FLUID MECHANICS I

SEMM 2313

FLUID DYNAMICS

The motion of a fluid is usually extremely complex.

The study of a fluid at rest, or in relative equilibrium, was simplified by the absence of shear stress, but when a fluid flows over a solid surface or other boundary, whether stationary or moving, the velocity of the fluid in contact with the boundary must be the same that the boundary, and a velocity gradient is created at right angle to the boundary.

The resulting change of velocity from layer to layer of fluid flowing parallel to the boundary gives rise to shear stresses in the fluid.

Individual particles of fluid move as a result of the action of forces set up by differences of pressure of elevation.

Their motion is controlled by their inertia and the effect of the shear stresses exerted by the surrounding fluid.

The resulting motion is not easily analysed mathematically, and it is often necessary to supplement theory by experiment.

MOTION OF FLUID PARTICLES AND STREAMS

1. Streamline is an imaginary curve in the fluid across which, at a given instant, there is no flow.

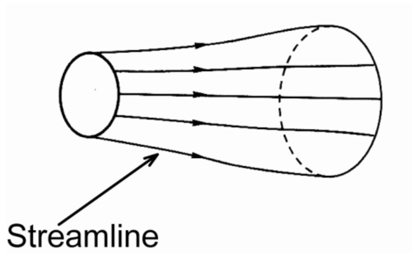


Figure 1

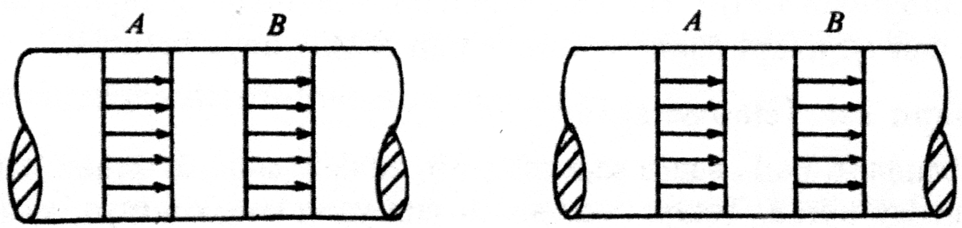
2. Steady flow is one in which the velocity, pressure and cross-section of the stream may vary from point to point but do not change with time.

   If, at a given point, conditions do change with time, the flow is described as unsteady flow.

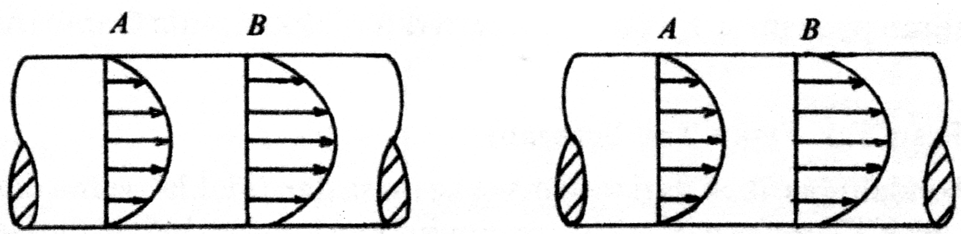
3. Uniform flow occur if the velocity at a given instant is the same in magnitude and direction at every point in the fluid.

   If, at the given instant, the velocity changes from point to point, the flow is described as non-uniform flow.

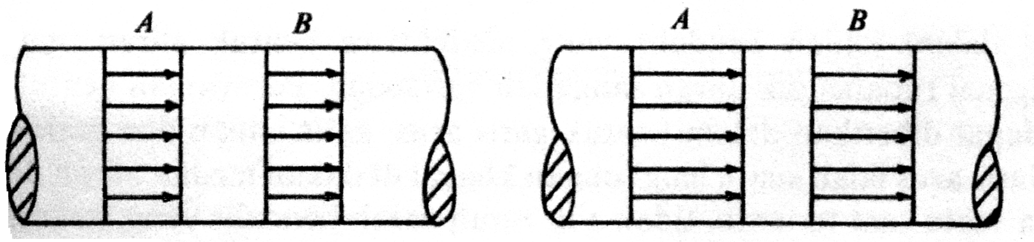
At time T1 At time T2



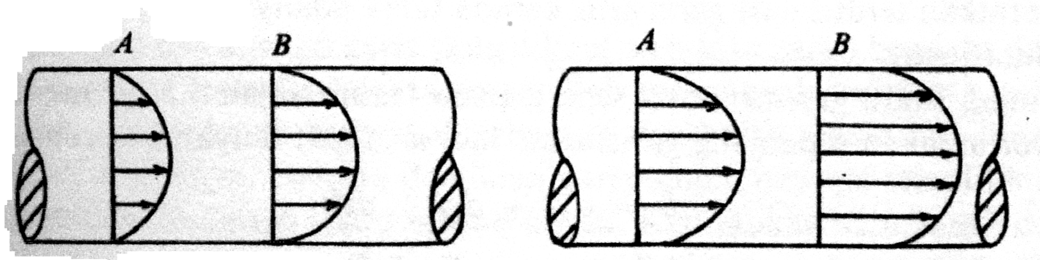
(i) Steady Flow and uniform



(ii) Steady Flow and non-uniform



(iii) Unsteady Flow and uniform



(iv) Unsteady flow and non-uniform

4. Real fluid is a fluid which when it flows past a boundary, the fluid immediately in contact with the boundary will have the same velocity as the boundary.

   Ideal fluid is a fluid which is assumed to have no viscosity and in which there are no shear stresses.

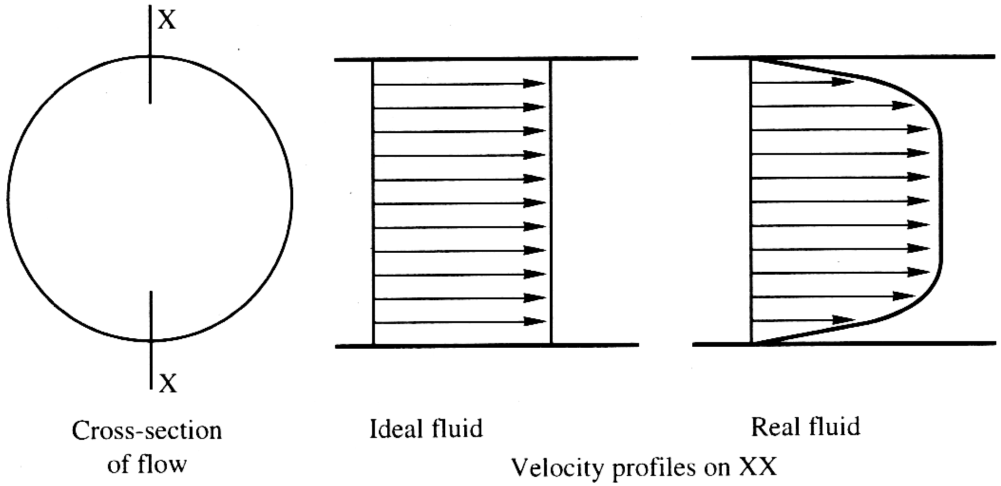


Figure 3

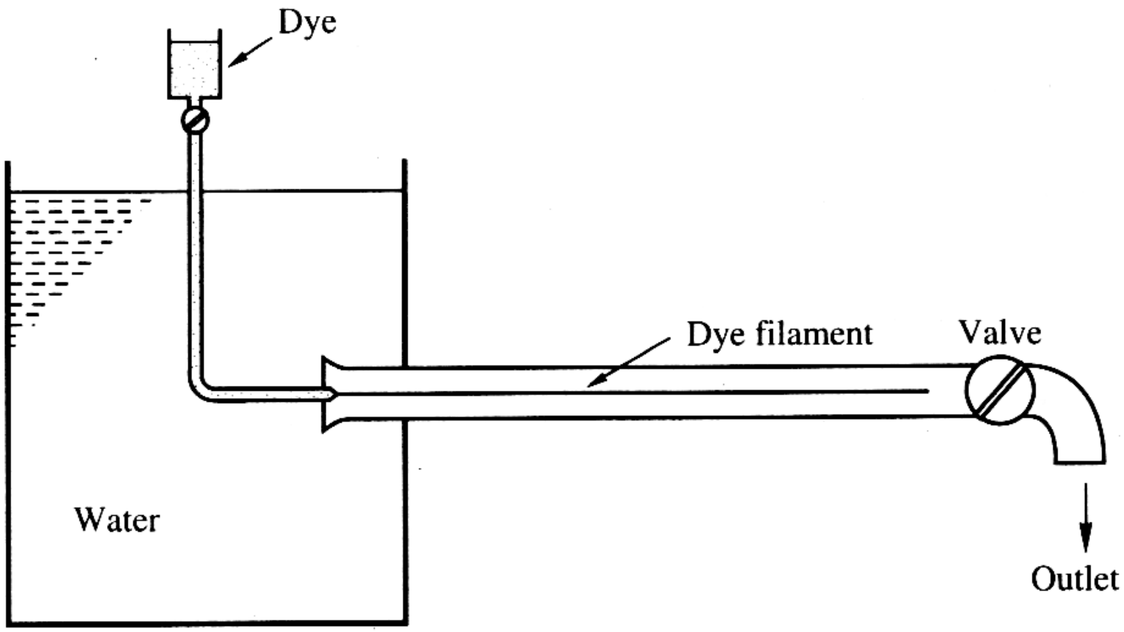
5. Compressible fluid is a fluid which its density will change with pressure.

6. Laminar flow, sometimes known as streamline flow, occurs when a fluid flows in parallel layers, with no disruption between the layers.

   Turbulent flow is a flow regime characterized by chaotic, stochastic property changes.

From the observation done by Osborne Reynolds in 1883, in straight pipes of constant diameter, flow can be assumed to be turbulent if the Reynolds number, Re, exceeds 4000.

$$\mathrm{Re} = \frac{\rho v D}{\mu}$$



$$\mathrm{Re} = \frac{\rho v D}{\mu}$$

CONTINUITY EQUATION

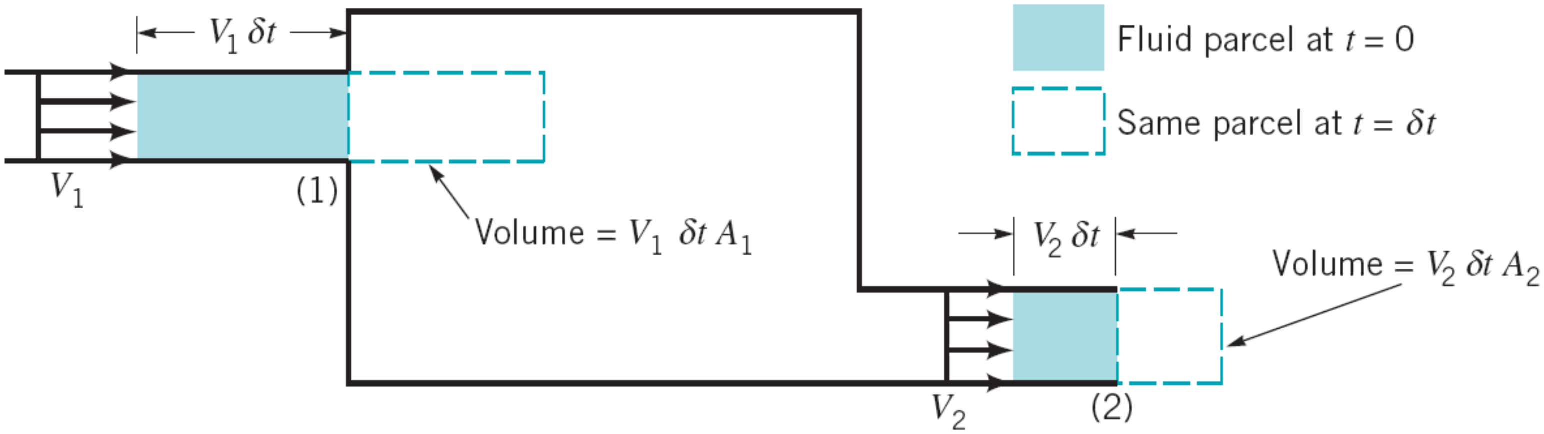


Figure 1

Consider a fluid flowing through a fixed volume that has one inlet and one outlet as shown in Figure 1.

