CHAPTER 4
LAMINAR BOUNDARY LAYER

Analysis for laminar boundary layer on thin flat plate can be conducted by using the Von Karman equation and / or Blasius exact solution.

In thin flat plate, there are no pressure gradient in the boundary layer. Because of that, there can be no separation.

Flow separation or boundary layer separation is the detachment of a boundary layer from a surface into a wake. Separation occurs in flow that is slowing down, with pressure increasing, after passing the thickest part of a streamline body or passing through a widening passage.

Flow separation occurs when there is a change of velocity or pressure. That is the reason why there is no flow separation occurs on the thin flat plate.

An image to show the separation that occur on the cylinder surface.

From the previous chapter, we know that the maximum velocity for flow around a cylinder is at 90° or at $\theta = \frac{\pi}{2}$. The distribution of velocity and pressure for the upper flow of the cylinder can be shown as follows.

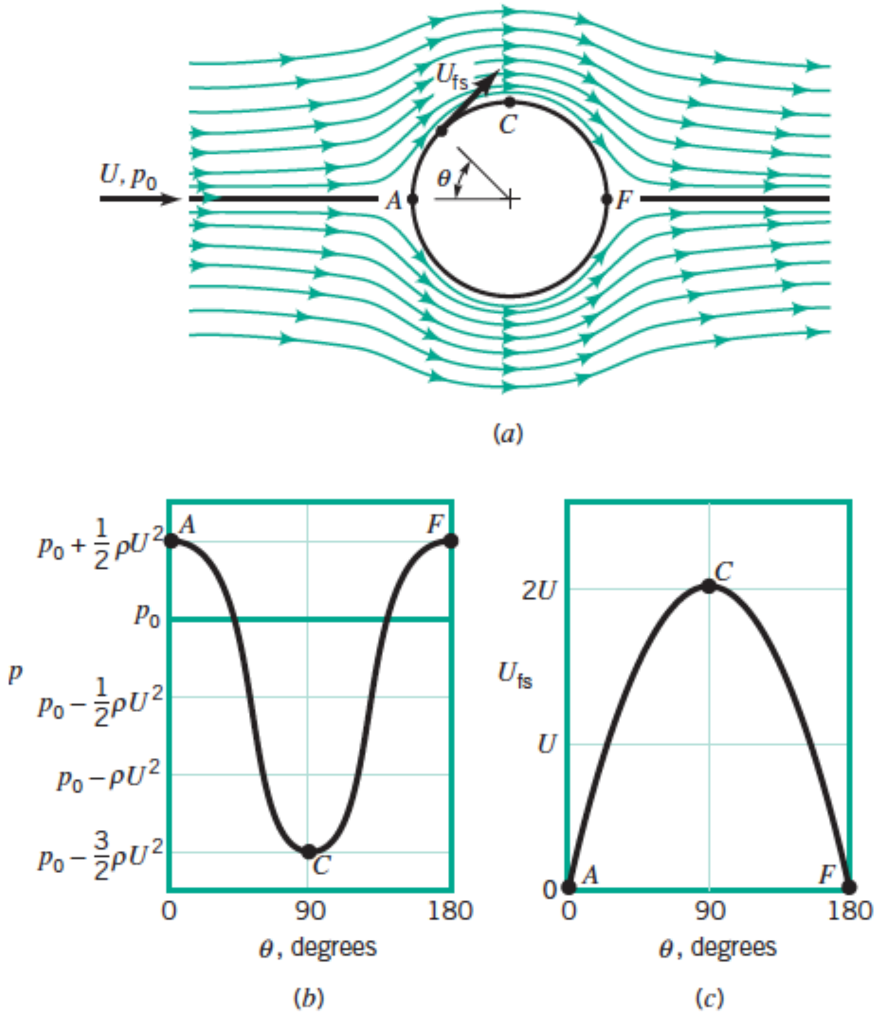


FIGURE 9.11 **Inviscid flow past a circular cylinder: (*a*) streamlines for the flow if there were no viscous effects, (*b*) pressure distribution on the cylinder's surface, and (*c*) free-stream velocity on the cylinder's surface.**

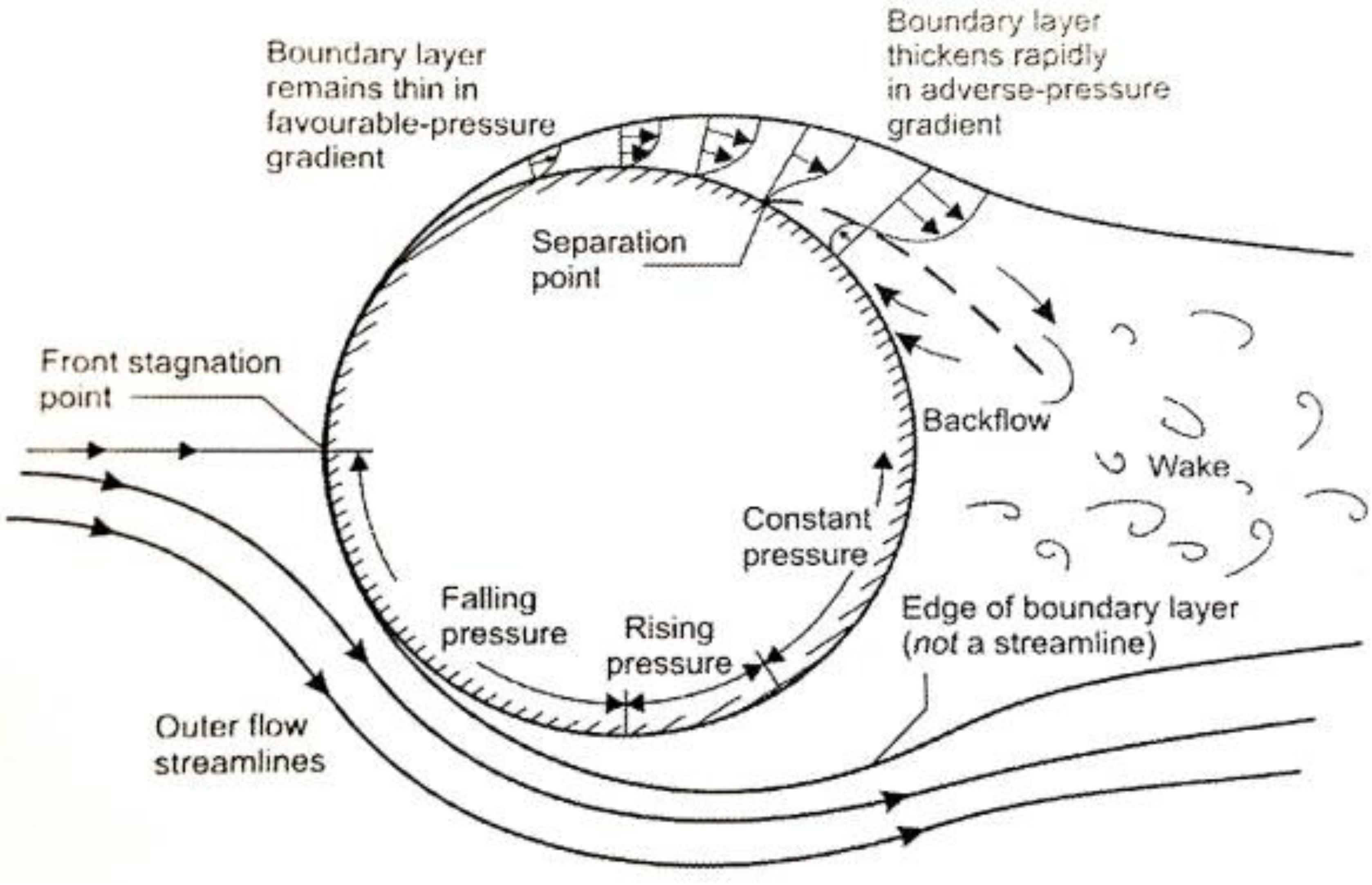
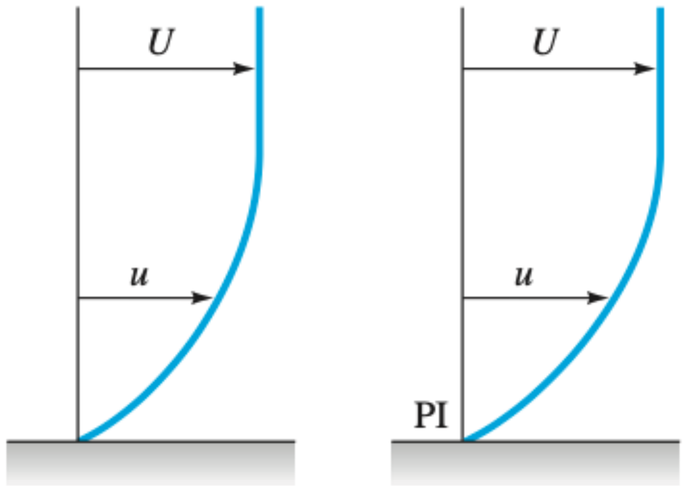

