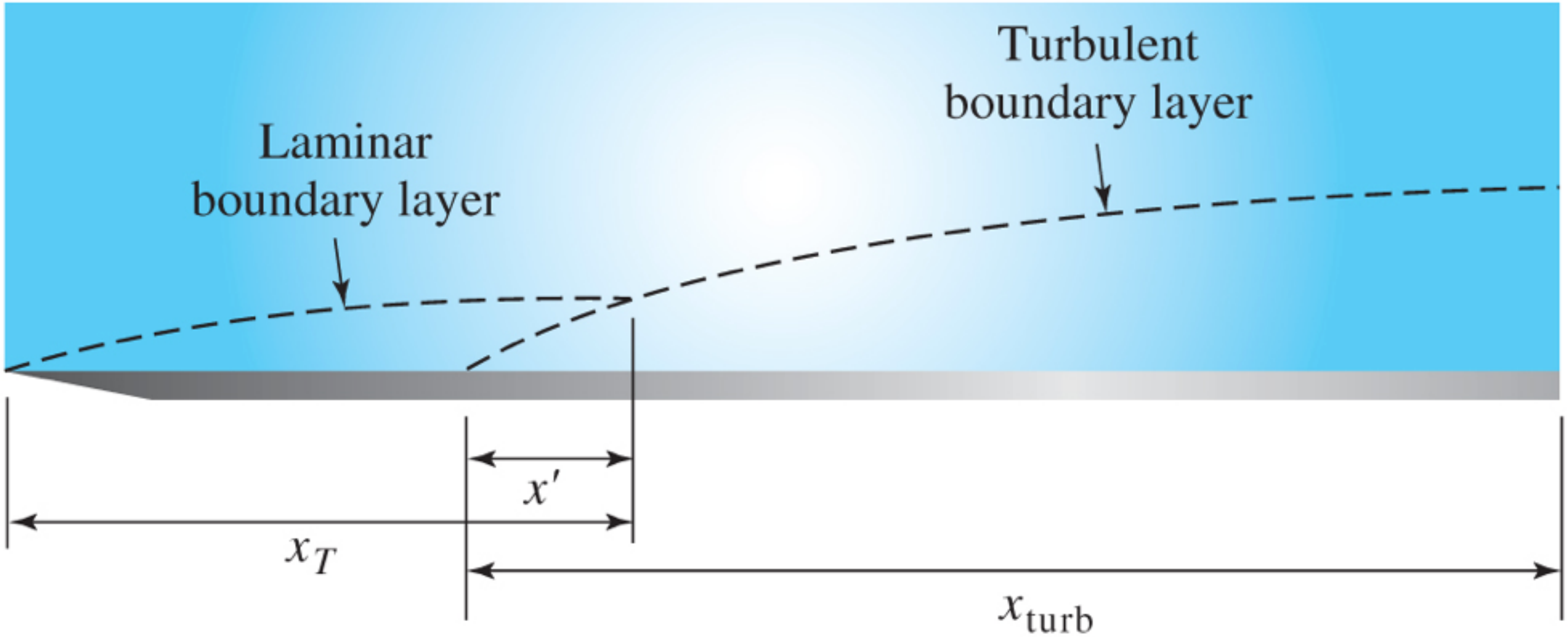
BOUNDARY LAYER

LAMINAR & TURBULENT BOUNDARY LAYER


Turbulent
boundary layer
Laminar
boundary layer
x'
x_T
x_{turb}

EXAMPLE

Estimate the boundary layer thickness at the end of a 4-m-long flat surface if the air free stream velocity
$U = 5m/s$. Predict the drag force if the surface is 5m wide.

(a) Neglect the laminar boundary layer
(b) Account the laminar boundary layer using $Re_{Critical} = 5 \times 10^5$

In this case, the velocity profile for laminar and turbulent boundary layer is not mentioned. To simplify this calculation, we could take:

Blasius exact solution as representative for laminar boundary layer

and

“one-seventh power law” as representative for turbulent boundary layer.

(a) Neglect the laminar boundary layer.

Reynolds number at the edge of the plate:

$$Re = \frac{\rho U x}{\mu} = \frac{U x}{\upsilon} = \frac{5 \times 4}{1.6 \times 10^{-5}} = 1.25 \times 10^{6}$$

$$\upsilon = \text{kinematic viscosity of air}$$

$Re > 5 \times 10^{5}$, (fully) turbulent boundary layer is occurred.

BL thickness, δ :

$$\delta = \frac{0.38x}{(Re)^{\frac{1}{5}}} = 0.0917\ (m)$$

$$x = \text{Length of flat plate} = 4\ \text{m}$$

Drag force, F_D

$$F_D = C_D \cdot \frac{1}{2}\rho A U^2 = 1.28\ (N)$$

$$C_D = \frac{0.073}{(Re)^{\frac{1}{5}}}$$

(b) Laminar boundary layer is accounted

BL thickness, δ :

Determine the length of laminar boundary layer:

$$Re_{Critical} = 5 \times 10^5 = \frac{\rho U x}{\mu} = \frac{U x}{\upsilon}$$

$$x = 1.6\ (m)$$

Laminar boundary layer was occurred for 1.6 m from total plate length of 4 m. It is 40% from the whole plate length. This value is too large to be neglected. It needs to be accounted in the calculation.

First, we need to determine the thickness of the boundary layer at the end of laminar boundary layer.

