt{Re}}$
$\frac{u}{U} = 2\left(\frac{y}{\delta}\right) - 2\left(\frac{y}{\delta}\right)^3 + \left(\frac{y}{\delta}\right)^4$	$\frac{5.84x}{\sqrt{Re}}$	$\frac{0.686}{\sqrt{Re}}$	$\frac{1.372}{\sqrt{Re}}$
$\frac{u}{U} = \sin\left(\frac{\pi}{2}\frac{y}{\delta}\right)$	$\frac{4.795x}{\sqrt{Re}}$	$\frac{0.654}{\sqrt{Re}}$	$\frac{1.310}{\sqrt{Re}}$
Blasius exact solution	$\frac{5x}{\sqrt{Re}}$	$\frac{0.664}{\sqrt{Re}}$	$\frac{1.328}{\sqrt{Re}}$

A BLASIUS EXACT SOLUTION

A Blasius exact solution equation for laminar flat-plate boundary layer problem derived from Navier-Stokes equations is:

$$ff'' + 2f''' = 0$$

or

$$f \cdot \frac{d^2 f}{d\eta^2} + 2 \cdot \frac{d^3 f}{d\eta^3} = 0$$

where;

$$f = f(\eta) \ , \qquad f'' = \frac{d^2 f(\eta)}{d\eta^2} \ , \qquad f''' = \frac{d^3 f(\eta)}{d\eta^3}$$

$$\eta = y\sqrt{\frac{U\rho}{\mu x}} = y\sqrt{\frac{U}{\upsilon x}}$$

With suitable boundary conditions, the above equation had been solved by 4th-order Runge-Kutta numerical integration and the results is tabulated in table below.

$\eta = y\sqrt{\frac{U}{\upsilon x}}$	$f(\eta)$	$f'(\eta) = \frac{u}{U}$	$f''(\eta)$
0.0	0.0000	0.0000	0.3321
0.5	0.0415	0.1659	0.3309
1.0	0.1656	0.3298	0.3230
1.5	0.3701	0.4868	0.3026
2.0	0.6500	0.6298	0.2668
2.5	0.9964	0.7513	0.2174
3.0	1.3969	0.8461	0.1614
3.5	1.8378	0.9131	0.1078
4.0	2.3059	0.9555	0.0642
4.5	2.7903	0.9795	0.0340
5.0	3.2834	0.9916	0.0159
5.5	3.7807	0.9969	0.0066
6.0	4.2798	0.9990	0.0024
6.5	4.7795	0.9997	0.0008
7.0	5.2794	0.9999	0.0002
7.5	5.7794	1.0000	0.0001
8.0	6.2794	1.0000	0.0000

Determine:

$$\delta\ , \qquad C_d\ , \qquad C_D$$

Boundary layer thickness, δ :

Limit for boundary layer:

$$u = 0.99U, \qquad y = \delta$$

Assumption:

$$f' = \frac{u}{U} = 0.99$$

From table, the nearest value is $\eta = 5$,
where $f' = 0.992$

$$\eta = 5 = \delta\sqrt{\frac{U}{\upsilon x}} \qquad \text{Eq.(1)}$$

$$Re = \frac{Ux}{\upsilon} \Rightarrow \frac{Re}{x} = \frac{U}{\upsilon} \qquad \text{Eq.(2)}$$

From Eq.(1) and Eq.(2);

$$5 = \delta\sqrt{\frac{U}{vx}} = \delta\sqrt{\frac{U}{v}\cdot\frac{1}{x}} = \delta\sqrt{\frac{Re}{x}\cdot\frac{1}{x}}$$

$$= \delta\sqrt{\frac{Re}{x^2}} = \frac{\delta}{x}\sqrt{Re}$$

$$\delta = \frac{5x}{\sqrt{Re}}$$

Displacement thickness, δ^* :

$$\eta = y\sqrt{\frac{U}{\upsilon x}} \Rightarrow \frac{d\eta}{dy} = \sqrt{\frac{U}{\upsilon x}} \Rightarrow dy = d\eta\sqrt{\frac{\upsilon x}{U}}$$

$$\delta^* = \int_0^{\delta}\left(1 - \frac{u}{U}\right) dy$$

$$= \int_0^{5}(1 - f') \cdot d\eta\sqrt{\frac{\upsilon x}{U}}$$

$$\delta^* = \sqrt{\frac{\upsilon x}{U}}\left(\int_0^{5} 1 \cdot d\eta - \int_0^{5}(f') \cdot d\eta\right)$$

