CHAPTER 6
TUTORIAL FOR TURBULENT BOUNDARY LAYER

QUESTION 1

A timber is pulled by a boat along the river with force 200N as shown in Figure 1. Only half of the timber above the water. Determine the drag force due to front surface of the timber.

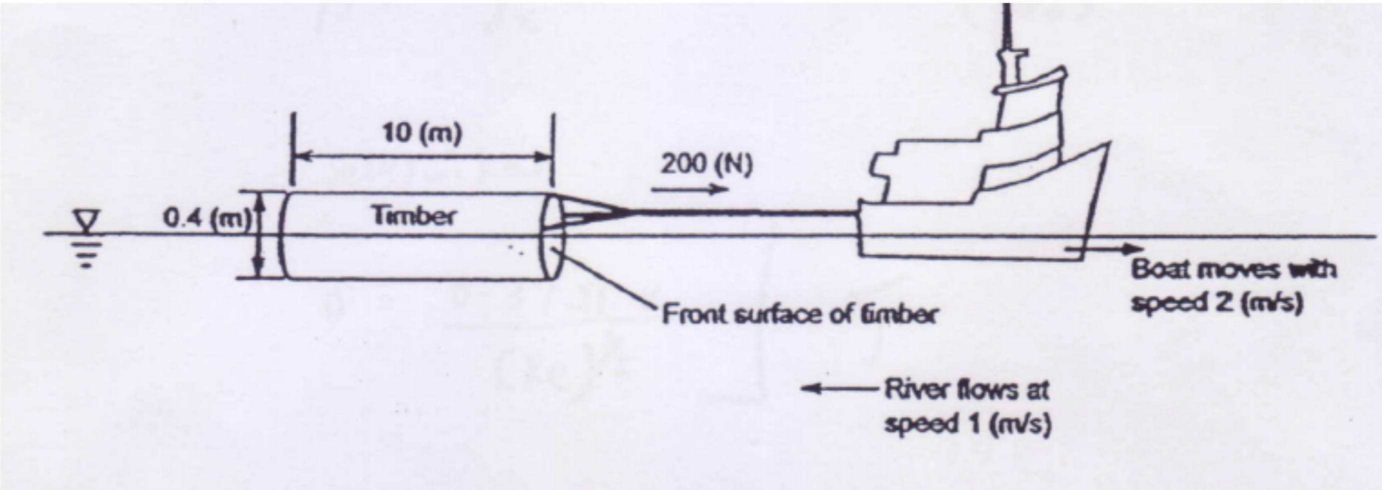


Figure Q2

Know that: $\rho_{water} = 1000\ (kg/m^3)$, $\mu_{water} = 1 \times 10^{-3}\ (N.s/m^2)$

Assume that laminar boundary layer is fulfilled the Blasius assumption and turbulent boundary layer is fulfilled the one-seventh power law.

Laminar BL : $$C_{D(laminar)} = \frac{1.328}{\sqrt{Re}}$$

Turbulent BL : $$C_D = \frac{0.072}{Re^{1/5}}$$

Reynold number at the end of timber:

$$Re = \frac{\rho U x}{\mu} = \frac{(1000)(2+1)(10)}{1 \times 10^{-3}} = 3 \times 10^7 \text{ (turbulent)}$$

Length of laminar BL:

$$Re_L = 5 \times 10^5 = \frac{\rho U x}{\mu}$$

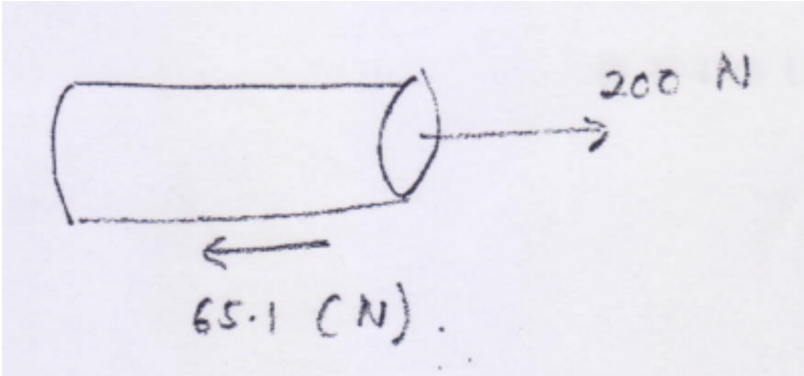
$$x = \frac{(5 \times 10^5)(1 \times 10^{-3})}{(1000)(2+1)} = 0.17\ (m)$$

Length of laminar BL is less than 10% from total length of timber
Laminar BL can be ignored.

