BOUNDARY LAYER

TURBULENT BOUNDARY LAYER
POWER-LAW FORM

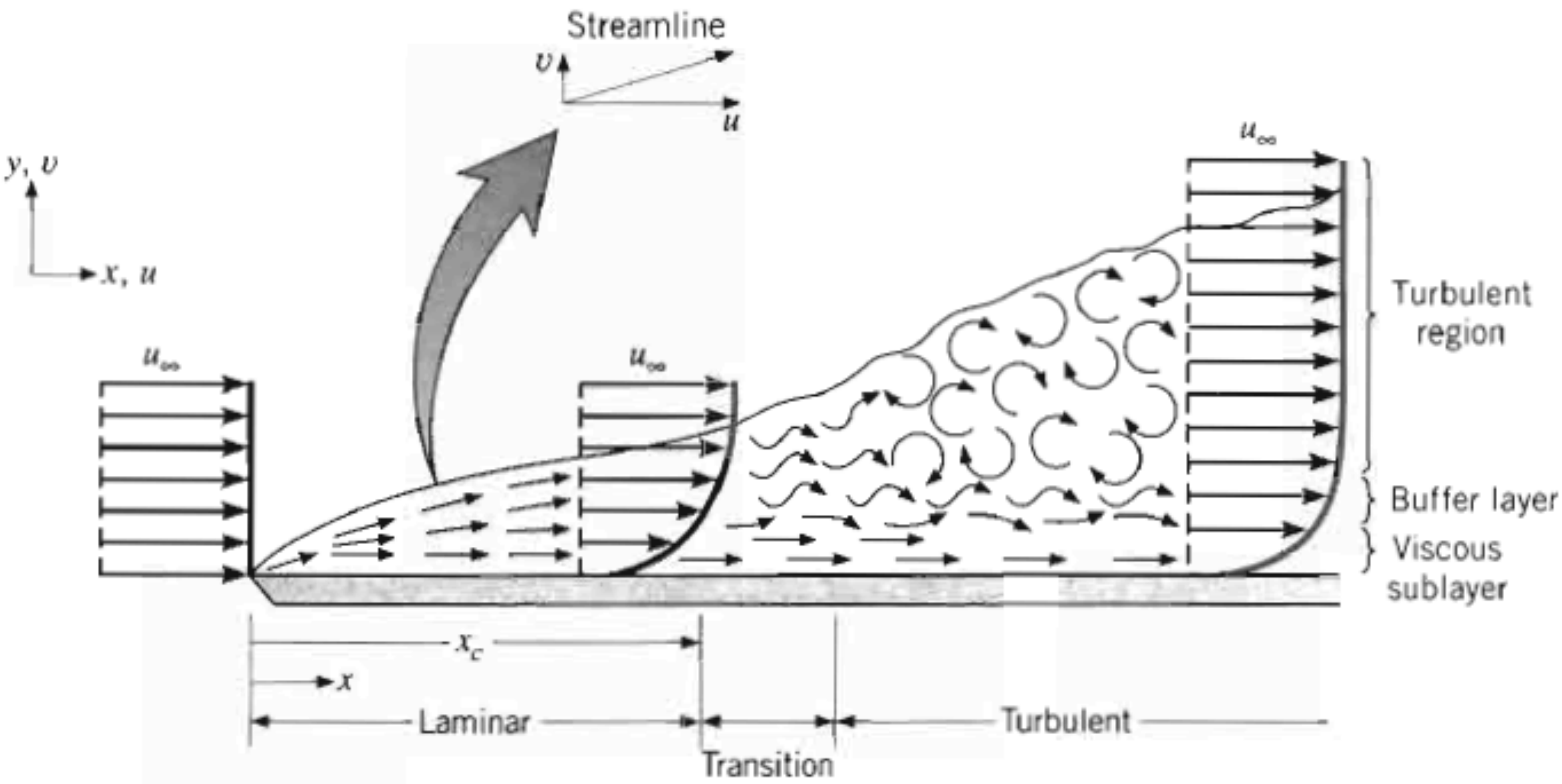
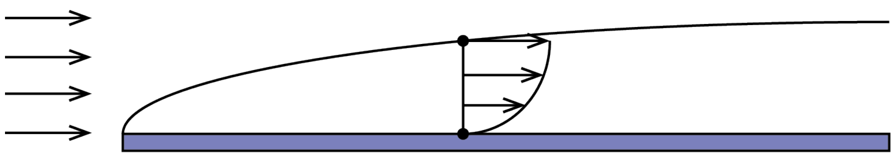
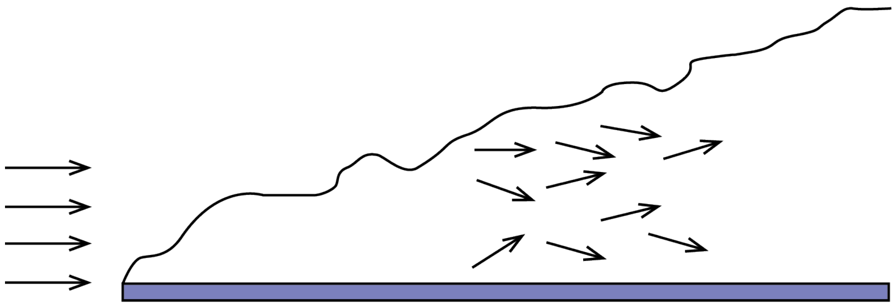


