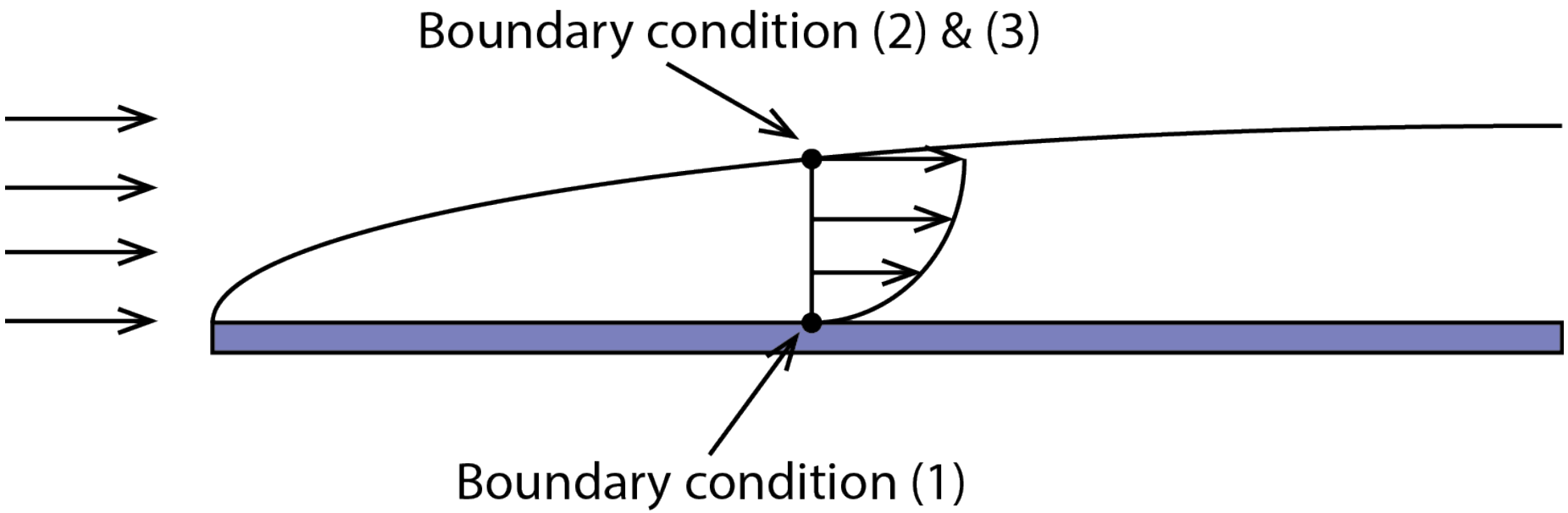
QUESTION 2

If velocity distribution in a laminar boundary layer over a flat plate is assumed to be given by second order polynomial, determine its form using the necessary boundary conditions.



ANSWER

Second order polynomial

$$u = a + by + cy^2$$

Boundary condition:

(1)

At $y = 0 \quad u = 0$

$$u = a + by + cy^2$$

$$0 = a + b(0) + c(0)^2$$

$$a = 0$$

Velocity profile becomes:

$$u = by + cy^2 \qquad (1)$$

(2)

At $y = \delta \quad u = U$

$$u = by + cy^2$$

$$U = b(\delta) + c(\delta)^2 \qquad (2)$$

(3)

At $y = \delta$, $\frac{du}{dy} = 0$

$$u = by + cy^2$$

$$\frac{du}{dy} = b + 2cy$$

$$\left(\frac{du}{dy}\right)_{y=\delta} = 0 = b + 2c\delta$$

$$b = -2c\delta \qquad (3)$$

By using Eq.(2) and Eq.(3)

$$U = b(\delta) + c(\delta)^2$$

$$= -2c\delta(\delta) + c(\delta)^2$$

$$= -2c\delta^2 + c(\delta)^2$$

$$U = c\delta^2$$

$$c = -\frac{U}{\delta^2}$$

From Eq.(3)

$$b = -2\left(-\frac{U}{\delta^2}\right)\delta$$

$$b = \frac{2U}{\delta}$$

Velocity profile can be simplified as:

$$u = b(y) + c(y)^2$$

$$= \frac{2U}{\delta}(y) - \frac{U}{\delta^2}(y)^2$$

$$= 2U\left(\frac{y}{\delta}\right) - U\left(\frac{y}{\delta}\right)^2$$

$$\frac{u}{U} = 2\left(\frac{y}{\delta}\right) - \left(\frac{y}{\delta}\right)^2$$