Boundary layer
remains thin in
favourable-pressure
gradient
Boundary layer
thickens rapidly
in adverse-pressure
gradient
Separation
point
Front stagnation
point
Backflow
Wake
Constant
pressure
Falling
pressure
Rising
pressure
Edge of boundary layer
(*not* a streamline)
Outer flow
streamlines

Figure 7.7 illustrates the general case. In a favourable gradient (Fig. 7.7a) the profile is very rounded, there is no point of inflection, there can be no separation, and laminar profiles of this type are very resistant to a transition to turbulence.

In a zero-pressure gradient (Fig. 7.7b), such as a flat-plate flow, the point of inflection is at the wall itself. There can be no separation, and the flow will undergo transition at Re number greater than about 3×10^6.



(*a*) Favorable gradient:

$$\frac{dU}{dx} > 0$$

$$\frac{dp}{dx} < 0$$

No separation, PI inside wall

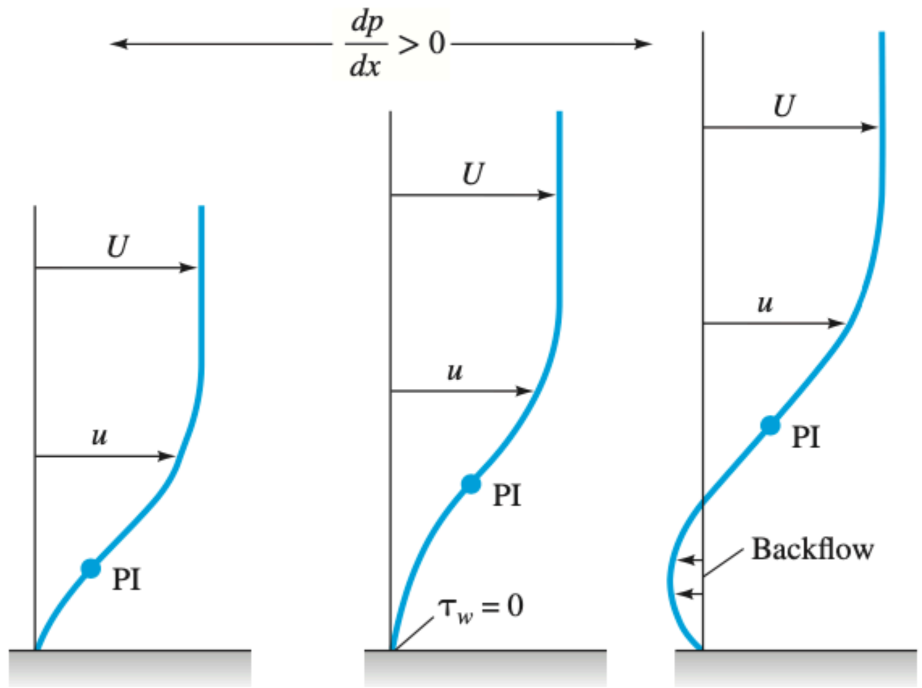
(*b*) Zero gradient:

$$\frac{dU}{dx} = 0$$

$$\frac{dp}{dx} = 0$$

No separation, PI at wall

In an adverse gradient (Fig. 7.7c to 7.7e), a point of inflection (PI) occurs in the boundary layer, its distance from the wall increasing with the strength of the adverse gradient. For a weak gradient (Fig. 7.7c) the flow does not actually separate, but it is vulnerable to transition to turbulence at Re number as low as 10^5.



(c) Weak adverse gradient:

$\frac{dU}{dx} < 0$

$\frac{dp}{dx} > 0$

No separation, PI in the flow

(d) Critical adverse gradient:

Zero slope at the wall:

Separation

(e) Excessive adverse gradient:

Backflow at the wall:

Separated flow region

At a moderate gradient, a critical condition (Fig. 7.7d) is reached where the wall shear is exactly zero $\left(\frac{\partial u}{\partial y} = 0\right)$. This is defined as the separation point $(\tau_{wall} = 0)$, because any stronger gradient will actually cause backflow at the wall (Fig. 7.7e): the boundary layer thickens greatly, and the main flow breaks away, or separates, from the wall (Fig. 7.2b).

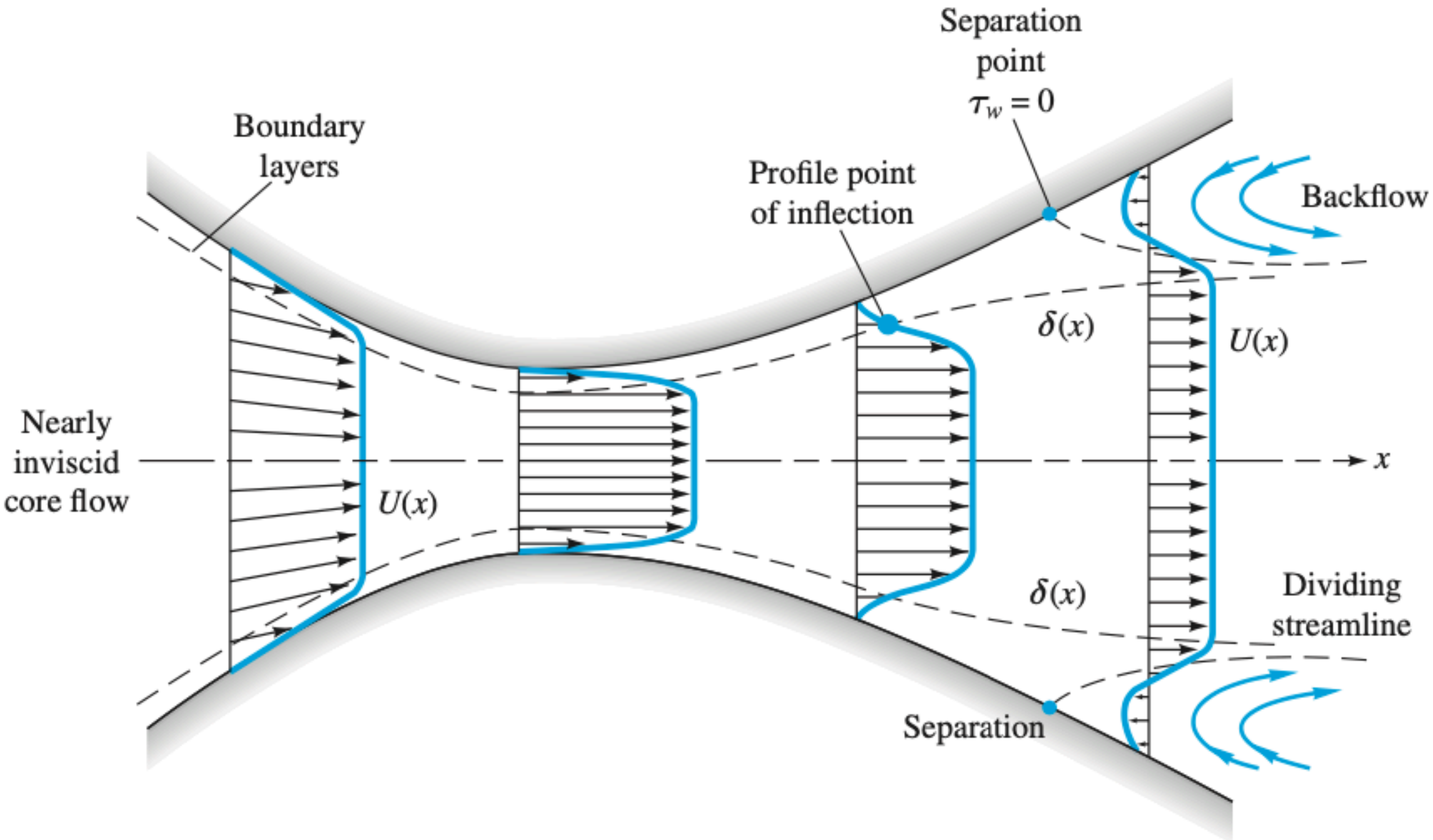
The flow profiles of Fig. 7.7 usually occur in sequence as the boundary layer progresses along the wall of a body. For example, in Fig. 7.2a, a favourable gradient occurs on the front of the body, zero pressure gradient occurs just upstream of the shoulder, and an adverse gradient occurs successively as we move around the rear of the body.