Drag force cause by side surface

$$A = \frac{1}{2} \times 2\pi r \times L = 6.286\ m^2$$

$$F_D = C_D \frac{1}{2}\rho U^2 A = \frac{0.072}{Re^{1/5}} \times \frac{1}{2}\rho U^2 (6.286) = 65.1\ (N)$$



Total pulling force = Drag force at side + Drag force at front surface

$$200 = 65.1 + x$$

$$x = 134.9\ N$$

QUESTION 2

Figure 2 shows a flat plate 3 m × 1.5 m is held in water moving at 5 m/s parallel to its length. Determine the power required to overcome the drag.

Take $\rho = 1000\ kg/\ m^3$ and $\mu = 0.001\ Ns/\ m^2$

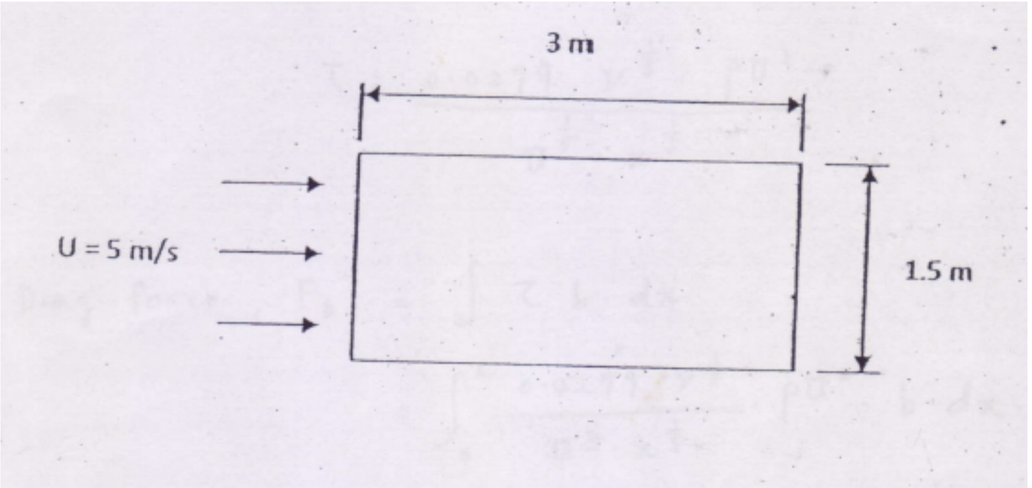


FIGURE Q2

Reynolds number:

$$Re = \frac{\rho U x}{\mu} = \frac{(1000)(5)(3)}{0.001} = 1.5 \times 10^7 = \text{Turbulent}$$

Length of x_{cr}

$$5 \times 10^5 = \frac{\rho U x_{cr}}{\mu}$$

$$x_L = \frac{5 \times 10^5 \mu}{\rho U} = \frac{(5 \times 10^5)(0.001)}{(1000)(5)} = 0.1\, m$$

Laminar BL is only 3.3% from the total plate length. The effect of laminar BL can be ignored.

Drag coefficient for turbulent BL is : $C_D = \frac{0.074}{(Re)^{1/5}} = \frac{0.074}{(1.5 \times 10^7)^{1/5}} = 2.71 \times 10^{-3}$

Drag force : $F_D = C_D \frac{1}{2} \rho U^2 A = 152.4\ (N)$

Total drag force : $F_D = 152.4\ \times\ 2 = 304.8\ (N)$

Power : $\text{Power} = \text{Drag}\ \times\ \text{Velocity} = 304.8\ \times\ 5 = 1524\ (\text{Watt})$

Question 3

Water at 20°C and 1 atm flows at 2 m/s past a smooth flat plate 1 m long and 60 cm wide.

Estimate:
(a) The trailing edge boundary layer thickness
(b) The trailing edge wall shear stress
(c) The drag force of one side of the plate.