If the flow is steady so that there is no additional accumulation of fluid within the volume, the rate at which the fluid flows into the volume must equal the rate at which it flows out of the volume.

Otherwise, mass would not be conserved.
The mass flowrate from an outlet is given as below;

$$\dot{m} = \rho Q = \rho A V$$

$\dot{m}$: Mass flowrate

Q : Volume flowrate

A : Outlet area

V : Average velocity

To conserve mass, the inflow rate must equal the outflow rate. If the inlet is designated as (1) and the outlet as (2), it follows that;

$$\dot{m}_1 = \dot{m}_2$$

(Mass flow rate at inlet = Mass flow rate at outlet)
It unit is kg/s

Thus, conservation of mass requires;

$$\rho_1 A_1 v_1 = \rho_2 A_2 v_2$$

If the density remains constant, then $\rho_1 = \rho_2$,

And the above equation becomes the continuity equation for incompressible flow, and shown as;

$$A_1 v_1 = A_2 v_2$$

$$Q_1 = Q_2$$

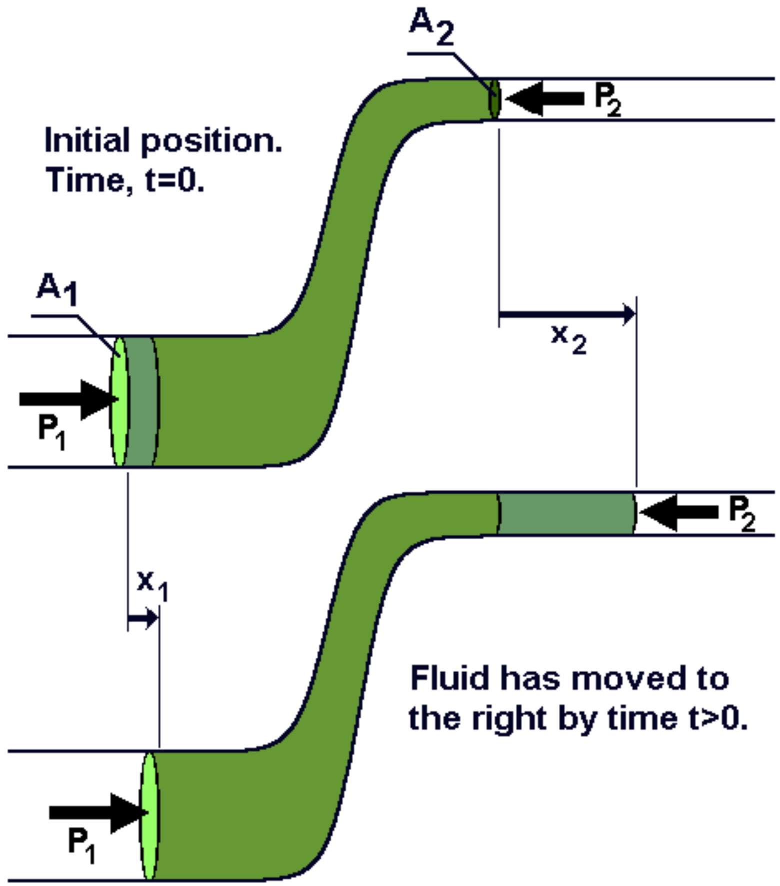
(Volume flow rate at inlet = Volume flow rate at outlet)
It unit is m^3/s

THE BERNOULLI EQUATION FOR AN INCOMPRESSIBLE, STEADY FLUID FLOW

In 1738 Daniel Bernoulli (1700-1782) formulated the famous equation for fluid flow that bears his name. The Bernoulli Equation is a statement derived from conservation of energy and work-energy ideas that come from Newton's Laws of Motion.

An important and highly useful special case is where friction is ignored and the fluid is incompressible. This is not as unduly restrictive as it might first seem. The absence of friction means that the fluid flow is steady. That is, the fluid does not stick to the pipe sides and has no turbulence. Most common liquids such as water are nearly incompressible, which meets the second condition.

Consider the case of water flowing though a smooth pipe. Such a situation is depicted in the figure below. We will use this as our working model and obtain Bernoulli's equation employing the work-energy theorem and energy conservation.



We examine a fluid section of mass m traveling to the right as shown in the schematic above. The total work (W) done in moving the fluid is

$$W_{total} = W_1 + W_2 = F_1 x_1 - F_2 x_2 \qquad \text{Eq.(1)}$$

Where F denotes a force and an x a displacement. The second term picked up its negative sign because the force and displacement are in opposite directions.

Pressure is the force exerted over the cross-sectional area, or $P = F/A$. Rewriting this as
$F = PA$ and substituting into Eq.(1) we find that

$$\Delta W = P_1 A_1 x_1 - P_2 A_2 x_2 \qquad \text{Eq.(2)}$$

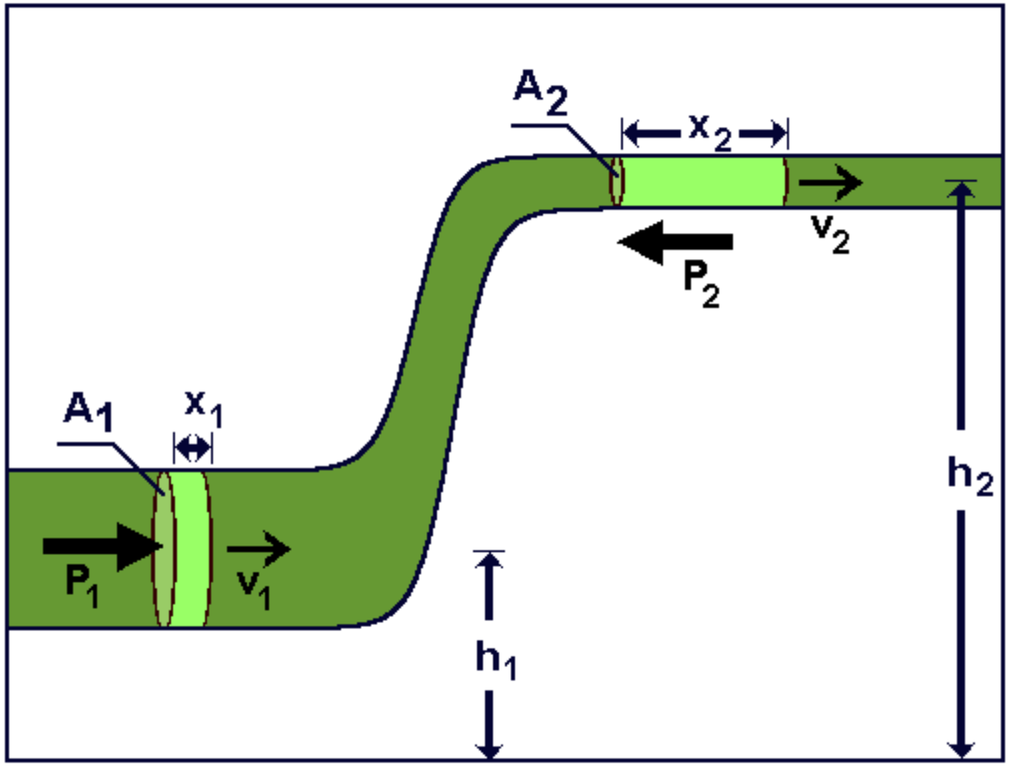
The displaced fluid volume V is the cross-sectional area A times the thickness x. This volume remains constant for an incompressible fluid, so

$$V = A_1 x_1 = A_2 x_2 \qquad \text{Eq.(3)}$$

Using Eq.(3) in Eq.(2) we have

$$\Delta W = (P_1 - P_2)V \qquad \text{Eq.(4)}$$

Since work has been done, there has been a change in the mechanical energy of the fluid segment. This energy change is found with the help of the next diagram.



The energy change between the initial and final positions is given by

$$\Delta E = \begin{bmatrix} Potential\ energy \\ and \\ Kinetik\ energy \end{bmatrix}_{outlet} - \begin{bmatrix} Potential\ energy \\ and \\ Kinetik\ energy \end{bmatrix}_{inlet} \qquad \text{Eq.(5)}$$

or

$$\Delta E = \left(mgh_2 + \frac{1}{2}mv_2^2\right) - \left(mgh_1 + \frac{1}{2}mv_1^2\right) \qquad \text{Eq.(5)}$$

Here, the the kinetic energy $K = mv^2/2$ where m is the fluid mass and v is the speed of the fluid. The potential energy $U = mgh$ where g is the acceleration of gravity, and h is average fluid height.

The work-energy theorem says that the net work done is equal to the change in the system energy. This can be written as

$$\Delta E = \Delta W \tag{Eq.(6)}$$

Substitution of Eq.(4) and Eq.(5) into Eq.(6) yields

$$(P_1 - P_2)V = \left(mgh_2 + \frac{1}{2}mv_2^2\right) - \left(mgh_1 + \frac{1}{2}mv_1^2\right) \tag{Eq.(7)}$$

Dividing Eq.(7) by the fluid volume, $\forall$, and replace $m/\forall = \rho$ gives us

$$(P_1 - P_2) = \rho g h_2 + \frac{1}{2}\rho v_2^2 - \rho g h_1 - \frac{1}{2}\rho v_1^2 \qquad \text{Eq.(8)}$$

Rearrange Eq.(8), gives us Eq.(9)

$$P_1 + \frac{1}{2}\rho v_1^2 + \rho g h_1 = P_2 + \frac{1}{2}\rho v_2^2 + \rho g h_2 \qquad \text{Eq.(9)}$$

or

$$\frac{P_1}{\rho g} + \frac{v_1^2}{2g} + h_1 = \frac{P_2}{\rho g} + \frac{v_2^2}{2g} + h_2 \qquad \text{Eq.(9)}$$

Some text book use: $h = z$

$$\frac{P_1}{\rho g} + \frac{v_1^2}{2g} + z_1 = \frac{P_2}{\rho g} + \frac{v_2^2}{2g} + z_2 \qquad \text{Eq.(9)}$$

Finally, note that Eq.(9) is true for any two positions. Therefore, Equation (10) is commonly referred to as ***Bernoulli's equation***. Keep in mind that this expression was restricted to ***incompressible fluids and smooth fluid flows.***

$$\frac{P}{\rho g} + \frac{v^2}{2g} + z = \text{constant} \qquad \text{Eq.(10)}$$

It is called Bernoulli equation.