A second practical example is the flow in a duct consisting of a nozzle, throat, and diffuser, as in Fig. 7.8. The nozzle flow is a favourable gradient and never separates, nor does the throat flow where the pressure gradient is approximately zero.

But the expanding-area diffuser produces low velocity and increasing pressure, an adverse gradient. If the diffuser angle is too large, the adverse gradient is excessive, and the boundary layer will separate at one or both walls, with backflow, increased losses, and poor pressure recovery.

In the diffuser literature this condition is called diffuser stall, a term used also in airfoil aerodynamics to denote airfoil boundary layer separation. Thus, the boundary layer behaviour explains why a large-angle diffuser has heavy flow losses and poor performance.

Presently boundary layer theory can compute only up to the separation point, after which it is invalid. Techniques are now developed for analysing the strong interaction effects caused by separated flows.



Nozzle:	*Throat*:	*Diffuser*:
Decreasing pressure and area	Constant pressure and area	Increasing pressure and area
Increasing velocity	Velocity constant	Decreasing velocity
Favorable gradient	Zero gradient	Adverse gradient (boundary layer thickens)

THE CORRELATION METHOD OF THWAITES

Von Karman solution can be used for laminar and turbulent boundary layer.
However, Blasius solution only can be used for laminar boundary layer.
Von Karman and Blasius used for flat plat problem or situation that does not have pressure gradient (separation will not occur).

For curved surface (sphere or circular surface), there is a pressure gradient and flow separation have probability to occur. In this case, Thwaites equation is proposed to be used. At the same time, Thwaites equation could predict the separation point.

From Von Karman integral method:

$$\tau_w = \rho U^2 \frac{d\theta}{dx} = c_d \cdot \frac{1}{2} \rho U^2$$

Thwaites proposed this correlation:

$$\frac{\tau_w}{\rho U^2} = c_d \cdot \frac{1}{2} = \frac{d\theta}{dx} + (2 + H)\frac{\theta}{U}\frac{dU}{dx} \qquad \text{Eq.(1)}$$

Where,

$$\theta(x) = \theta = \text{Momentum thickness}$$

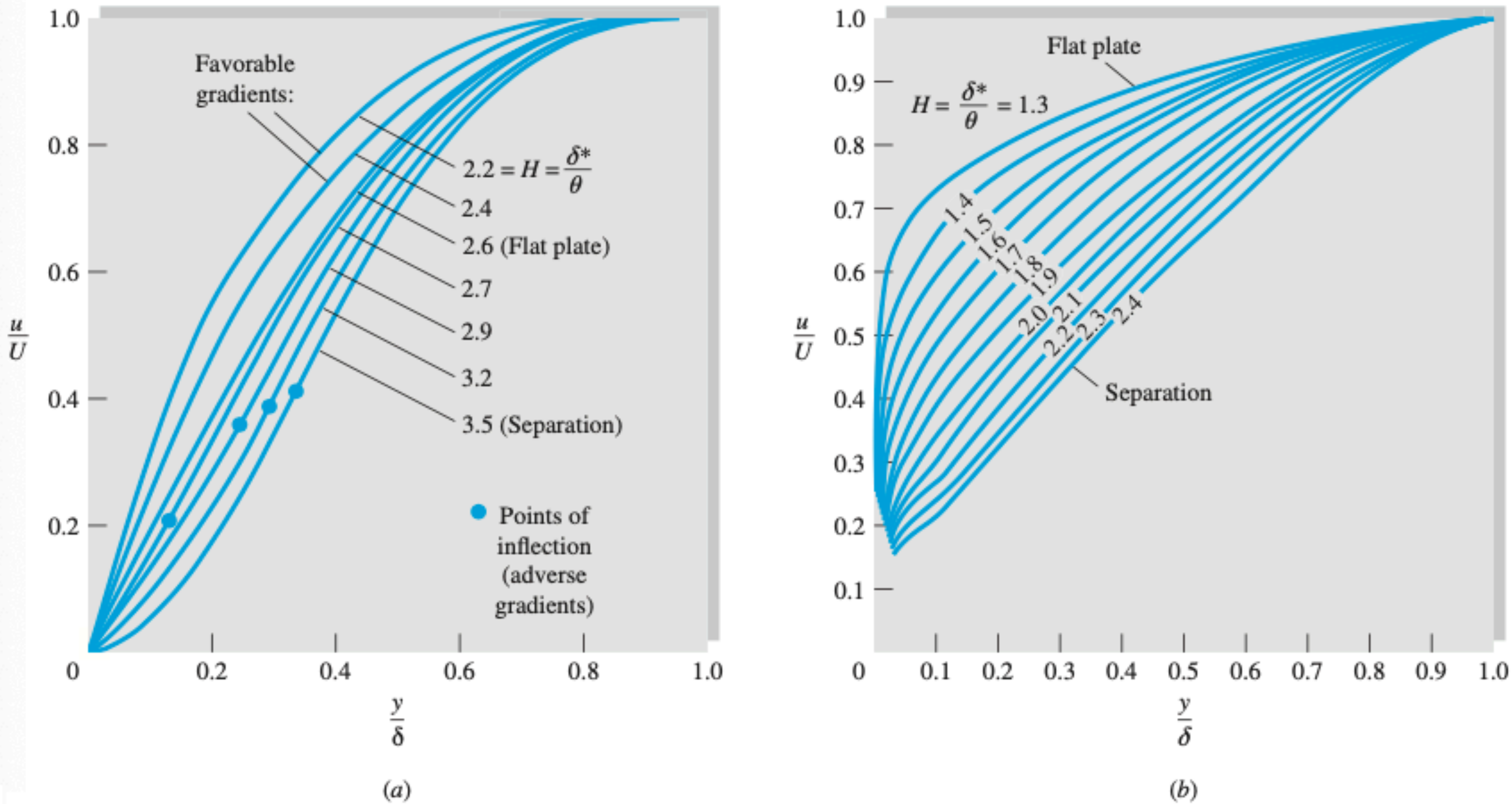
$$H(x) = H = \frac{\delta^*(x)}{\theta(x)} = \text{Shape factor}$$

$$-\frac{dU}{dx} = +\frac{dp}{dx} = \text{Adverse gradient}$$

The higher the H, the stronger the adverse gradient, and separation occurs approximately at:

$$H \approx 3.5 \text{ for laminar flow}$$

$$H \approx 2.4 \text{ for turbulent flow}$$



Velocity profiles with pressure gradient: (*a*) laminar flow; (*b*) turbulent flow with adverse gradients.