Fluid data :

$$\rho = 1000\ (kg/m^3)$$
$$\mu = 0.001\ (kg/m \cdot s)$$

Reynolds number :

$$Re_L = \frac{\rho U L}{\mu} = 2 \times 10^6 \quad (Turbulent)$$

Reynolds transition :

$$Re_{tr} = 5 \times 10^5 = \frac{\rho U L}{\mu}$$
$$L = 0.25\ \ (m)$$

Length of laminar boundary layer need to be considered.

Power :

Assume that laminar BL is fulfilled the Blasius solution and Turbulent BL is one-seventh power law.

Laminar BL : $$\delta = \frac{5x}{\sqrt{Re_{cr}}} = 1.768 \times 10^{-3} \;\; (m)$$

Turbulent BL : $$\delta = 1.768 \times 10^{-3} = \frac{0.3721 x_1}{(Re_{cr})^{1/5}}$$

$$x_1 = 0.066 \;\; (m)$$

Total length for turbulent BL : $$x_{turbulent} = (1.0 - 0.25) + (0.066) = 0.816 \;\; (m)$$

Reynolds for turbulent BL only : $$Re_T = \frac{\rho U L}{\mu} = 1{,}632{,}000$$

BL thickness at trailing edge : $$\delta = \frac{0.3721 x}{(Re_T)^{1/5}} = 0.0174 \;\; (m)$$

Shear stress at trailing edge : $$\tau_w = \frac{0.0289 \rho U^2}{(Re_T)^{1/5}} = 6.60 \;\; (N/m^2)$$

Drag force for the one side of plate:

$Re_{Critical} = 5 \times 10^5$	$C_D = \frac{0.073}{(Re)^{\frac{1}{5}}} - \frac{1700}{(Re)}$

$$C_D = \frac{0.073}{(Re_L)^{\frac{1}{5}}} - \frac{1700}{(Re_L)} = 0.00316$$

$$F_D = (C_D)\frac{1}{2}\rho A U^2 = 3.792 \ (N)$$

Question 4

Figure Q1 shows a uniform flow past a flat plate. Determine the wall shear stress τ_w between AD=0.5 m. the velocity profile is laminar $\frac{u}{U} = \left(\frac{y}{\delta}\right)^{1/2}$, $\delta_{AB} = 0.15\ m$ and $\delta_{ACD} = 0.16\ m$. Take $\rho = 1.208\ kg/m^3$ and $\mu = 1.81 \times 10^{-5}\ Ns/m^2$ for air.