To use it correctly, we must constantly remember the basic assumptions used in its derivation:

1. Viscous effects are assumed negligible
2. The flow is assumed to be steady
3. The flow is assumed to be incompressible
4. The equation is applicable along a streamline

If Bernoulli equation is integrated along the streamline between any two points indicated by suffixes 1 and 2;

$$\frac{P_1}{\rho g}+\frac{v_1^2}{2g}+z_1=\frac{P_2}{\rho g}+\frac{v_2^2}{2g}+z_2$$

Physical Interpretation

Integration of the equation of motion to give eq.2 actually corresponds to the work-energy principle often used in the study of dynamics.The work done on a particle by all forces acting on the particle is equal to the change of the kinetic energy of the particle.

Each of the terms in this equation has the units of energy per weight ($LF/F = L$) or length (feet, meters) and represents a certain type of head.

The elevation term, z, is related to the potential energy of the particle and is called the elevation head.

The pressure term, $P/\rho g$, is called the pressure head and represents the height of a column of the fluid that is needed to produce the pressure P.

The velocity term, $v^2/2g$, is the velocity head and represents the vertical distance needed for the fluid to fall freely (neglecting friction) if it is to reach velocity v from rest.

The Bernoulli equation states that the sum of the pressure head, the velocity head, and the elevation head is constant along a streamline.

Static, Stagnation, Dynamic and Total Pressure

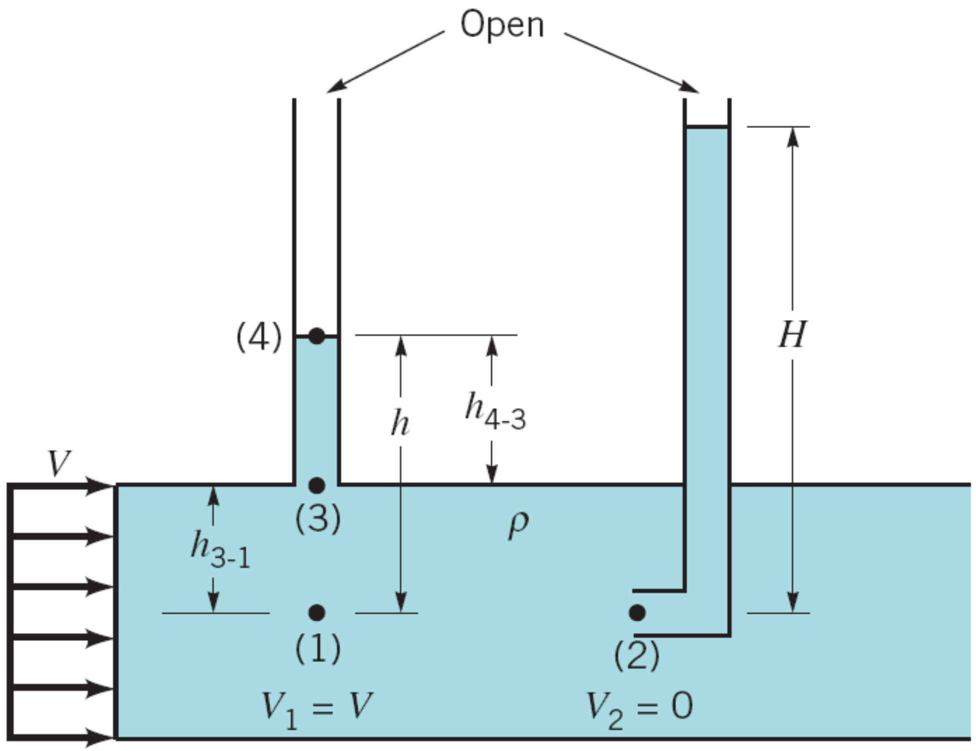


Figure 6

The second term in the Bernoulli equation, $v^2/2g$, is termed the dynamic pressure.

Its interpretation can be seen in Figure 6 by considering the pressure at the end of a small tube inserted into the flow and pointing upstream.

After the initial transient motion has died out, the liquid will fill the tube to a height of H as shown. The fluid in the tube, including that at its tip, (2), will be stationary. That is, $v_2 = 0$, or point (2) is a <u>stagnation point.</u>

If we apply the Bernoulli equation between points (1) and (2), using $v_2 = 0$ and assuming that $z_1 = z_2$, we find that

$$p_2 = p_1 + \frac{1}{2}\rho v_1^2$$

Hence, the pressure at the stagnation point is greater than the static pressure, p_1, by an amount $\frac{1}{2}\rho v_1^2$, the <u>dynamic pressure</u>.

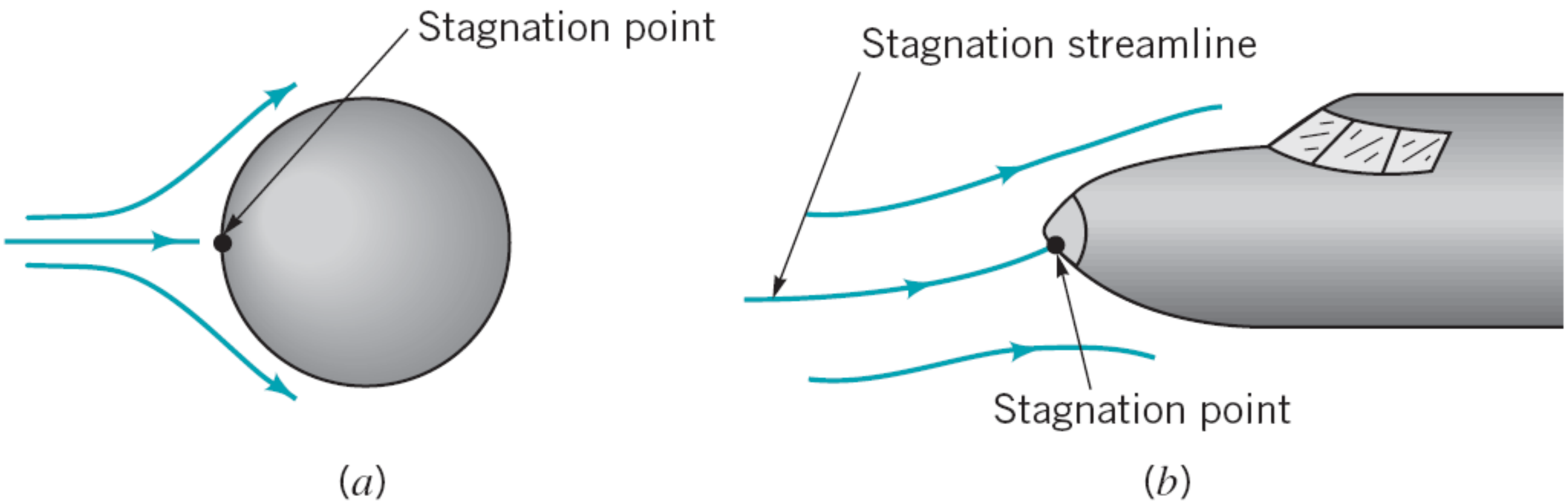


Figure 7

It can be shown that there is a stagnation point on any stationary body that is placed into a flowing fluid.

Some of the fluid flows "over" and some "under" the object. The dividing line (or surface for two-dimensional flows) is termed the stagnation streamline and terminates at the stagnation point on the body.

For symmetrical objects (such as a sphere) the stagnation point is clearly at the tip or front of the object as shown in Figure 7(a).

For nonsymmetrical objects such as the airplane shown in Figure 7(b), the location of the stagnation point is not always obvious.

Knowledge of the values of the static and stagnation pressures in a fluid implies that the fluid speed can be calculated.

This is the principle on which the Pitot-static tube is based H. de Pitot (1695–1771), as shown in Figure 8.

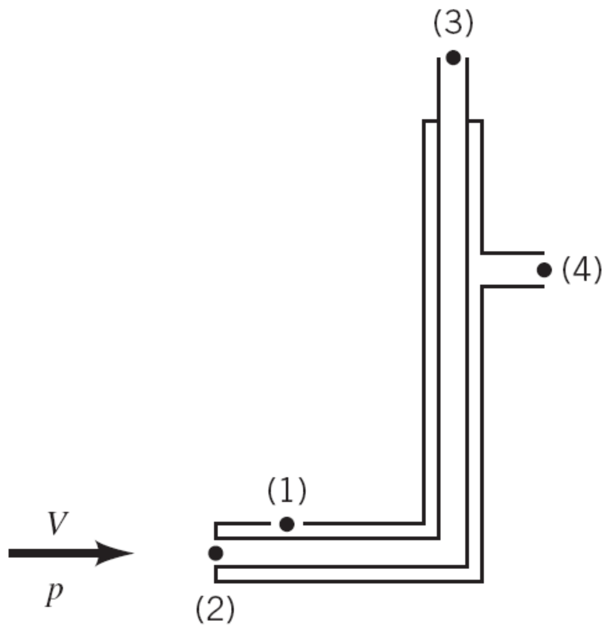


Figure 8

Free jets

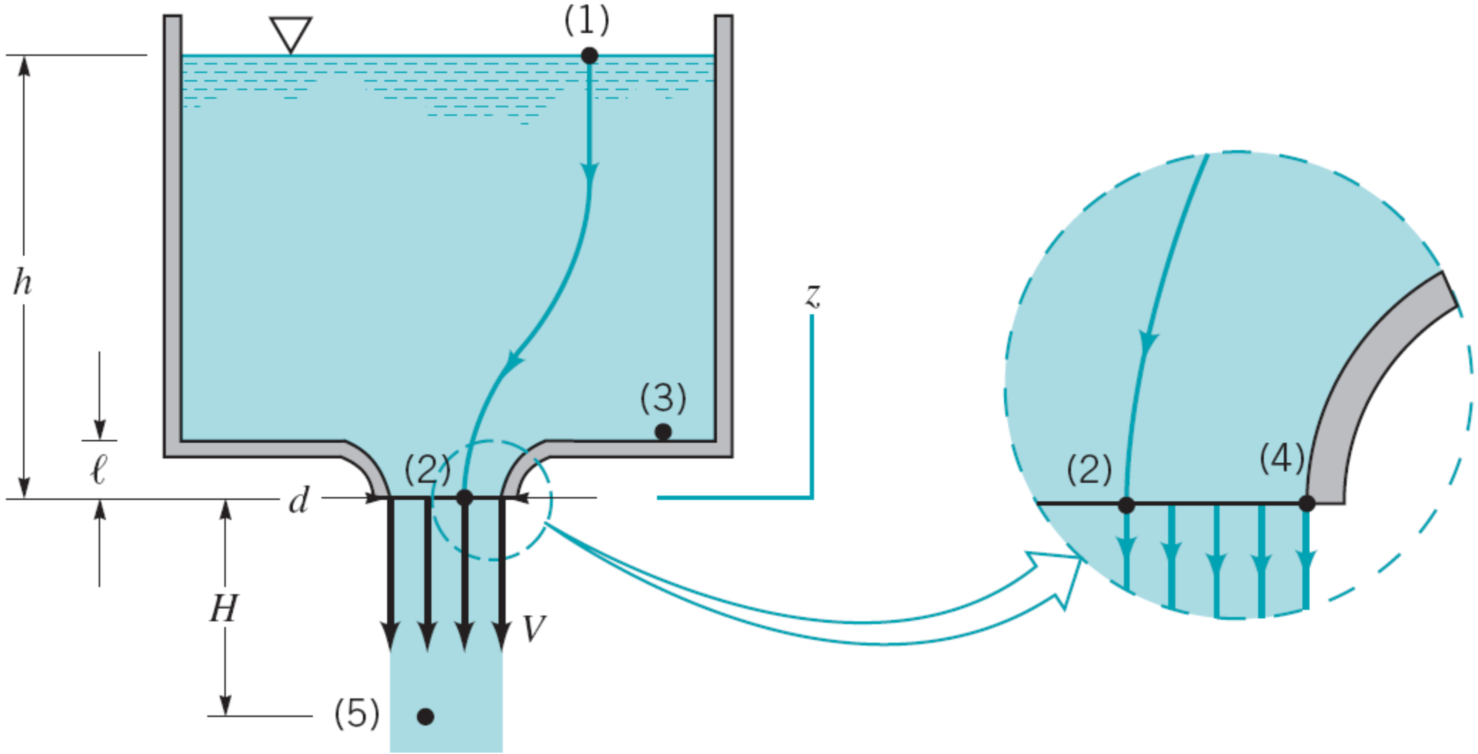


Figure 1

From the fact, we found that;