For laminar flow, a simple and effective method was developed by Thwaites, who found that Eq.(1) can be correlated by a single dimensionless momentum thickness variable λ , defined as:

$$\lambda = \frac{\theta^2}{\upsilon}\frac{dU}{dx}$$

Using a straight-line fit to his correlation, Thwaites was able to integrate Eq.(1) in closed form, with the result:

$$\theta^2 = \theta_0^2\left(\frac{U_0}{U}\right)^6 + \frac{0.45\upsilon}{U^6}\int_0^x U^5 dx$$

Where:

$$\theta_0 = \text{Momentum thickness at } \; x = 0$$

$$\upsilon = \text{kinematic viscosity}$$

Separation was found to occur at a particular value of λ :

$$\lambda = -0.09$$

Finally, Thwaites correlated values of the dimensionless shear stress, S with λ , and his graphed result can be curve-fitted as follows:

$$S(\lambda) = \frac{\tau_w}{\mu U} \approx (\lambda + 0.09)^{0.62}$$

This parameter is related to the skin friction by the identity.

$$S \approx \frac{1}{2} C_d \cdot Re_\theta$$

For a flat plate, $U =$ constant, $\lambda = 0$, $\theta_0 = 0$

$$\theta^2 = \frac{0.45 \upsilon x}{U}$$

Or

$$\frac{\theta}{x} = \frac{0.671}{\sqrt{Re}}$$

Thwaites equation:

$$\theta^2 = \frac{0.45\upsilon}{U^6}\int_0^x U^5 \cdot dx$$

$$\lambda = \frac{\theta^2}{\upsilon}\frac{dU}{dx}$$

EXAMPLE

The example of velocity profile for a laminar boundary layer on a curved surface is given as:

TABLE 4-5
Laminar-separation-point prediction by Thwaites' method

		Thwaites	
$U(x)$	x_{sep} (exact)	x_{sep}	Error, %
Howarth (1938)			
$1 - x$	0.120	0.123	+2.5
Tani (1949)			
$1 - x^2$	0.271	0.268	−1.1
$1 - x^4$	0.462	0.449	−2.8
$1 - x^8$	0.640	0.621	−3.0
Terrill (1960)			
$\sin(x)$	1.823	1.800	−1.3
Curle (1958)			
$x - x^3$	0.655	0.648	−1.1
Görtler (1957)			
$\cos(x)$	0.389	0.384	−1.3
$(1 - x)^{1/2}$	0.218	0.221	+1.3
$(1 - x)^2$	0.0637	0.0652	+2.4
$(1 + x)^{-1}$	0.151	0.158	+4.6
$(1 + x)^{-2}$	0.0713	0.0739	+3.6

Let say:

$$U(x) = U = 1 - x^2$$

$$U^5 = 1 - 5x^2 + 10x^4 - 10x^6 + 5x^8 - x^{10}$$

From Thwaites equation:

$$\theta^2 = \frac{0.45\upsilon}{U^6}\int_0^x U^5 dx$$

$$\lambda = \frac{\theta^2}{\upsilon}\frac{dU}{dx}$$

Solve the momentum thickness :

$$\theta^2 = \frac{0.45\upsilon}{U^6}\int_0^x U^5 dx = \frac{0.45\upsilon}{U^6}\int_0^x (1 - 5x^2 + 10x^4 - 10x^6 + 5x^8 - x^{10})dx$$

$$\theta^2 = \frac{0.45\upsilon}{U^6}\left(x - \frac{5}{3}x^3 + 2x^5 - \frac{10}{7}x^7 + \frac{5}{9}x^9 - \frac{x^{11}}{11}\right)$$

Assume that :

$$A = \left(x - \frac{5}{3}x^3 + 2x^5 - \frac{10}{7}x^7 + \frac{5}{9}x^9 - \frac{x^{11}}{11}\right)$$

$$\theta^2 = \frac{0.45\upsilon}{U^6}\left(x - \frac{5}{3}x^3 + 2x^5 - \frac{10}{7}x^7 + \frac{5}{9}x^9 - \frac{x^{11}}{11}\right)\theta^2 = \frac{0.45\upsilon A}{U^6}$$

Solve the lamda :

$$\lambda = \frac{\theta^2}{\upsilon}\frac{dU}{dx} = \frac{0.45\upsilon A}{U^6 \upsilon} \cdot \frac{dU}{dx} = \frac{0.45\upsilon A}{U^6 \upsilon} \cdot \frac{d}{dx}(1 - x^2) = \frac{0.45\upsilon A(-2x)}{U^6 \upsilon} = \frac{-0.9xA}{U^6}$$

We know that separation occur at :

$$\lambda = -0.09$$

Then, we could calculate the value of x .

Substitute the value of A :

$$\lambda = \frac{-0.9x}{U^6}\left(x - \frac{5}{3}x^3 + 2x^5 - \frac{10}{7}x^7 + \frac{5}{9}x^9 - \frac{x^{11}}{11}\right)$$

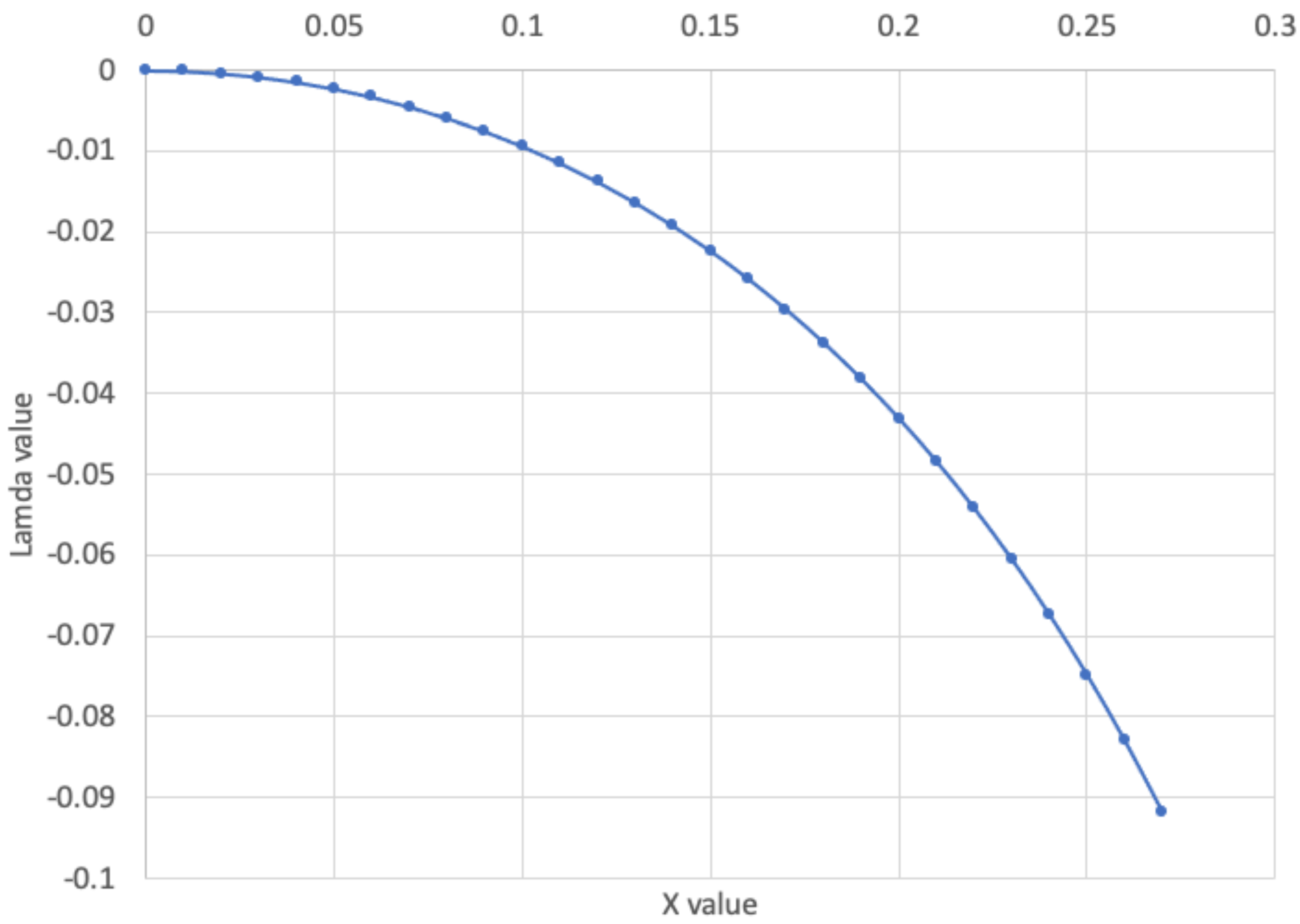
It can be simplified as :

$$\lambda = \frac{-0.9}{(1 - x^2)^6}\left(x^2 - \frac{5}{3}x^4 + 2x^6 - \frac{10}{7}x^8 + \frac{5}{9}x^{10} - \frac{x^{12}}{11}\right)$$