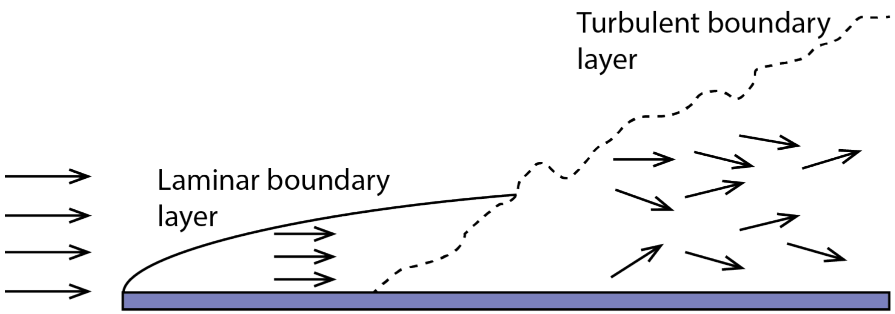
FIGURE 6.6 Velocity boundary layer development on a flat plate.



(Fully) Laminar boundary layer



(Fully) Turbulent boundary layer



Laminar & Turbulent boundary layer

For laminar boundary layer, the velocity profiles are:

$$\frac{u}{U} = \frac{y}{\delta}$$

$$\frac{u}{U} = 2\left(\frac{y}{\delta}\right) - \left(\frac{y}{\delta}\right)^2$$

$$\frac{u}{U} = \frac{3}{2}\left(\frac{y}{\delta}\right) - \frac{1}{2}\left(\frac{y}{\delta}\right)^3$$

$$\frac{u}{U} = 2\left(\frac{y}{\delta}\right) - 2\left(\frac{y}{\delta}\right)^3 + \left(\frac{y}{\delta}\right)^4$$

$$\frac{u}{U} = \sin\left(\frac{\pi}{2}\frac{y}{\delta}\right)$$

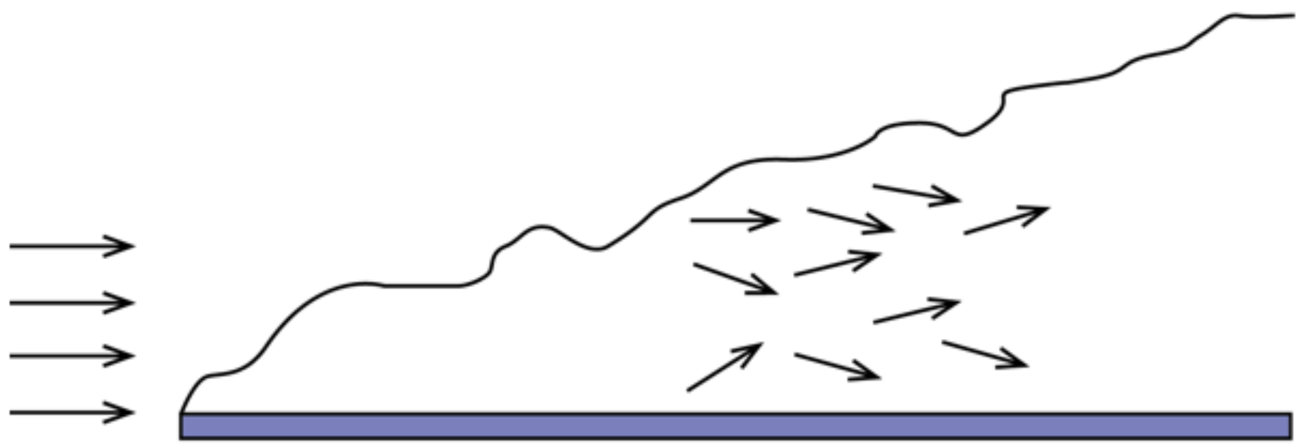
Basic assumption: All region is laminar boundary layer.