$$= \sqrt{\frac{\upsilon x}{U}}\left([\eta]_0^5 - [f(\eta)]_0^5\right)$$

$$= \sqrt{\frac{\upsilon x}{U}}(5 - 3.2834)$$

$$\delta^* = \sqrt{\frac{\upsilon x}{U}}(1.7166)$$

Known that: $5 = \delta\sqrt{\frac{U}{\upsilon x}} \Rightarrow \frac{\delta}{5} = \sqrt{\frac{\upsilon x}{U}}$

$$\delta^* = \frac{\delta}{5}(1.7166)$$

Substitute: $\delta = \frac{5x}{\sqrt{Re}}$

$$\delta^* = \frac{1.7166x}{\sqrt{Re}}$$

Momentum thickness, θ :

$$\theta = \int_0^{\delta} \frac{u}{U}\left(1 - \frac{u}{U}\right) dy$$

$$= \int_0^{5} f'(1 - f') \cdot d\eta \sqrt{\frac{\upsilon x}{U}}$$

$$= \sqrt{\frac{\upsilon x}{U}} \left(\int_0^{5} f'(1 - f') \cdot d\eta \right)$$

$$= \frac{\delta}{5} \sum_0^{5} f'(1 - f') \cdot \Delta\eta$$

Pengiraan di atas dibuat berdasarkan table.

$\Delta\eta = 0.5$		
η	f'	$f'(1-f') \cdot \Delta\eta$
0	0	0
0.5	0.1659	0.069188595
1	0.3298	0.11051598
1.5	0.4868	0.12491288
2	0.6298	0.11657598
2.5	0.7513	0.093424155
3	0.8461	0.065107395
3.5	0.9131	0.039674195
4	0.9555	0.021259875
4.5	0.9795	0.010039875
5	0.9916	0.00416472
		0.65486365

$$\theta = \frac{\delta}{5}\sum_{0}^{5} f'(1-f')\cdot\Delta\eta$$

$$= \frac{\delta}{5}(0.65486365)$$

Substitute: $\delta = \frac{5x}{\sqrt{Re}}$

$$\theta = \frac{0.6548x}{\sqrt{Re}}$$

This value is slightly different from the official value because the number of data was limited.

The official value of momentum thickness is:

$$\theta = \frac{0.664x}{\sqrt{Re}}$$

Shear wall stress, τ :

Known that :

$$\frac{u}{U} = \frac{df}{d\eta} = f'^{(\eta)} = f' \;\Rightarrow\; u = U \cdot f'$$

$$\frac{du}{d\eta} = U \cdot df'$$

$$\eta = y\sqrt{\frac{U}{\upsilon x}} \;\Rightarrow\; \frac{d\eta}{dy} = \sqrt{\frac{U}{\upsilon x}}$$

$$\tau = \mu\left(\frac{du}{dy}\right)_{y=0}$$

$$= \mu\left(\frac{du}{d\eta} \cdot \frac{d\eta}{dy}\right)_{\eta=0}$$

$$= \mu \left(U \cdot df' \cdot \sqrt{\frac{U}{\upsilon x}} \right)_{\eta=0}$$

$$= \mu \cdot U \sqrt{\frac{U}{\upsilon x}} (df')_{\eta=0}$$

$$\tau = \mu \cdot U \sqrt{\frac{U}{\upsilon x}} \left((f'')_{\eta=0} \right)$$

$$= \mu \cdot U \sqrt{\frac{U}{\upsilon x}} \cdot (0.3321)$$

$$\tau = \frac{0.3321 \rho U^2}{\sqrt{Re}}$$

Local friction coefficient, C_d

$$C_d = \frac{\tau}{\frac{1}{2}\rho U^2}$$

$$= \frac{\frac{0.3321\rho U^2}{\sqrt{Re}}}{\frac{1}{2}\rho U^2}$$

$$C_d = \frac{0.6642}{\sqrt{Re}}$$

Drag coefficient, C_D

$$C_D = \frac{\int \tau \cdot B \cdot dx}{\frac{1}{2}\rho U^2 \cdot B \cdot L}$$

$$= \frac{\int \rho U^2 \cdot \frac{d\theta}{dx} \cdot B \cdot dx}{\frac{1}{2}\rho U^2 \cdot B \cdot L}$$

$$= \frac{\int d\theta}{\frac{1}{2} \cdot L} = \frac{2\theta}{L}$$

Diketahui $\theta = \frac{0.6548x}{\sqrt{Re}}$

$$C_D = \frac{1.3096}{\sqrt{Re}}$$

If the official value of momentum thickness is used,

$$\theta = \frac{0.664x}{\sqrt{Re}}$$

The value of drag coefficient will become:

$$C_D = \frac{1.328}{\sqrt{Re}}$$