$\lambda = -0.09 \quad \text{at} \quad x = 0.268$

We solve this by using excel :

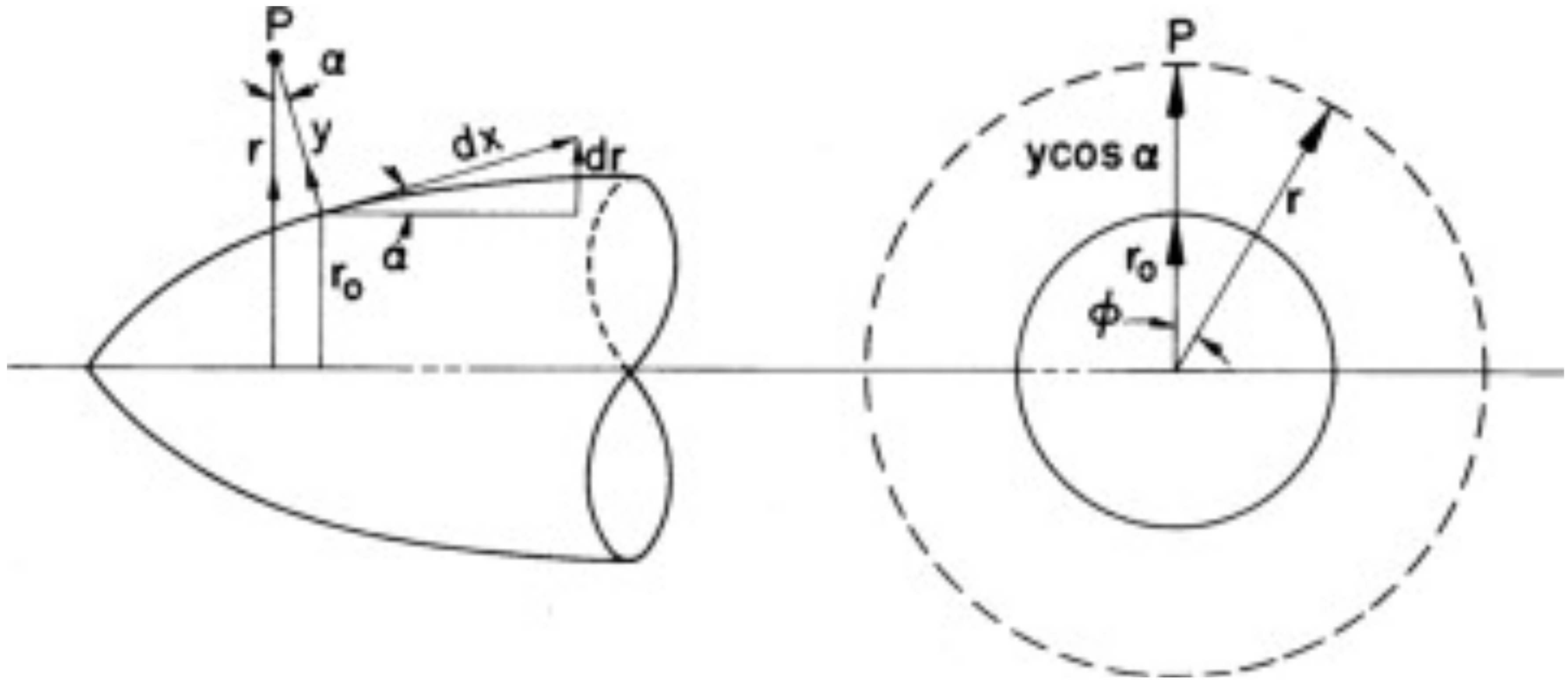
B	C	D	E	F	G	H	I	J	K	L	M	N	O
x	lamda	(1-x^2)^	x^2	x^4	x^6	x^8	x^10	x^12	5//3	2x^6	10//7	5//9	1//11
0	0	1	0	0	0	0	0	0	0	0	0	0	0
0.01	-9.0039E-05	0.999	1E-04	1E-08	1E-12	1E-16	1E-20	1E-24	2E-08	2E-12	1E-16	6E-21	9E-26
0.02	-0.00036062	0.998	4E-04	2E-07	6E-11	3E-14	1E-17	4E-21	3E-07	1E-10	4E-14	6E-18	4E-22
0.03	-0.00081317	0.995	9E-04	8E-07	7E-10	7E-13	6E-16	5E-19	1E-06	1E-09	9E-13	3E-16	5E-20
0.04	-0.00145003	0.99	0.002	3E-06	4E-09	7E-12	1E-14	2E-17	4E-06	8E-09	9E-12	6E-15	2E-18
0.05	-0.00227456	0.985	0.003	6E-06	2E-08	4E-11	1E-13	2E-16	1E-05	3E-08	6E-11	5E-14	2E-17
0.06	-0.00329109	0.979	0.004	1E-05	5E-08	2E-10	6E-13	2E-15	2E-05	9E-08	2E-10	3E-13	2E-16
0.07	-0.00450503	0.971	0.005	2E-05	1E-07	6E-10	3E-12	1E-14	4E-05	2E-07	8E-10	2E-12	1E-15
0.08	-0.00592286	0.962	0.006	4E-05	3E-07	2E-09	1E-11	7E-14	7E-05	5E-07	2E-09	6E-12	6E-15
0.09	-0.00755222	0.952	0.008	7E-05	5E-07	4E-09	3E-11	3E-13	1E-04	1E-06	6E-09	2E-11	3E-14
0.1	-0.00940199	0.941	0.01	1E-04	1E-06	1E-08	1E-10	1E-12	2E-04	2E-06	1E-08	6E-11	9E-14
0.11	-0.01148235	0.93	0.012	1E-04	2E-06	2E-08	3E-10	3E-12	2E-04	4E-06	3E-08	1E-10	3E-13
0.12	-0.0138049	0.917	0.014	2E-04	3E-06	4E-08	6E-10	9E-12	3E-04	6E-06	6E-08	3E-10	8E-13
0.13	-0.01638276	0.903	0.017	3E-04	5E-06	8E-08	1E-09	2E-11	5E-04	1E-05	1E-07	8E-10	2E-12
0.14	-0.01923069	0.888	0.02	4E-04	8E-06	1E-07	3E-09	6E-11	6E-04	2E-05	2E-07	2E-09	5E-12
0.15	-0.02236529	0.872	0.023	5E-04	1E-05	3E-07	6E-09	1E-10	8E-04	2E-05	4E-07	3E-09	1E-11
0.16	-0.02580509	0.856	0.026	7E-04	2E-05	4E-07	1E-08	3E-10	0.001	3E-05	6E-07	6E-09	3E-11
0.17	-0.02957083	0.839	0.029	8E-04	2E-05	7E-07	2E-08	6E-10	0.001	5E-05	1E-06	1E-08	5E-11
0.18	-0.0336856	0.821	0.032	0.001	3E-05	1E-06	4E-08	1E-09	0.002	7E-05	2E-06	2E-08	1E-10
0.19	-0.03817516	0.802	0.036	0.001	5E-05	2E-06	6E-08	2E-09	0.002	9E-05	2E-06	3E-08	2E-10
0.2	-0.04306819	0.783	0.04	0.002	6E-05	3E-06	1E-07	4E-09	0.003	1E-04	4E-06	6E-08	4E-10
0.21	-0.04839661	0.763	0.044	0.002	9E-05	4E-06	2E-07	7E-09	0.003	2E-04	5E-06	9E-08	7E-10
0.22	-0.054196	0.743	0.048	0.002	1E-04	5E-06	3E-07	1E-08	0.004	2E-04	8E-06	1E-07	1E-09
0.23	-0.06050595	0.722	0.053	0.003	1E-04	8E-06	4E-07	2E-08	0.005	3E-04	1E-05	2E-07	2E-09
0.24	-0.06737063	0.701	0.058	0.003	2E-04	1E-05	6E-07	4E-08	0.006	4E-04	2E-05	4E-07	3E-09
0.25	-0.07483926	0.679	0.063	0.004	2E-04	2E-05	1E-06	6E-08	0.007	5E-04	2E-05	5E-07	5E-09
0.26	-0.08296683	0.657	0.068	0.005	3E-04	2E-05	1E-06	1E-07	0.008	6E-04	3E-05	8E-07	9E-09
0.27	-0.09181474	0.635	0.073	0.005	4E-04	3E-05	2E-06	2E-07	0.009	8E-04	4E-05	1E-06	1E-08
0.28	-0.10145171	0.613	0.078	0.006	5E-04	4E-05	3E-06	2E-07	0.01	1E-03	5E-05	2E-06	2E-08
0.29	-0.11195469	0.59	0.084	0.007	6E-04	5E-05	4E-06	4E-07	0.012	0.001	7E-05	2E-06	3E-08
0.3	-0.12341002	0.568	0.09	0.008	7E-04	7E-05	6E-06	5E-07	0.014	0.001	9E-05	3E-06	5E-08