TURBULENT BOUNDARY LAYER

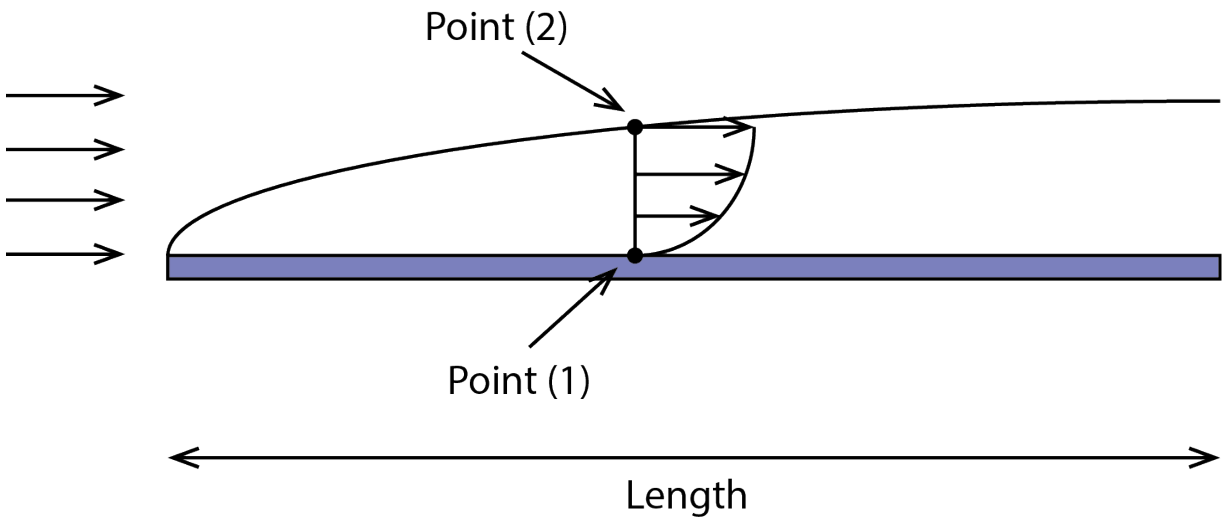
For turbulent boundary layer flow, we have 2 methods for obtaining the information desired.

Method #1:
Power-law form.
Easier.
Less accurate compared to method 2.

Method #2:
Empirical form.
More complicated.
More accurate compared to method 1.



(Fully) Turbulent boundary layer



$$Re = \frac{\rho U x}{\mu} = \frac{\rho U L}{\mu}$$

ρ = Density of moving fluid
U = Free stream velocity
x = Length of flat plate
μ = Dynamic viscosity of fluid

TURBULENT BOUNDARY LAYER: Power-law form

In this method, we fit the data for the velocity profile with a power-law equation.

The power-law form is:

$$\frac{u}{U} = \left(\frac{y}{\delta}\right)^{\frac{1}{n}}$$

$n = 7 \quad for \quad 5 \times 10^5 < Re < 10^7$

$n = 8 \quad for \quad 10^7 < Re < 10^8$

$n = 9 \quad for \quad 10^8 < Re < 10^9$

The power-law velocity profile will give very poor result when solving using Newton's law of viscosity.
Thus,

$$\tau = \left(\frac{du}{dy}\right)_{y=0} \text{ : cannot be used for turbulent BL}$$

$$\text{For} \quad \frac{u}{U} = \left(\frac{y}{\delta}\right)^{\frac{1}{7}} \quad \Longrightarrow \quad \frac{du}{dy} = \frac{U}{\delta^{\frac{1}{7}}} \cdot \frac{1}{7} \cdot y^{-\frac{6}{7}}$$

$$At \quad y = 0\,, \qquad \frac{du}{dy} = 0$$

This will ruin the calculation of shear stress.

We need to use an empirical relation introduced by Blasius formula.

Common velocity profile for turbulent is one-seventh power law:

$$\frac{u}{U} = \left(\frac{y}{\delta}\right)^{\frac{1}{7}}$$

the shear stress relation is ;

$$\tau = 0.0226\rho U^2 \left(\frac{\mu}{\rho U \delta}\right)^{\frac{1}{4}}$$

This shear stress relation will be slightly different when velocity profile changed. This relation will be given in the question. In real case, it must be calculated using empirical method.

Boundary layer thickness, δ :

Known that :

$$\theta = \int_0^\delta \frac{u}{U}\left(1 - \frac{u}{U}\right) dy = \frac{7}{72}\delta$$

Substitute in Von Karman equation :

$$\tau = \rho U^2 \frac{d\theta}{dx} = \rho U^2 \frac{d}{dx}\left(\frac{7}{72}\delta\right)$$

$$\tau = \rho U^2 \frac{d\delta}{dx}\left(\frac{7}{72}\right) \qquad \text{......... Eq.(1)}$$

Then, if we solved using the Newton's law of viscosity

$$\tau = \left(\frac{du}{dy}\right)_{y=0} = 0$$

It produces zero answers. Calculation cannot be continued. The value of shear stress cannot be zero because no slip condition occurs. Therefore, the value of shear stress must be obtained from the results of Blasius solution.