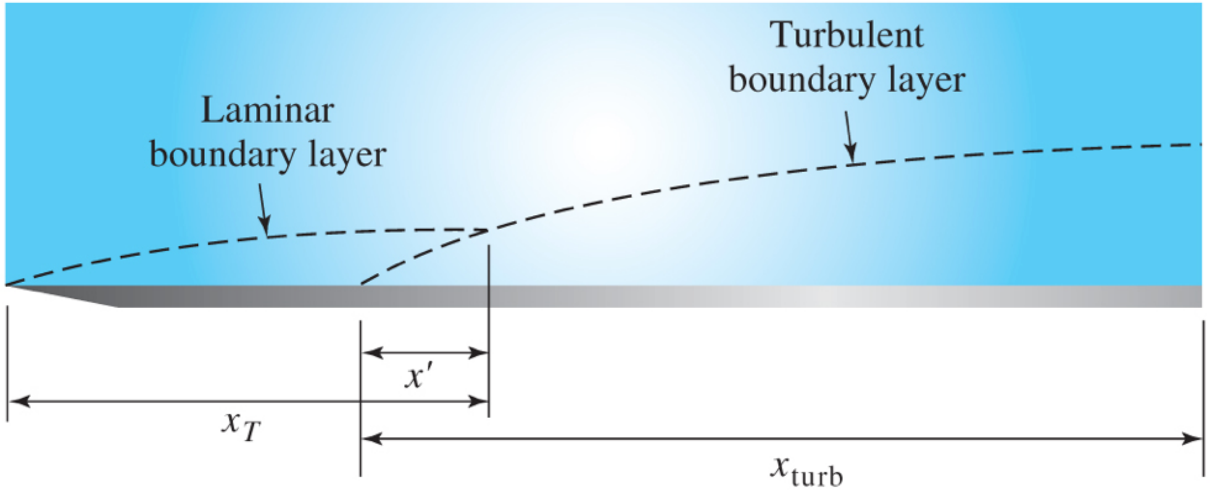
From Blasius equation:

$$\delta = \frac{5x}{\sqrt{Re}} = 0.0113\ (m)$$

From "one-seventh" power law, turbulent boundary layer thickness is:

$$\delta = \frac{0.38x}{(Re)^{\frac{1}{5}}}$$

Substitute value $\delta = 0.0113\ (m)$ in the turbulent boundary layer thickness, we could get $x = 0.292\ (m)$



From figure, we could predict the starting point of turbulent boundary layer by extending $x = 0.292\ (m)$ to the left from the meeting point of laminar and turbulent boundary layer.

It is shows as x' in the figure above.

Equation can be written as:

$$x_{turbulent} = \left(x_{flat\ plate}\right) - \left(x_{Transition}\right) + \left(x'\right)$$

$$x_{turb} = 4 - 1.6 + 0.292 = 2.69\ (m)$$

It is means that, to calculate the turbulent boundary layer by using turbulent boundary layer calculation method, we only need to consider plate length of 2.69 m only.

It is because, in laminar or turbulent theoretical, we develop their equations by assuming boundary layer starts at $x = 0,\ \ \delta = 0$.

Substitute the value of $x_{turb} = 2.69\ (m)$ in the turbulent boundary layer thickness, we could find the answer.

$$Re = \frac{\rho U x}{\mu} = \frac{Ux}{\upsilon} = \frac{5 \times 2.69}{1.6 \times 10^{-5}} = 840{,}625$$

$$\delta = \frac{0.38x}{(Re)^{\frac{1}{5}}} = \frac{0.38(2.69)}{(840{,}625)^{\frac{1}{5}}} = 0.0668\ (m)$$

This value is more accurate compare to the first calculation.

If laminar boundary layer is neglected, the boundary layer thickness at the end of the plate is 0.0917 m. It is 37% higher compare to the second calculation.

Drag force, F_D

To calculate the drag force, we cannot use drag force equation for turbulent boundary layer alone. We must use specific value of drag coefficient, C_D. It because the laminar boundary was influenced the formation of boundary layer.

In this case, we use C_D as shown below. In this calculation, we need to use the

$$Re = \frac{\rho U x}{\mu} = \frac{U x}{\upsilon} = \frac{5 \times 4}{1.6 \times 10^{-5}} = 1.25 \times 10^{6}$$

$$C_D = \frac{0.073}{(Re)^{\frac{1}{5}}} - \frac{1700}{(Re)} = \frac{0.073}{(1.25 \times 10^{6})^{\frac{1}{5}}} - \frac{1700}{(1.25 \times 10^{6})} = 0.00304$$

$$F_D = C_D \cdot \frac{1}{2} \rho A U^2 = (0.00304) \cdot \frac{1}{2}(1.164)(4 \times 5)(5^2) = 0.885\ (N)$$

The guideline to choose the value of C_D

$Re_{Critical} = 3 \times 10^5$	$C_D = \frac{0.073}{(Re)^{\frac{1}{5}}} - \frac{1060}{(Re)}$
$Re_{Critical} = 5 \times 10^5$	$C_D = \frac{0.073}{(Re)^{\frac{1}{5}}} - \frac{1700}{(Re)}$
$Re_{Critical} = 6 \times 10^5$	$C_D = \frac{0.073}{(Re)^{\frac{1}{5}}} - \frac{2080}{(Re)}$