$$z_1 = h \quad \text{and} \quad z_2 = 0$$

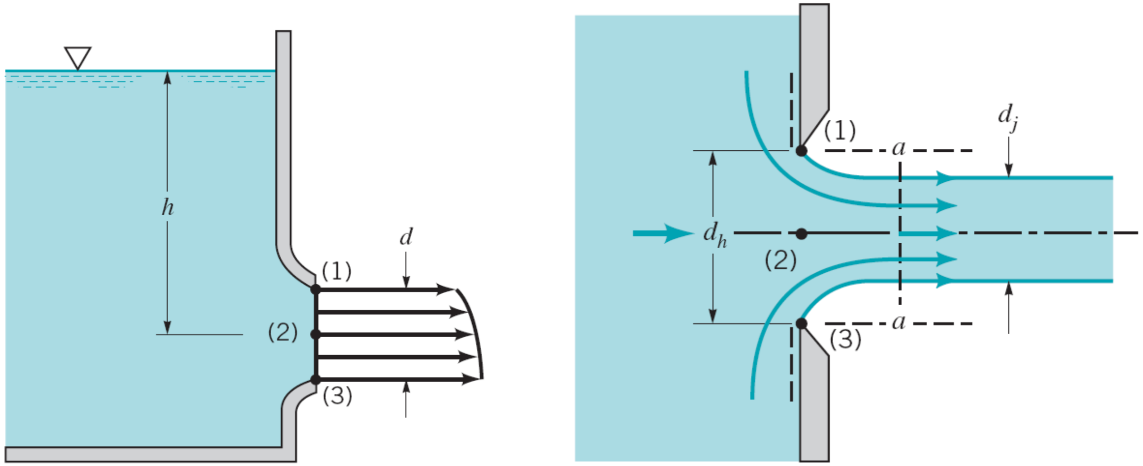
$$p_1 = p_2 = 0 \quad \text{and} \quad v_1 = 0$$

Thus, the fluid leaves as a “free jets” with;

$$v_2 = \sqrt{2gh}$$

This is introduced in 1643 by *Torricelli* (1608-1647)

Nozzle



(a) (b)

Figure 2

We can safely use the centerline velocity at point (2) as a reasonable *"average velocity"*, as shown in Figure 2(a).

If the exit is not a smooth, well-contoured nozzle, but rather a flat plate as shown in Figure 2(b), the diameter of the jet, d_j will be less that the diameter of the hole, d_h.

This phenomenon is called a vena contracta effect, is a result of the inability of the fluid to turn the sharp 90-degree corner indicated by the dotted line in the figure.

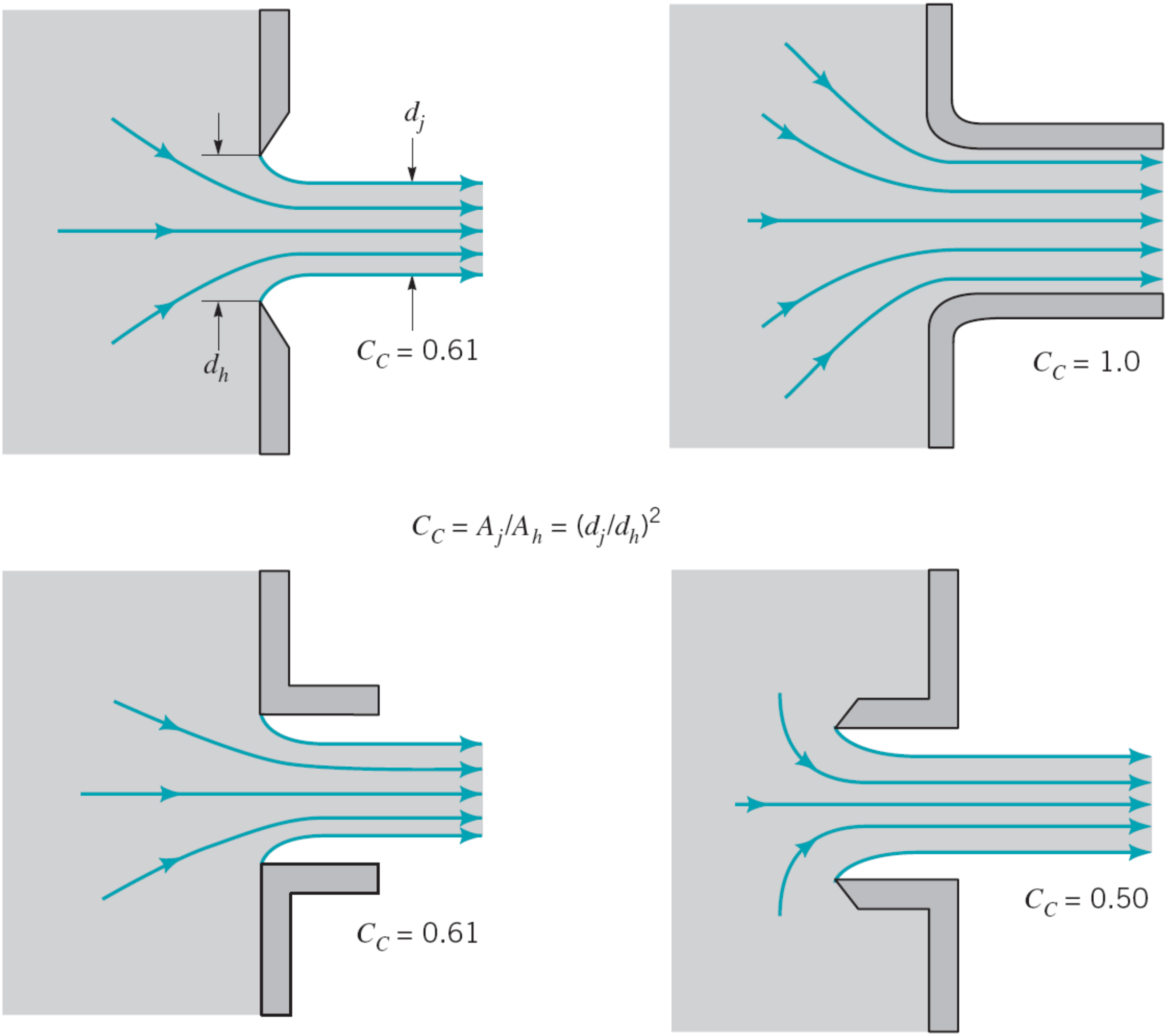

d_j
d_h
$C_C = 0.61$
$C_C = 1.0$
$C_C = A_j/A_h = (d_j/d_h)^2$
$C_C = 0.61$
$C_C = 0.50$

Figure 3

The vena contracta effect is a function of the geometry of the outlet. It can be obtained by experimental, and called as contraction coefficient, C_c.

$$C_c = \frac{A_j}{A_h}$$

with A_j is area of the jet
A_h is area of the hole

The pitot-static tube

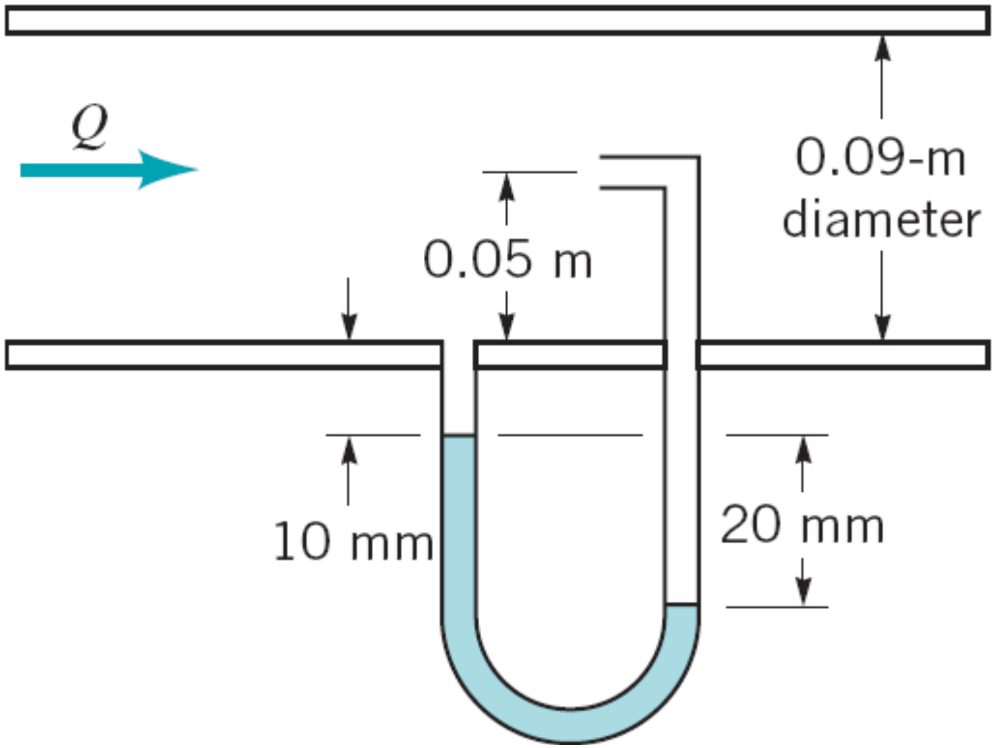


Figure 4

The specific gravity of the manometer fluid shown in Figure 4 is 1.07. Determine the volume flowrate, Q, if the flow is inviscid and incompressible and the flowing fluid is water.