0
0.05
0.1
0.15
0.2
0.25
0.3
0
-0.01
-0.02
-0.03
-0.04
-0.05
-0.06
-0.07
-0.08
-0.09
-0.1
Lamda value
X value

A THWAITES METHOD FOR AXISYMMETRIC FLOW

A flow pattern is said to be axisymmetric when it is identical in every plane that passes through a certain straight-line. The straight-line in question is referred to as the symmetry axis.



Thwaites equation for axisymmetric flow becomes like this :
It is also known as Rott-Crabtree method.

$$\theta^2 = \frac{0.45\upsilon}{r_0^2 U^6} \int_0^x r_0^2 U^5 dx$$

$$\lambda = \frac{\theta^2}{\upsilon} \frac{dU}{dx}$$

EXAMPLE

For potential freestream flow past a sphere, the velocity distribution is given as :

$$U = 1.5U_0 \sin\left(\frac{x}{a}\right)$$

$$r_0 = a \sin\left(\frac{x}{a}\right)$$

$$a = \text{Sphere radius}$$

$$x = \text{Stagnation point}$$

$$U_0 = \text{Inlet velocity of stream}$$

$$\frac{dU}{dx} = \frac{d}{dx}(U) = \frac{d}{dx}\left(1.5U_0 \sin\left(\frac{x}{a}\right)\right) = \frac{1.5U_0}{a}\cos\left(\frac{x}{a}\right)$$

$$\theta^2 = \frac{0.45\upsilon}{r_0^2 U^6}\int_0^x r_0^2 U^5 dx$$

$\upsilon =$ Kinematic viscosity of the flowing fluid

$$\frac{\theta^2}{\upsilon} = \frac{0.45}{r_0^2 U^6}\int_0^x r_0^2 U^5 dx = \frac{0.45}{\left(a \sin\left(\frac{x}{a}\right)\right)^2 \left(1.5U_0 \sin\left(\frac{x}{a}\right)\right)^6}\int_0^x \left(a \sin\left(\frac{x}{a}\right)\right)^2 \left(1.5U_0 \sin\left(\frac{x}{a}\right)\right)^5 dx$$

$$\frac{\theta^2}{\upsilon} = \frac{0.45}{1.5U_0 \sin\left(\frac{x}{a}\right)^8}\int_0^x \sin\left(\frac{x}{a}\right)^7 dx$$

Determine the lamda :

$$\frac{dU}{dx} = \frac{1.5U_0}{a} \cdot \cos\left(\frac{x}{a}\right)$$

$$\lambda = \frac{\theta^2}{\upsilon}\frac{dU}{dx} = \frac{0.45}{1.5U_0 \sin\left(\frac{x}{a}\right)^8}\int_0^x \sin\left(\frac{x}{a}\right)^7 dx \quad \times \quad \frac{1.5U_0}{a} \cdot \cos\left(\frac{x}{a}\right)$$

$$\lambda = \frac{0.45 \cos\left(\frac{x}{a}\right)}{a \sin\left(\frac{x}{a}\right)^8}\int_0^x \sin\left(\frac{x}{a}\right)^7 dx$$

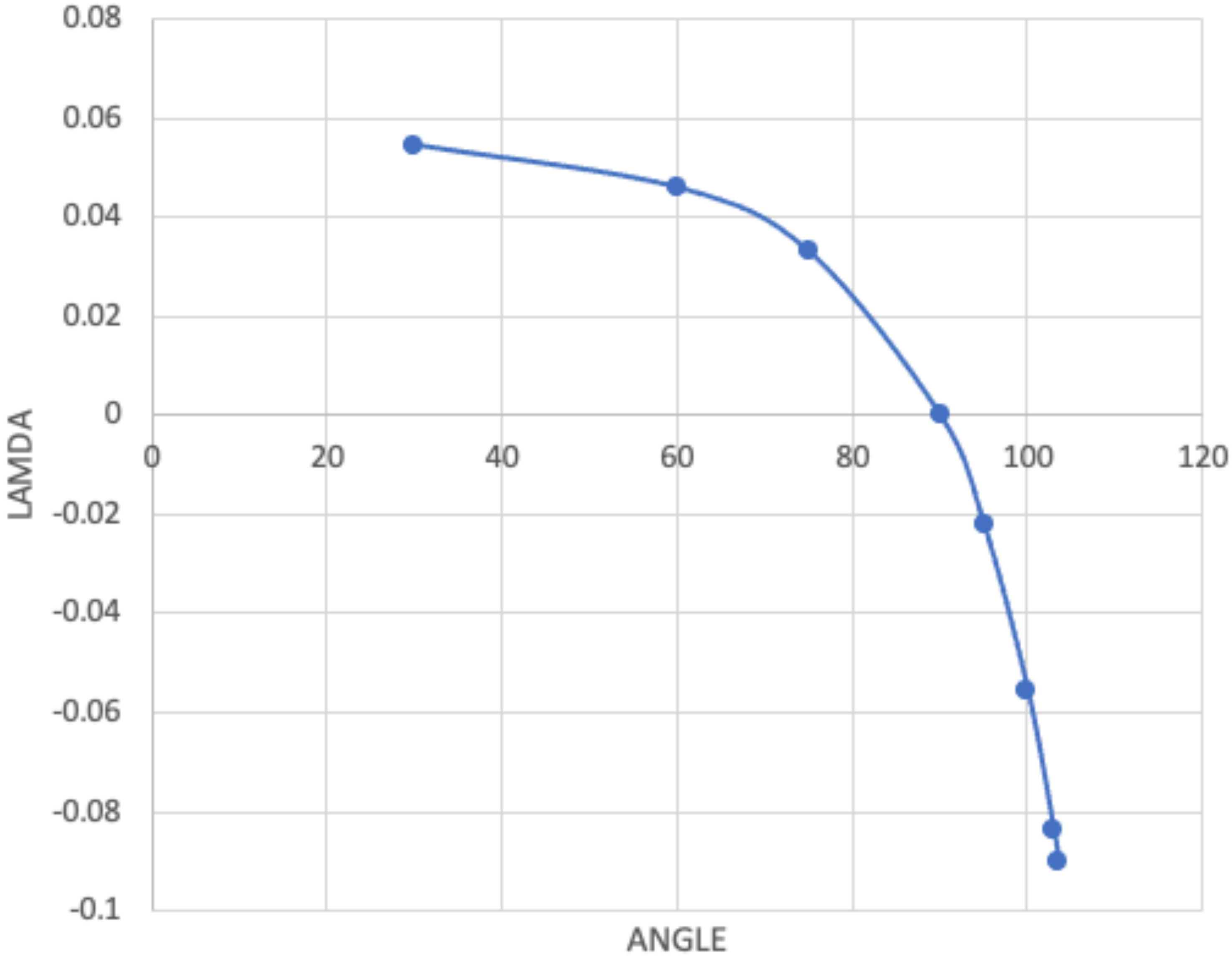
$$\lambda = \frac{0.13 \cos\left(\frac{x}{a}\right)}{\sin\left(\frac{x}{a}\right)^8}\left\{\left[\cos\left(\frac{x}{a}\right)\right]\left[5\left(\cos\left(\frac{x}{a}\right)\right)^6 - 21\left(\cos\left(\frac{x}{a}\right)\right)^4 + 35\left(\cos\left(\frac{x}{a}\right)\right)^2 - 35\right] + 16\right\}$$