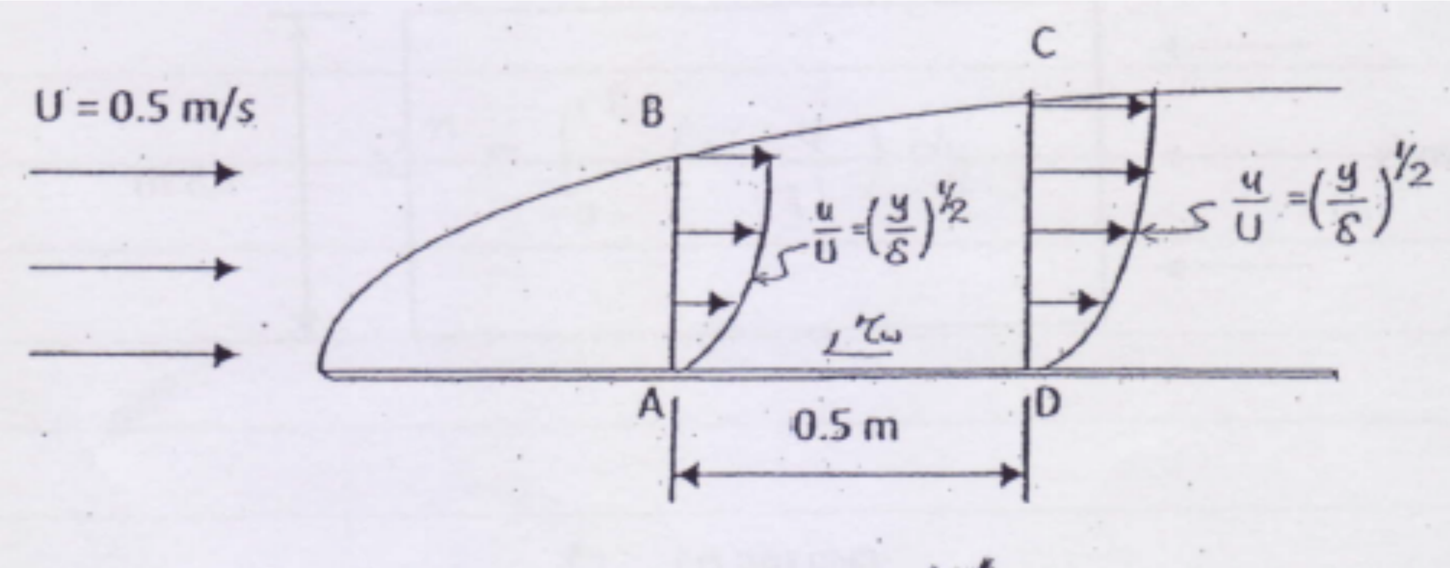
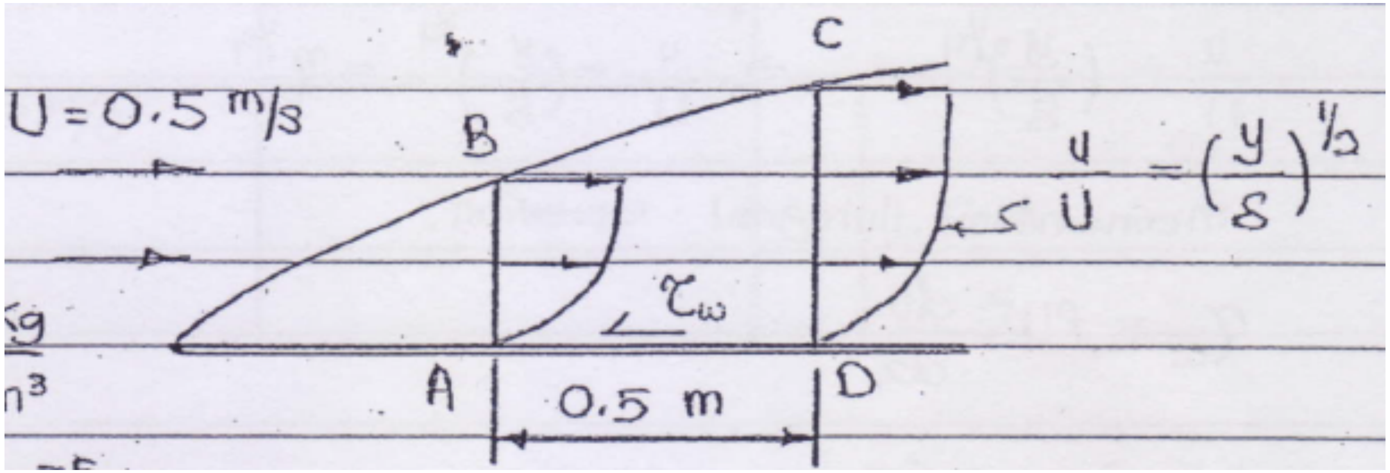


FIGURE Q1

Answer:



$\rho = 1.208\ kg/m^3$

$\mu = 1.81 \times 10^{-5}\ Ns/m^2$

$$\frac{u}{U} = \left(\frac{y}{\delta}\right)^{1/2} = \eta^{1/2}$$

$\delta_{AB} = 0.15\ (m)$

$\delta_{CD} = 0.16\ (m)$

$$d\delta = \delta_{CD} - \delta_{AB}$$
$$= 0.16 - 0.15$$
$$= 0.01\ (m)$$

$dx = 0.05\ (m)$

$$\tau_w = \rho U^2 \frac{d\theta}{dx}$$

$$= \rho U^2 \frac{d\delta}{dx}\left(\frac{1}{6}\right)$$

$$= (1.208)(0.5)^2\left(\frac{0.01}{0.5}\right)\left(\frac{1}{6}\right)$$

$$= 1.007 \times 10^{-3} \ (N/m^2)$$