The orifice nozzle / The nozzle meter

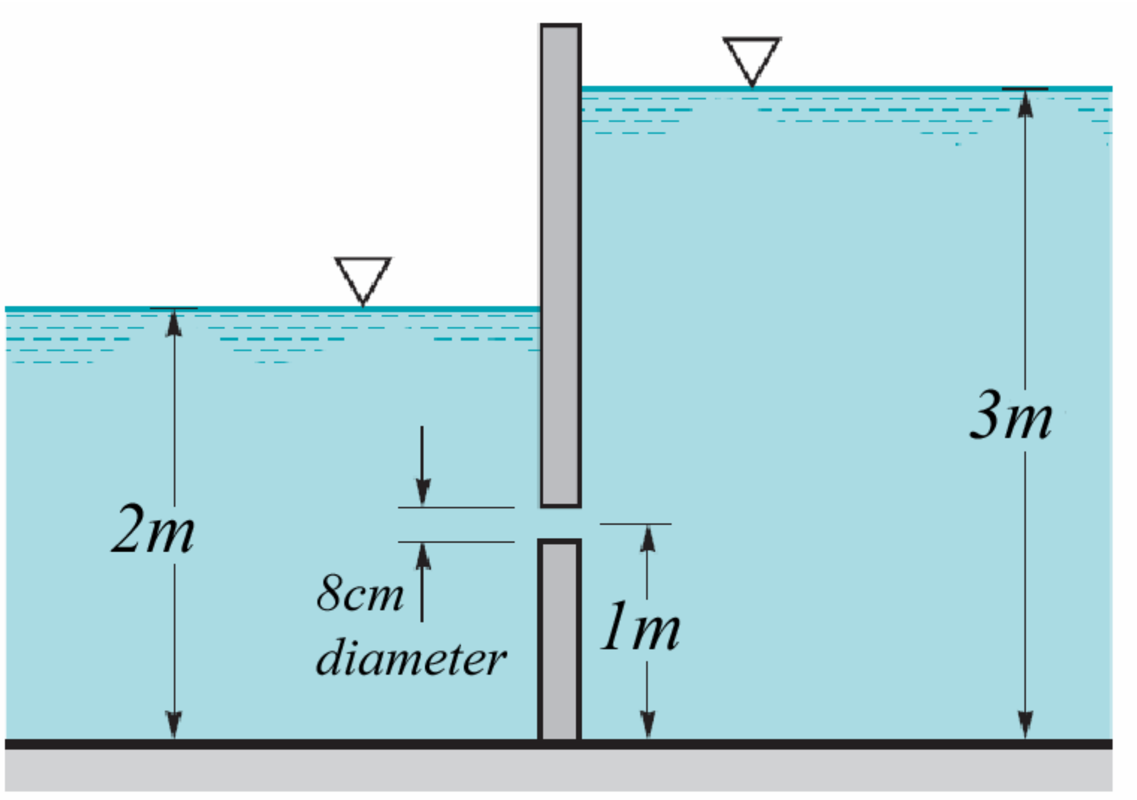
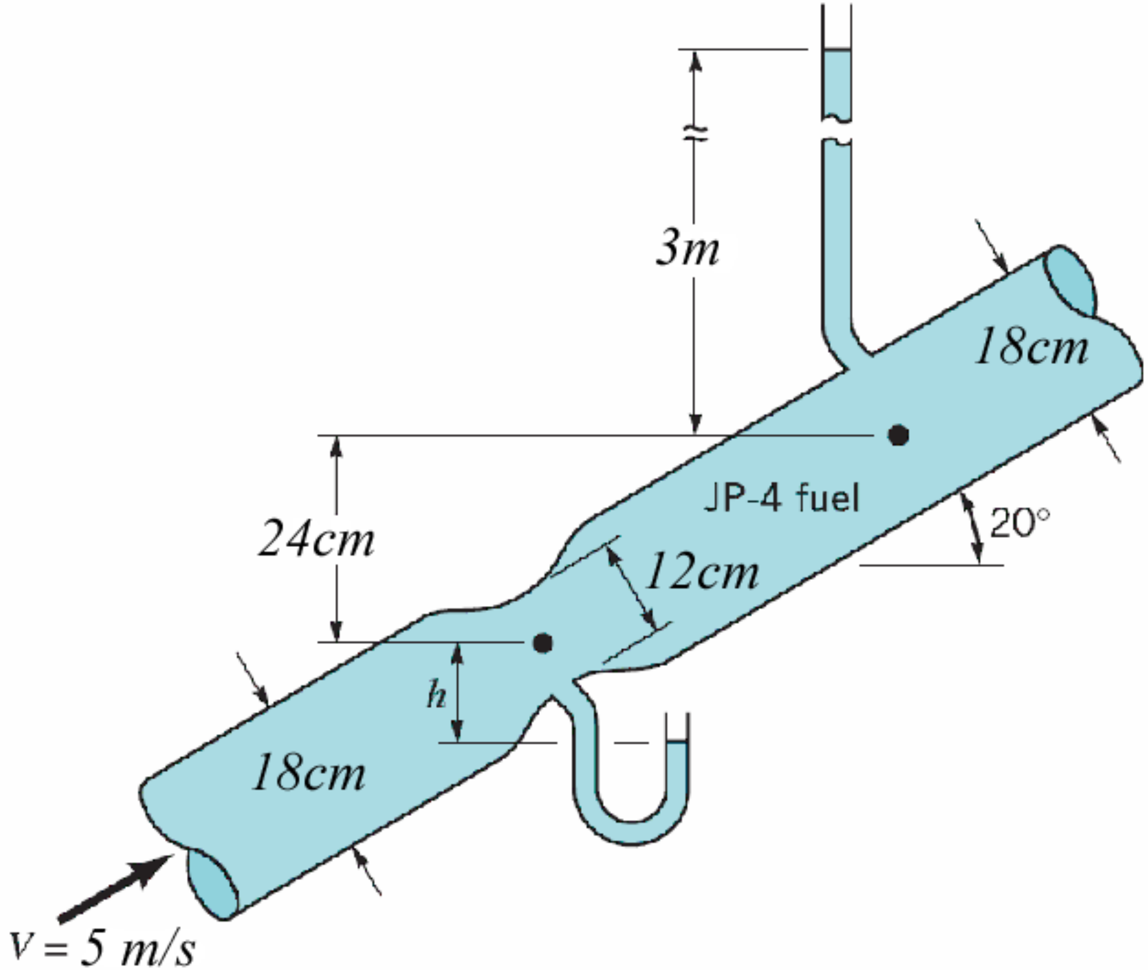


Figure 5

Determine the flowrate through the submerged orifice shown in Figure 5 if the contraction coefficient is
$C_c = 0.63$

The venturi meter



JP-4 fuel (*SG*=0.77) flows through the Venturi meter shown in Figure 6. Determine the elevation, *h*, of the fuel in the open tube connected to the throat of the Venturi meter.

A rectangular weir

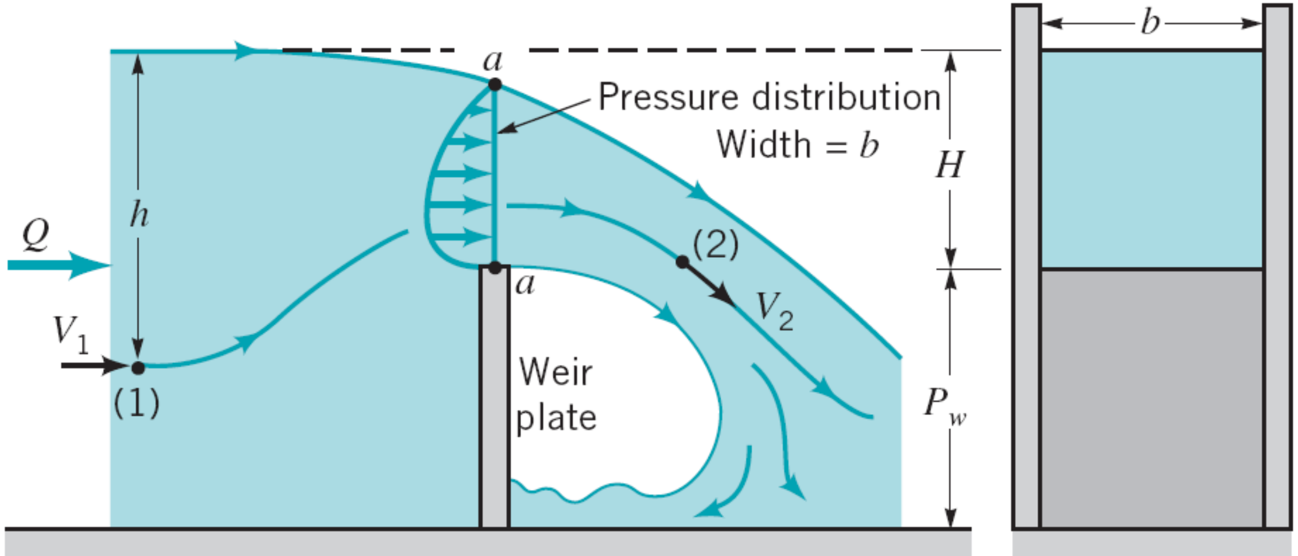


Figure 7

The volume flowrate, Q, follows that;

$$Q = C_1 H b\sqrt{2gH} = C_1 b H^{\frac{3}{2}}\left(\sqrt{2g}\right)$$

<u>A triangular weir</u>

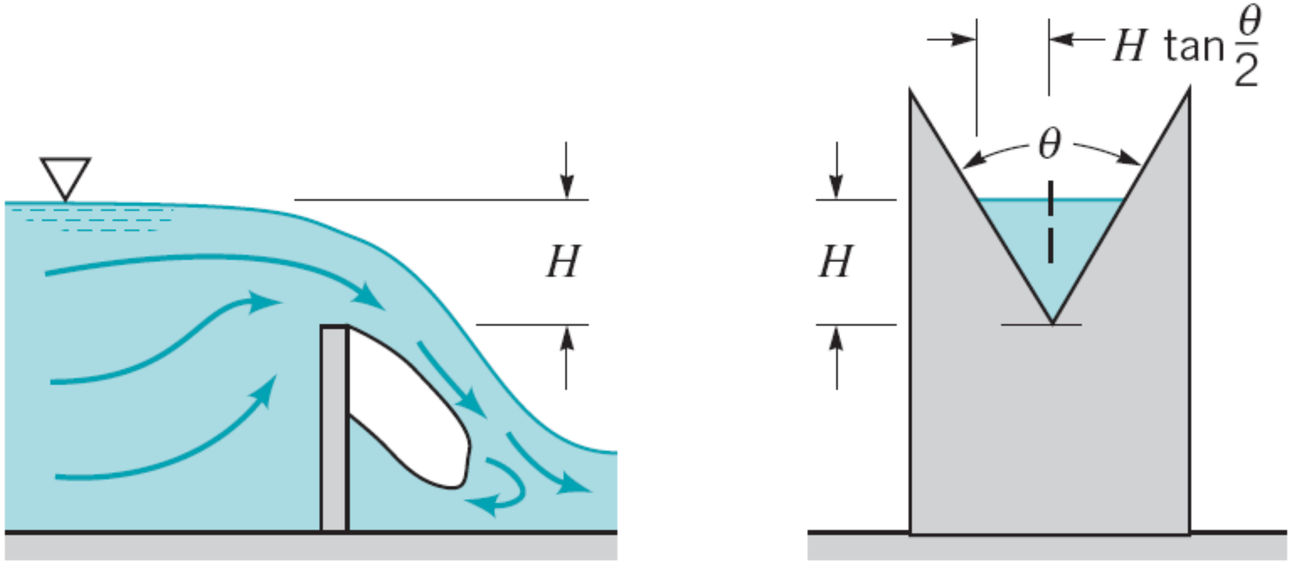


Figure 8

The volume flowrate, Q, follows that;

$$Q = C_2\left(\tfrac{1}{2}\tan\theta\right)\left(H\right)^{\frac{5}{2}}\left(\sqrt{2g}\right)$$

$$Q = C_2 \left(\tfrac{1}{2}\tan\theta\right)(H)^{\frac{5}{2}}\left(\sqrt{2g}\right)$$

$$Q = C_2 \left(\frac{1}{2}\tan\theta\right)(H)^{\frac{5}{2}}\left(\sqrt{2g}\right)$$

Here,

$$C_2 = \frac{8}{15}$$

The energy line and the hydraulic grade line

As discussed before, the Bernoulli equation is actually an energy equation representing the partitioning of energy for an inviscid, incompressible, steady flow.