Assume $\frac{x}{a} = X$

$$\lambda = \frac{0.13 \cos(X)}{\sin(X)^8}\{[\cos(X)][5(\cos(X))^6 - 21(\cos(X))^4 + 35(\cos(X))^2 - 35] + 16\}$$

Make a graph

X	LAMDA
30	0.0545
60	0.0459
75	0.033
90	0
95	-0.022
100	-0.0553
103	-0.0836
103.6	-0.09


0.08
0.06
0.04
0.02
0
-0.02
-0.04
-0.06
-0.08
-0.1
0
20
40
60
80
100
120
LAMDA
ANGLE

THE PLANE LAMINAR WAKE
FAR-FIELD APPROXIMATION

A wake is the defect is stream velocity behind an immersed body in a flow, a shown below. A slender plane body with zero lift, such as the airfoil parallel to the stream, usually produces a smooth wake whose velocity defect u_1 decays monotonically downstream.

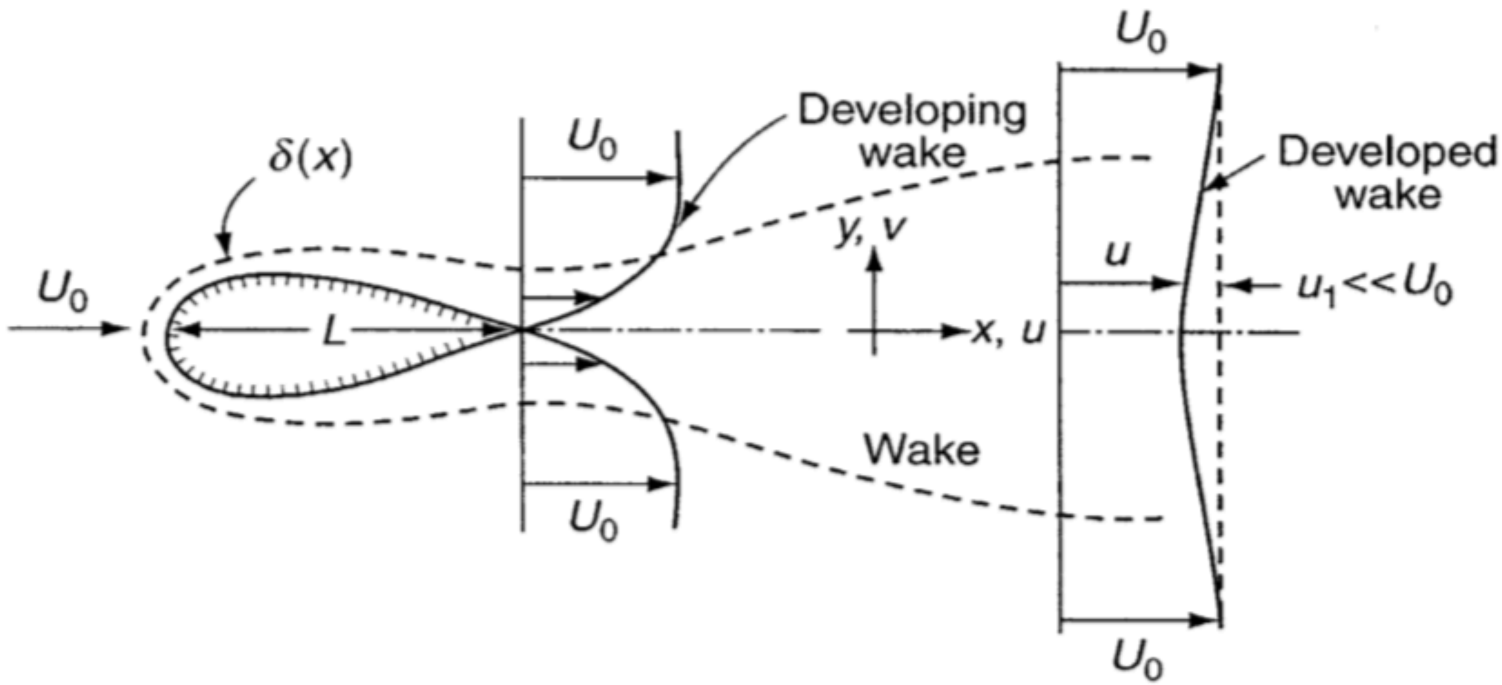
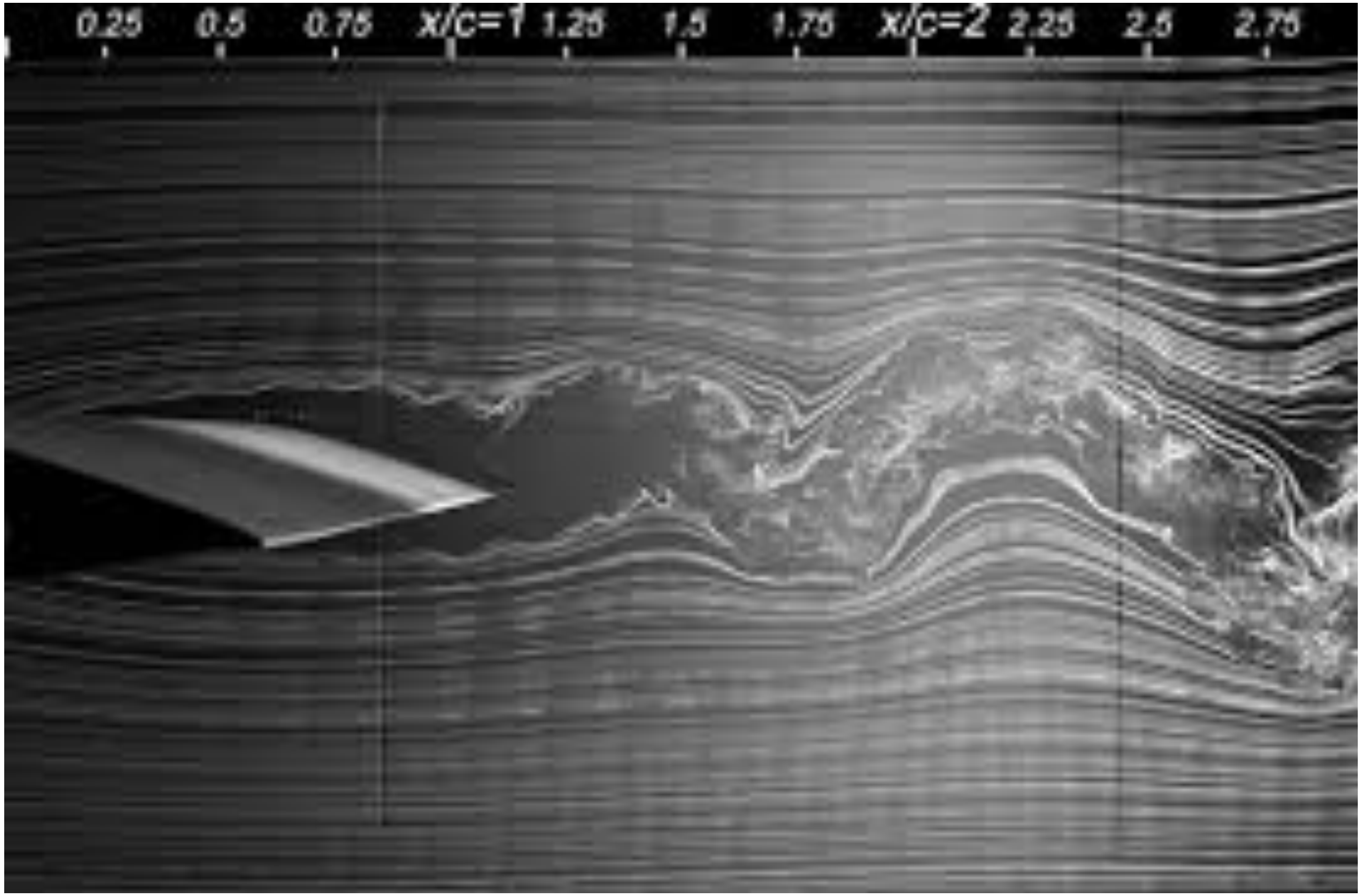


FIGURE 4-19
Flow in the wake of a body immersed in a stream.