Value of shear stress from Blasius calculation is :

$$\tau = 0.0226\rho U^2 \left(\frac{\mu}{\rho U \delta}\right)^{\frac{1}{4}}$$

$$\rho U^2 \frac{d\delta}{dx}\left(\frac{7}{72}\right) = 0.0226\rho U^2 \left(\frac{\mu}{\rho U \delta}\right)^{\frac{1}{4}}$$

$$\frac{d\delta}{dx}\left(\frac{7}{72}\right) = 0.0226 \left(\frac{\mu}{\rho U}\right)^{\frac{1}{4}} \left(\frac{1}{\delta}\right)^{\frac{1}{4}}$$

$$\delta^{\frac{1}{4}} \cdot d\delta = \left(\frac{72}{7}\right) 0.0226 \left(\frac{\mu}{\rho U}\right)^{\frac{1}{4}} \cdot dx$$

Integrating both sides :

$$\int \delta^{\frac{1}{4}} \cdot d\delta = \int \left(\frac{72}{7}\right) 0.0226 \left(\frac{\mu}{\rho U}\right)^{\frac{1}{4}} \cdot dx$$

$$\frac{4}{5}\delta^{\frac{5}{4}} = \left(\frac{72}{7}\right)(0.0226)\left(\frac{\mu}{\rho U}\right)^{\frac{1}{4}} x + C$$

At $x = 0,\ \delta = 0 \quad \Rightarrow \quad C = 0$

$$\frac{4}{5}\delta^{\frac{5}{4}} = \left(\frac{72}{7}\right)(0.0226)\left(\frac{\mu}{\rho U}\right)^{\frac{1}{4}} x$$

$$\delta^{\frac{5}{4}} = \left(\frac{5}{4}\right)\left(\frac{72}{7}\right)(0.0226)\left(\frac{\mu}{\rho U}\right)^{\frac{1}{4}} x = (0.2906)\left(\frac{\mu}{\rho U}\right)^{\frac{1}{4}} x$$

$$\delta = (0.3721)\left(\frac{\mu}{\rho U}\right)^{\frac{1}{5}} x^{\frac{4}{5}}$$

$$\delta = \frac{0.3721x}{(Re)^{\frac{1}{5}}}$$

Displacement thickness, δ^* :

Known that :

$$\frac{u}{U} = \left(\frac{y}{\delta}\right)^{\frac{1}{7}}$$

$$\delta^* = \int_0^{\delta} \left(1 - \frac{u}{U}\right) dy = \frac{1}{8}\delta$$

Substitute the value :

$$\delta = \frac{0.3721x}{(Re)^{\frac{1}{5}}}$$

$$\delta^* = \frac{0.0465x}{(Re)^{\frac{1}{5}}}$$

Momentum thickness, θ :

Known that :

$$\theta = \int_0^{\delta} \frac{u}{U}\left(1 - \frac{u}{U}\right) dy = \frac{7}{72}\delta$$

Substitute the value :

$$\delta = \frac{0.3721x}{(Re)^{\frac{1}{5}}}$$

$$\theta = \frac{0.036x}{(Re)^{\frac{1}{5}}}$$

Shear stress, τ :

Known that :

$$\tau = 0.0226\rho U^2 \left(\frac{\mu}{\rho U \delta}\right)^{\frac{1}{4}}$$

Substitute the value :

$$\delta = \frac{0.3721x}{(Re)^{\frac{1}{5}}}$$

$$\tau = \frac{0.0289\rho U^2}{(Re)^{\frac{1}{5}}}$$

Drag force, F_D :

Known that : $$F_D = \int_0^L \tau \cdot B \cdot dx$$

Substitute the value :

$$\tau = \frac{0.0289\rho U^2}{(Re)^{\frac{1}{5}}}$$

Solve the integration :

$$F_D = \frac{0.0361\rho U^2 \cdot B \cdot L}{(Re)^{\frac{1}{5}}}$$

Drag coefficient, C_D :

Known that :

$$C_D = \frac{F_D}{\frac{1}{2}\rho U^2 \cdot B \cdot L} = \frac{\frac{0.0361\rho U^2 \cdot B \cdot L}{(Re)^{\frac{1}{5}}}}{\frac{1}{2}\rho U^2 \cdot B \cdot L}$$

$$C_D = \frac{0.0722}{(Re)^{\frac{1}{5}}}$$