The sum of the various energies of the fluid remains constant as the fluid flows from one section to another.

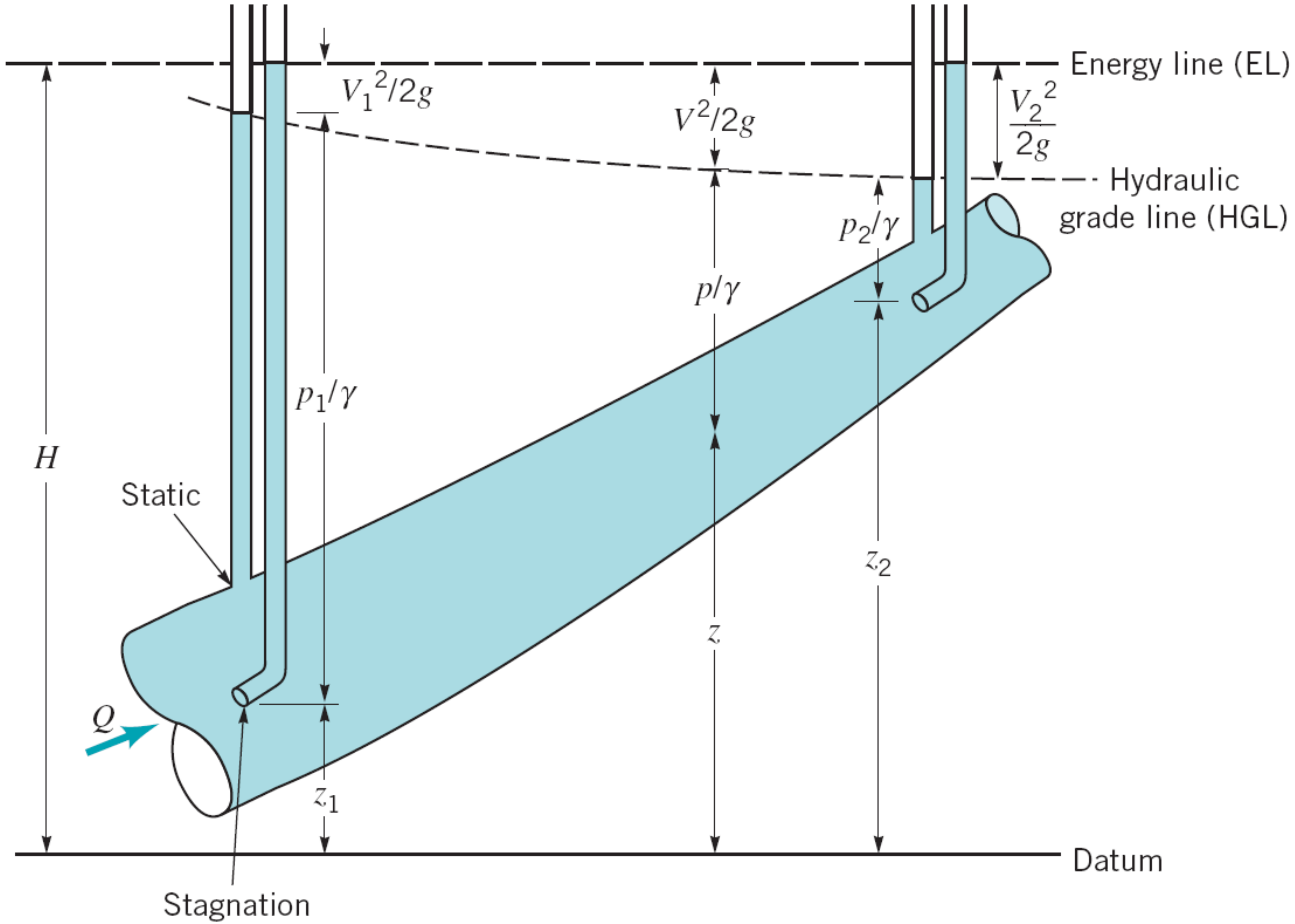
A useful interpretation of the Bernoulli equation can be obtained through the use of the concepts of the hydraulic grade line (HGL) and the energy line (EL).

This ideas represent a geometrical interpretation of a flow and can often be effectively used to better grasp the fundamental processes involved.

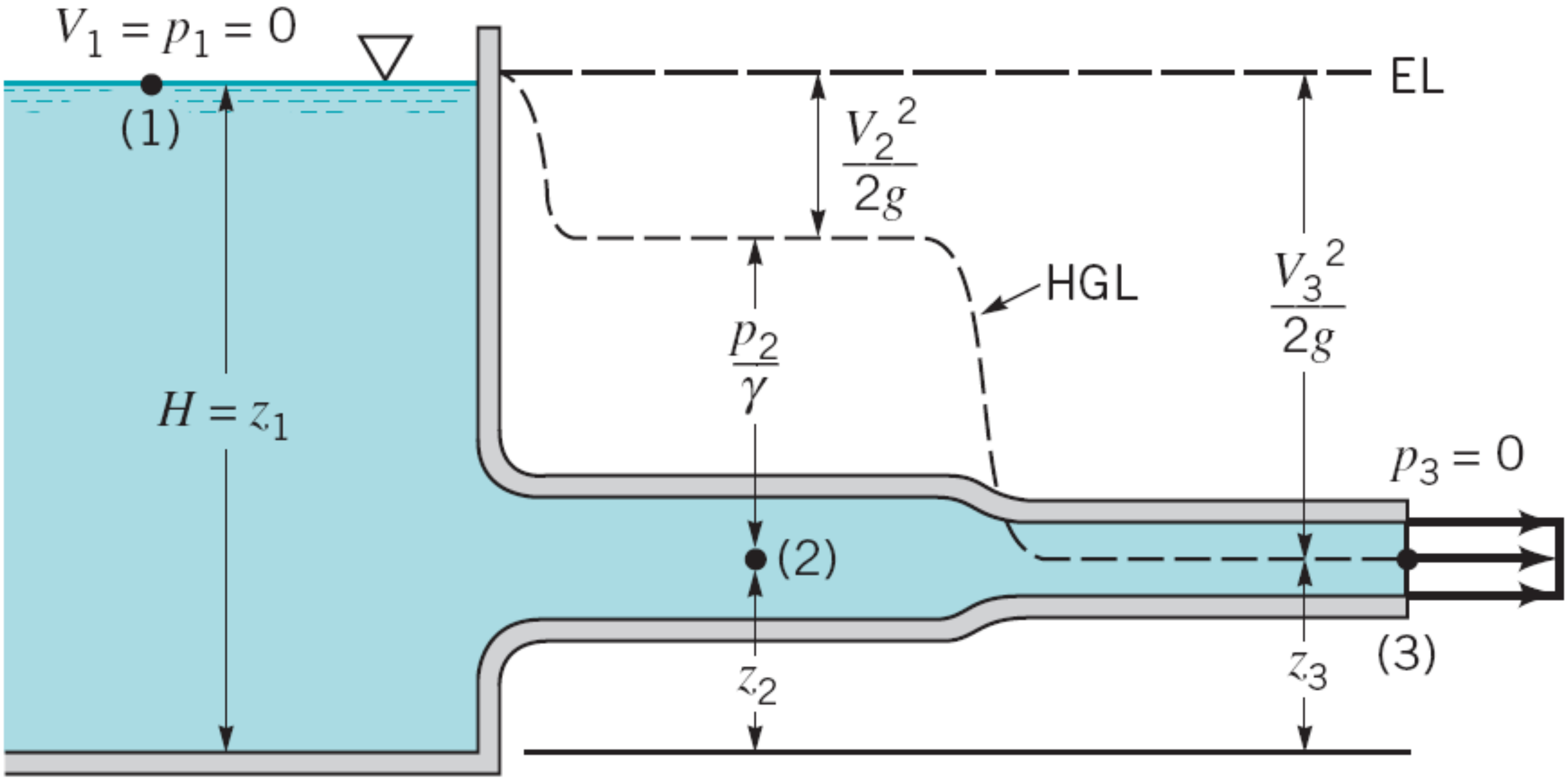
The energy line is a line that represents the total head available to the fluid. The elevation of the energy line can be obtained by measuring the stagnation pressure with a pitot tube.

The static pressure tap connected to the piezometer tube measures the sum of the pressure head and elevation head, and called piezometer head.

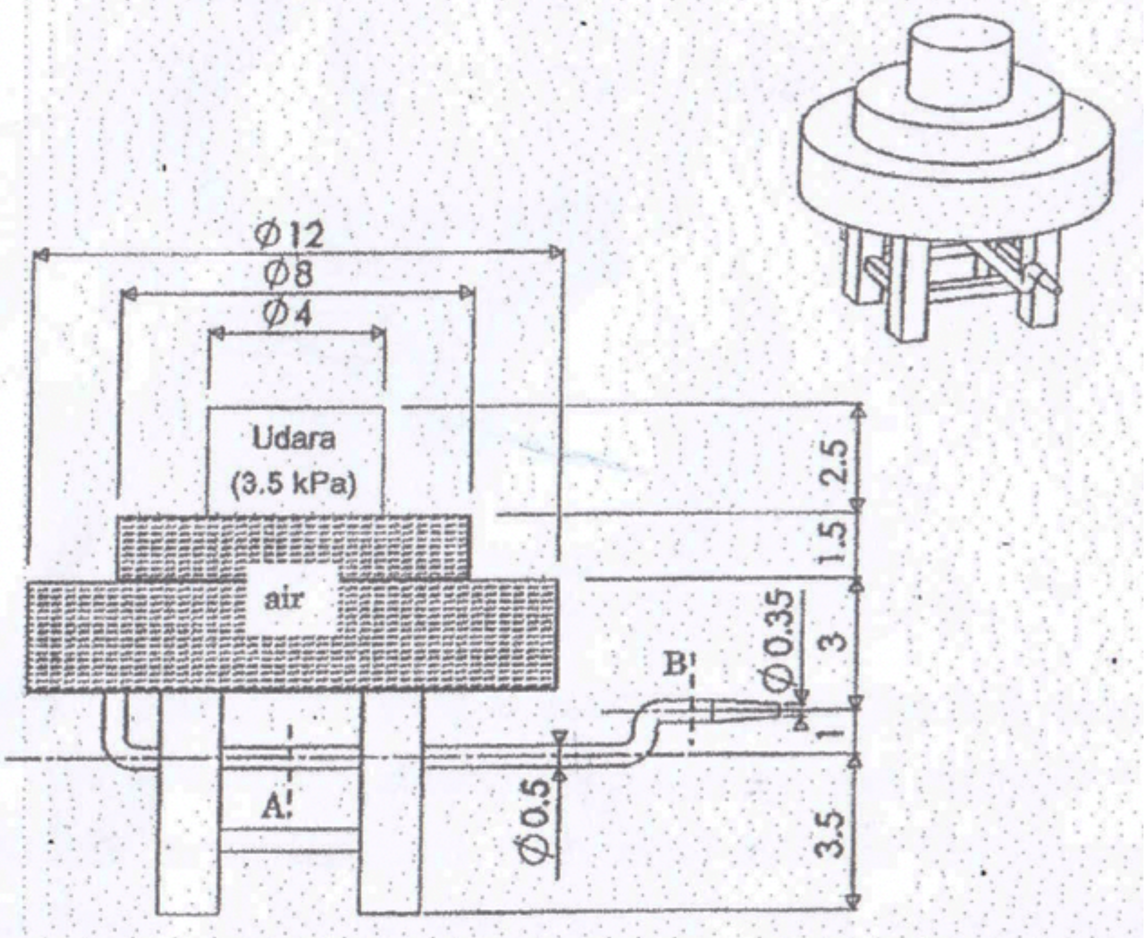
The locus provided by a series of piezometer taps is termed the hydraulic line.