0.25
0.5
0.75
x/c=1
1.25
1.5
1.75
x/c=2
2.25
2.5
2.75

Drag force can be written as:

$$F_D = C_D \cdot \frac{1}{2}\rho A U^2$$

The wake velocity may be written as:

$$\frac{u_1}{U_0} = C_D \left(\frac{Re_L}{16\pi}\right)^{1/2} \left(\frac{L}{x}\right)^{1/2} \exp\left(-\frac{U_0 y^2}{4x\upsilon}\right)$$

u_1 = Defect velocity
U_0 = Free stream velocity
υ = Kinematic viscosity of fluid

4-18

Air at 20°C and 1 atm flows at 1 m/s past a slender two-dimensional body, of length L=30cm, whose drag coefficient is 0.05 based on "plan" area bL. Assuming laminar flow at a point 3 m downstream of the trailing edge, estimate:

(a) The maximum wake velocity defect
(b) The "one-percent" wake thickness
(c) The wake thickness Reynolds number

At 20°C: $\rho = 1.205 \text{ kg/m}^3$, $\mu = 1.81 \times 10^{-5} \text{ kg/m} \cdot \text{s}$

(a) Body length Re number, Re_L

$$Re_L = \frac{\rho u L}{\mu} = 20{,}000$$

For $x = 3$ m, the centerline wake defect velocity is computed by applying Eq (4-112), at $y = 0$

$$\frac{u_1}{U_0} = C_D \left(\frac{Re_L}{16\pi}\right)^{1/2} \left(\frac{L}{x}\right)^{1/2} \exp\left(-\frac{U_0 y^2}{4xv}\right) = 0.315$$

$$u_1 = (1)(0.315)$$

$$u_1(y = 0) = 0.315 \text{ m/s} = \Delta u_{max}$$

(b) Wake half-thickness could be defined as the point where the defect velocity drops to 1% of its maximum velocity.

From Gaussian profile:

$$\exp\left(-\frac{Uy^2}{4\upsilon x}\right) = 0.01$$

$$-(y)^2{}_{1\%} = \frac{\ln(0.01)\, 4\upsilon x}{U} = \frac{\ln(0.01)\, 4x\mu}{U\rho}$$

$$(y)_{1\%} = 0.0288\ (\mathrm{m})$$

$$\text{wake "half} - \text{thickness"} = \frac{(y)_{1\%}}{2}$$

$$= 0.0144\ (\mathrm{m})$$

$$\text{width}, b = 2(y)_{1\%} = 0.0288\ (\mathrm{m})$$

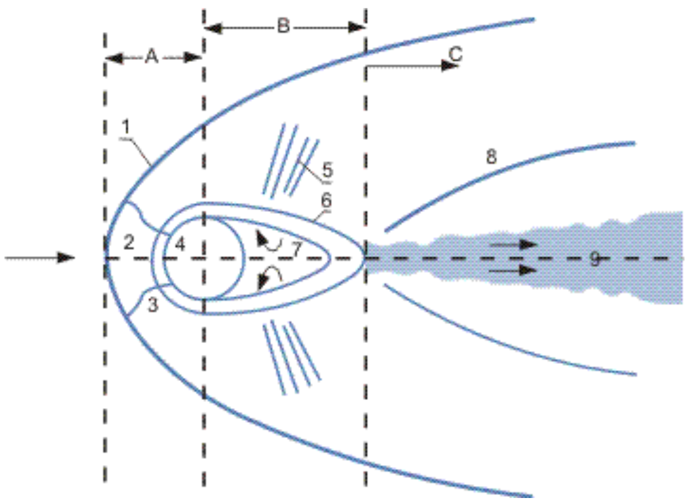
(c) Wake Reynolds number

$$Re = \frac{\rho \Delta u_{max} b}{\mu} = 604$$

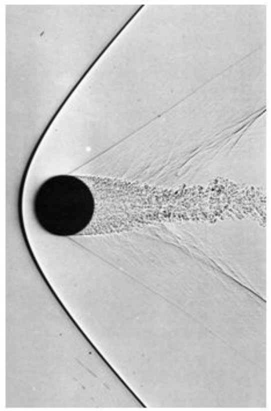
THE PLANE LAMINAR WAKE
AXISYMMETRIC WAKES

The wake defect velocity can be estimated by:

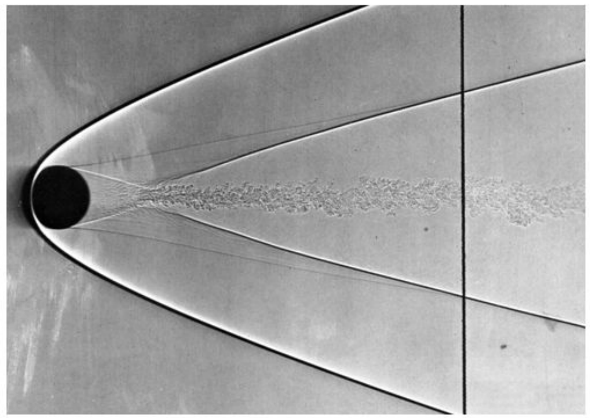
$$\frac{u_1}{U_0} = C_D\left(\frac{U_0 L}{8\pi\upsilon}\right)\left(\frac{L}{x}\right)\exp\left(-\frac{U_0 r^2}{4x\upsilon}\right)$$



Flow Past a Sphere



Mach = 1.53



Mach = 4.01

Pictures are from *"An Album of Fluid Motion"* by Van Dyke

4-41

Air at 20°C and 1 atm flows at 1m/s past a slender body of revolution, of length L=15cm, whose drag coefficient is 0.008 based on area (L^2). Assuming laminar flow at point 3m downstream of the trailing edge, estimate:

(a) The maximum wake velocity defect

(b) The one percent wake thickness

(c) The wake thickness Reynolds number

$$\rho = 1.205 \text{ kg/m}^3$$
$$\mu = 1.81 \times 10^{-5} \text{ kg/ms}$$
$$\upsilon = 1.5 \times 10^{-5} \text{ m}^2\text{/s}$$

(a) Eq (4-211)

$$\frac{u_1}{U_0} = C_D \left(\frac{U_0 L}{8\pi\upsilon}\right)\left(\frac{L}{x}\right) \exp\left(-\frac{U_0 r^2}{4x\upsilon}\right)$$

$$u_1 = max \text{ apabila } exp = 1$$

$$u_{max} = \frac{C_D {U_0}^2 L^2}{8\pi\upsilon x} = 15.9 \text{ cm/s}$$

(b) 1% thickness occur when the Gaussian profile in Eq 4-210 equal 0.01

$$\exp\left(-\frac{U_0 r^2}{4xv}\right) = 0.01$$

$$r = (r)_{1\%} = 0.0288 \text{ (m)}$$

Wake thickness

$$b = 2(r)_{1\%} = 5.76 \text{ (cm)}$$

(c)

$$Re = \frac{u_{max} b}{v} = 610$$


Photo by: Bel Air Aviation Denmark - Helicopter Services