Energy line (EL)
$V_1^2/2g$
$V^2/2g$
$\frac{V_2^2}{2g}$
Hydraulic grade line (HGL)
p_2/γ
p/γ
p_1/γ
H
Static
z_2
z
Q
z_1
Datum
Stagnation


$V_1 = p_1 = 0$
(1)
EL
$\frac{V_2^2}{2g}$
HGL
$\frac{p_2}{\gamma}$
$\frac{V_3^2}{2g}$
$H = z_1$
$p_3 = 0$
(2)
z_2
z_3
(3)

TUTORIAL 1



Rajah 1

Berdasarkan Rajah 1 di sebelah, kirakan ;

i. Halaju air yang keluar

ii. Tekanan pada keratan rentas A dan keratan rentas B.

TUTORIAL 2

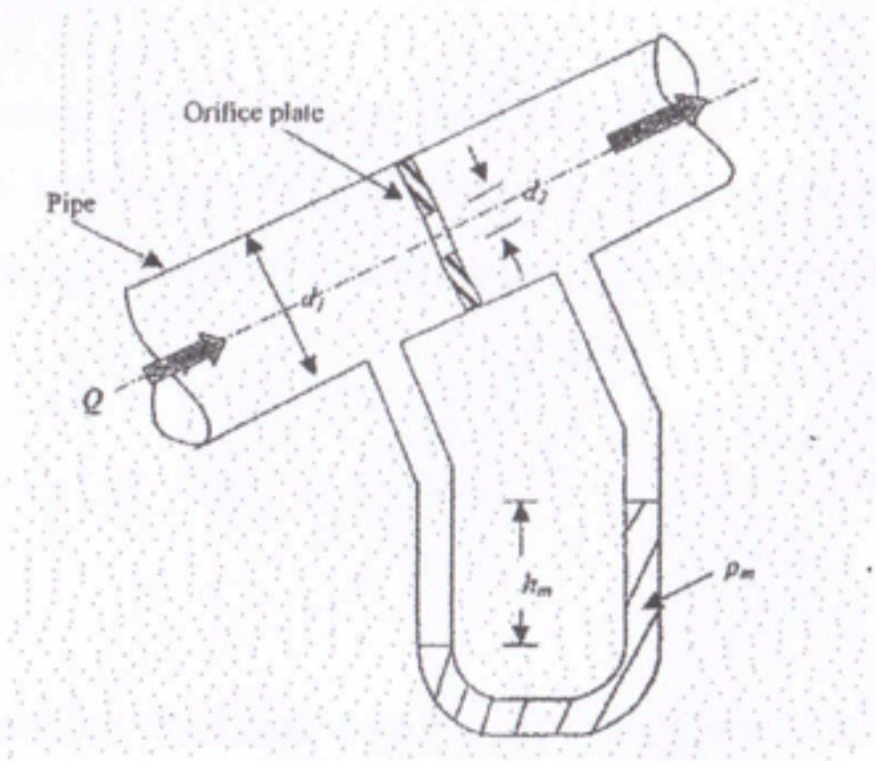
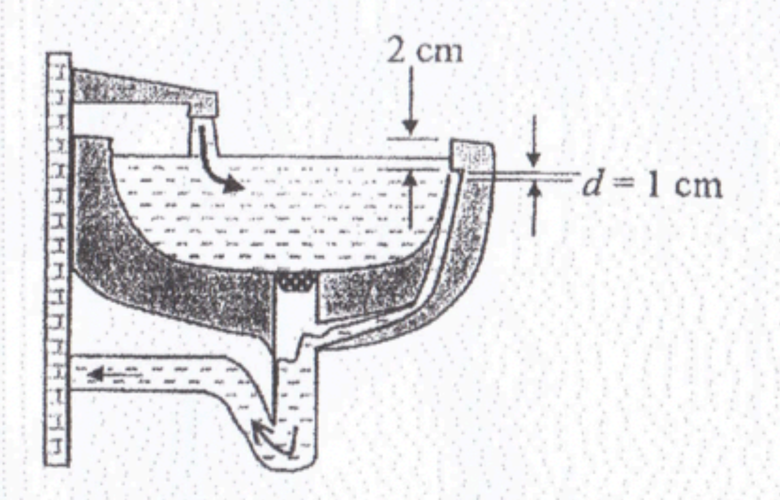


Figure 2

If the specific density of the flowing fluid is SG=0.9, manometric fluid is mercury, SG=13.6, determine the flowrate, Q. Given h_m=0.2m, d_1=100mm and d_2=30mm. Take Cc=0.6.

TUTORIAL 3



Rajah 3

Air mengalir masuk ke dalam sinki yang dilakarkan dalam Rajah 3 dengan kadar aliran 8 liter-per-minit. Jika salur keluar sinki ditutup, akhirnya air akan mengalir keluar melalui alur limpah yang terletak di bahagian tepi sinki. Oleh sebab satu alur limpah (diameter 1 cm) tidak mampu mengalirkan semua air keluar dalam kes ini, maka tentukan bilangan alur limpah (diameter 1 cm) yang diperlukan untuk memastikan air tidak melimpah keluar dari sinki.

TUTORIAL 4

Sebuah meter venture dipasang pada sebatang paip mengufuk. Tekanan yang diukur pada leher meter venture menunjukkan kesusutan 25% berbanding tekanan yang memasuki meter tersebut. Jika luas keratan rentas leher juga susut 25% daripada keratan rentas masukan, buktikan bahawa ;

$$P_1 - P_2 = \frac{\rho Q^2}{2}\left(\frac{A_1^2 - A_2^2}{A_1^2 A_2^2}\right)$$

dengan 1 ialah titik pada bahagian masukan dan 2 ialah titik pada leher meter venture meter.

Tentukan nilai P_1 jika kadar aliran ialah 1 m^3/s

TUTORIAL 5

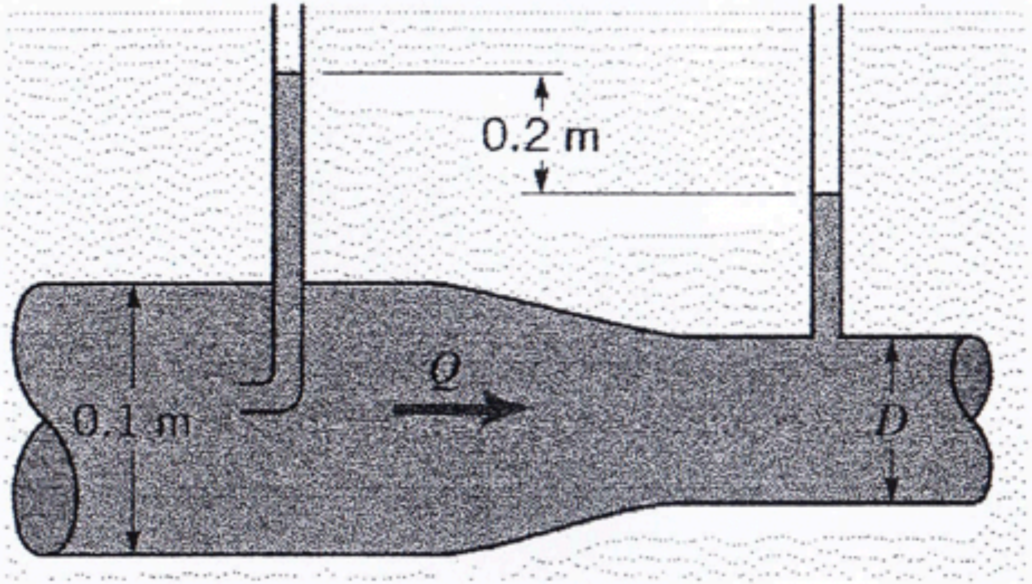


Figure 1

Water flows through the pipe contraction shown in Figure 1. For the given 0.2(m) difference in the manometer level, determine the flowrate as a function of the diameter of the small pipe D.

$$V_1 = 1.98 \quad m/s$$

$$Q = 0.0156 \quad m^3/s$$

TUTORIAL 6

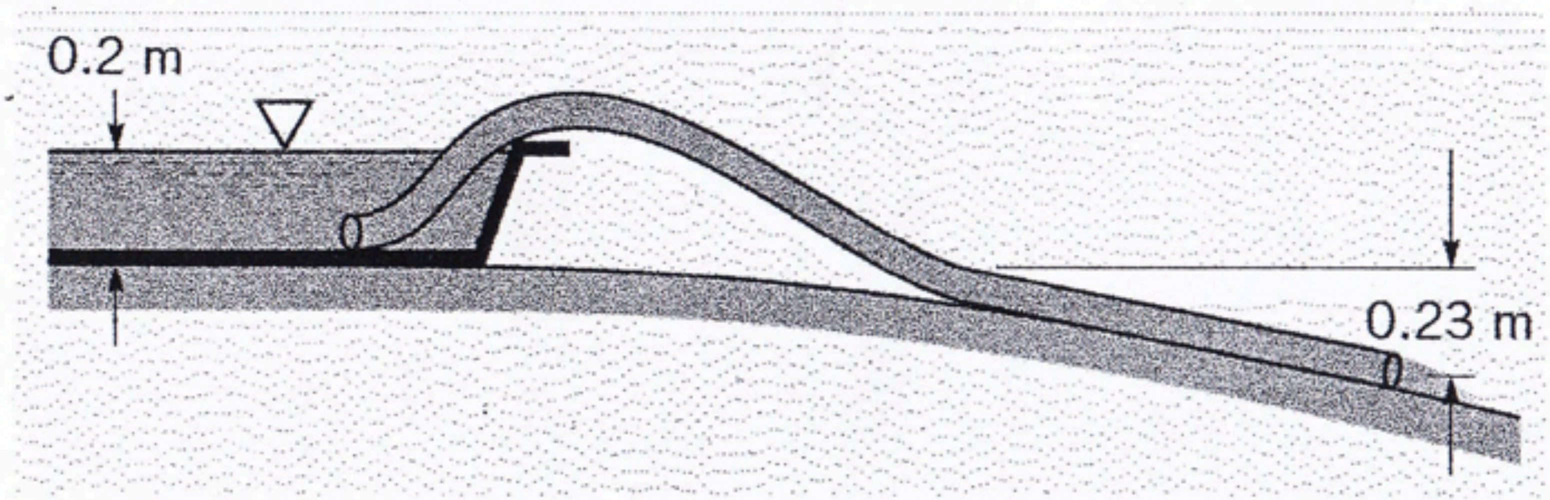


Figure 2

A smooth plastic, 10-m long garden hose with an inside diameter of 20-mm is used to drain a wading pool as is shown in Figure 2. If viscous effects are neglected, what is the flowrate from the pool.

$$V_2 = 2.90 \quad m/s$$

$$Q = 9.11 \times 10^{-4} \quad m^3/s$$

TUTORIAL 7

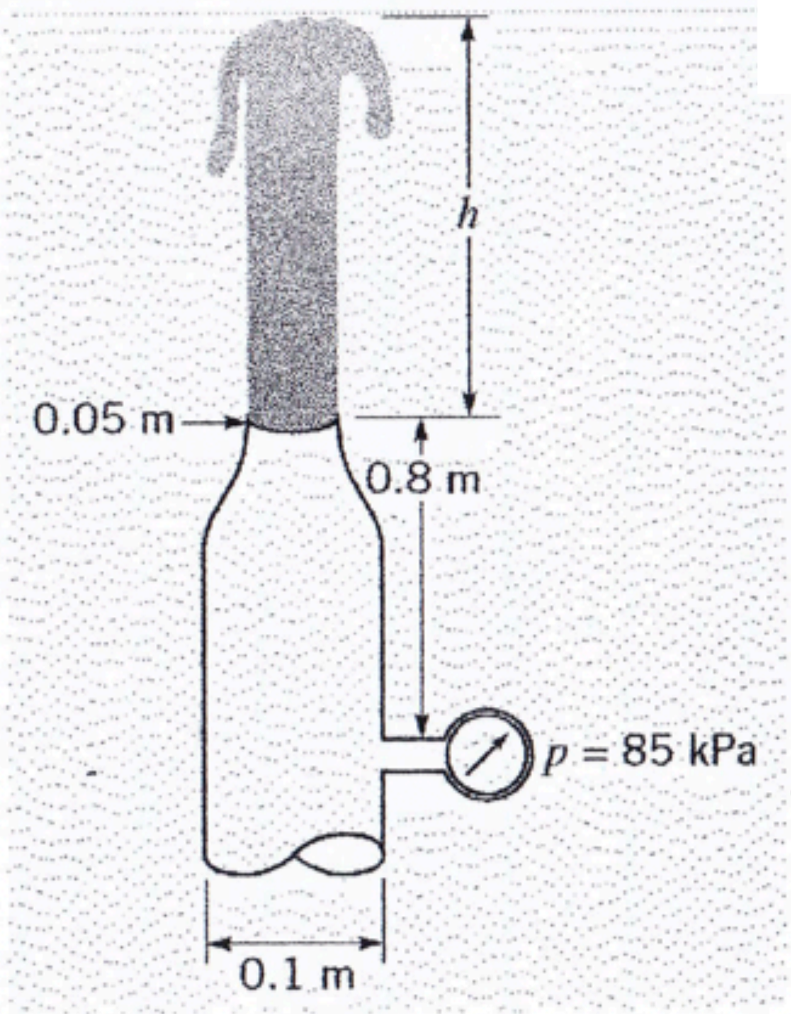


Figure 3

Water flows without viscous effect from the nozzle shown in Figure 3. Determine the flowrate and the height, h to which the water can flow.

TUTORIAL 8

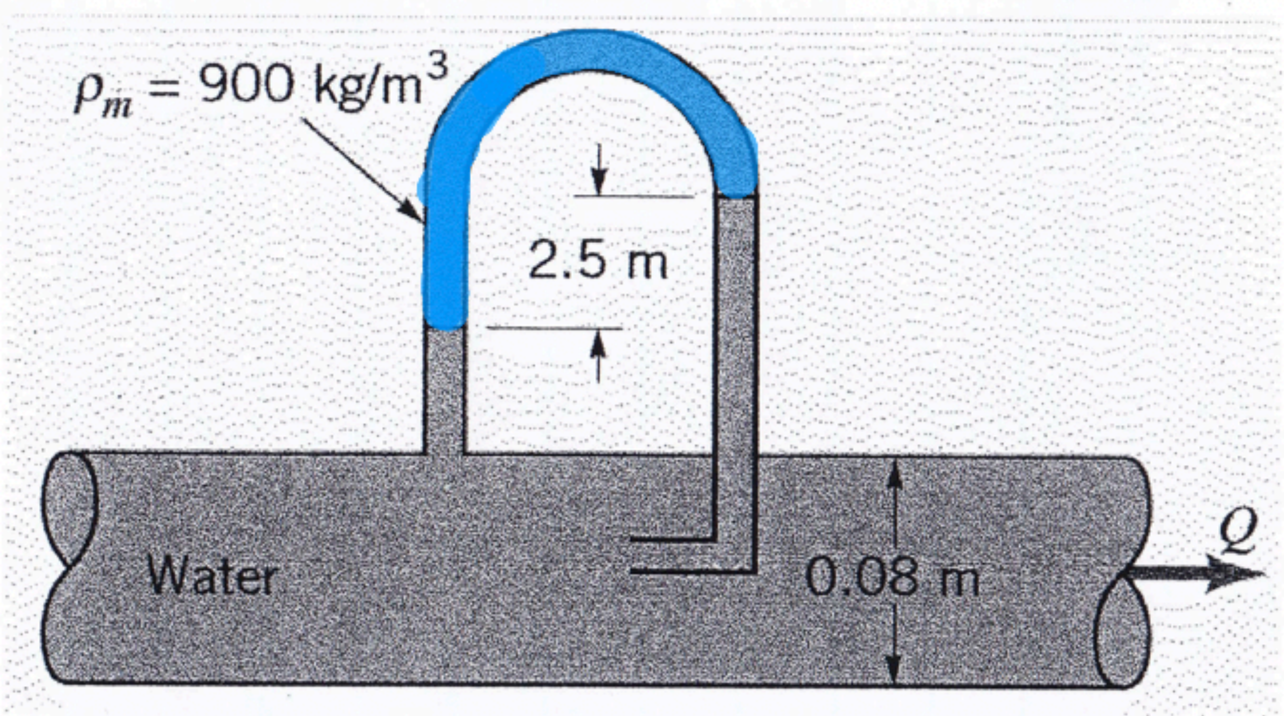


Figure 4

Determine the flowrate through the pipe shown in Figure 4.

$$V_1 = 2.20 \quad m/s$$

$$Q = 0.0111 \quad m^3/s$$

TUTORIAL 9

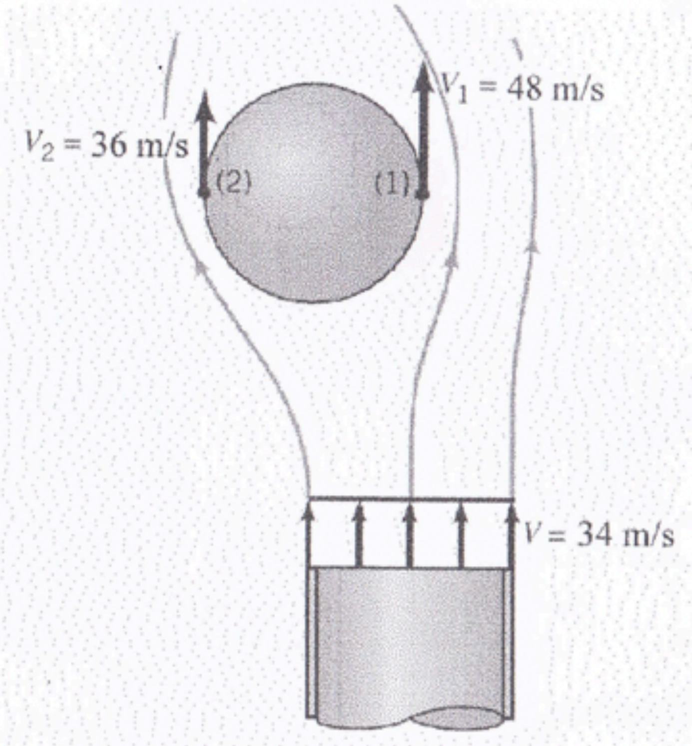


Figure 3

A 34m/s jet of air flows past a ball as shown in Figure-3. When the ball is not centered in the jet, the air velocity is greater on the side of the ball near the jet center [point (1)] than it is on the other side of the ball [point (2)]. Determine the pressure difference, $p_2 - p_1$, across the ball if $V_1 = 48$m/s and $V_2 = 36$m/s. Neglect gravity and viscous effects. Explain how this effect helps keep the ball centered on the jet.

TUTORIAL 10

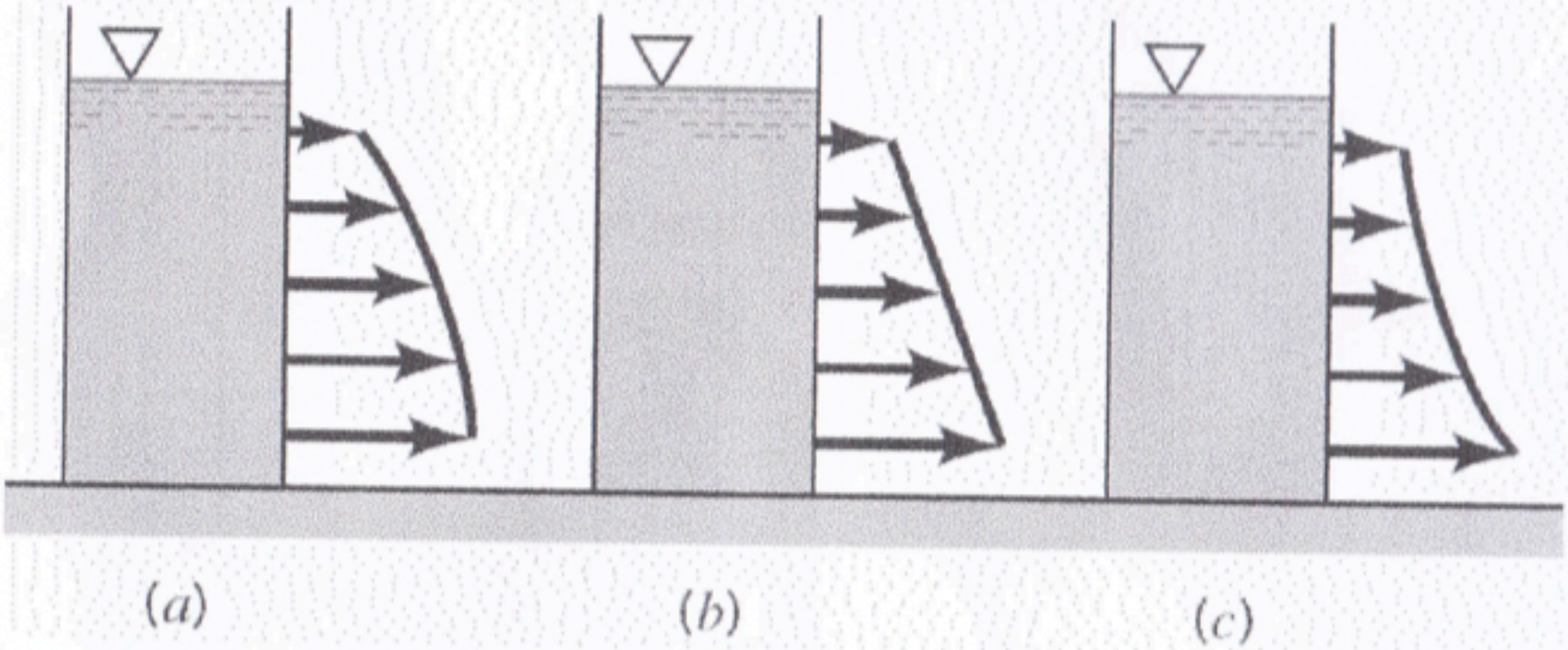


Figure 4

Several holes are punched into a tin can as shown in Figure 4. Which of the figures represents the variation of the water velocity as it leaves the holes? Justify your choice.