FLUID MECHANICS I

SEMM 2313

BUCKINGHAM PI THEOREM

Dimensional Analysis

It is used to determine the equation is right or wrong.

The calculation is depending on the unit or dimensional conditions of the equations.

For example;

$F = ma$

$F = MLT^{-2}$

Unit : $F = kg.m/s$

Buckingham Pi Theorem

If an equation involving k variables is dimensionally homogeneous, it can be reduced to a relationship among $k - r$ independent dimensionless products, where r is the minimum number of reference dimensions required to describe the variables.

For example, the function of f can be written as ;

$$f\,(\pi_1,\ \pi_2,\ \pi_3,\ \ldots\ldots,\ \pi_{k-r}) = 0$$

Or

$$\pi_1 = f\,(\pi_2,\ \pi_3,\ \pi_4,\ \ldots\ldots,\ \pi_{k-r}) = 0$$

The dimensionless products are frequently referred to as *"pi terms"*, and the theorem is called the Buckingham Pi Theorem.

Buckingham used the symbol π to represent a dimensionless product, and this notation is commonly used.

To summarize, the steps to be followed in performing a dimensional analysis using the method of repeating variables are as follows :

- Step 1 List all the variables that are involved in the problem.
- Step2 Express each of the variables in terms of basic dimensions.
- Step 3 Determine the required number of pi terms.
- Step 4 Select a number of repeating variables, where the number required is equal to the number of reference dimensions. (usually the same as the number of basic dimensions)
- Step 5 Form the pi term by multiplying one of the nonrepeating variables by the product of repeating variables each raised to an exponent that will make the combination dimensionless.
- Step 6 Repeat step 5 for each of the remaining nonrepeating variables.
- Step 7 Check all the resulting pi terms to make sure they are dimensionless.
- Step 8 Express the final form as a relationship among the pi terms and think about what it means.

Selection of Variables

One of the most important, and difficult, steps in applying dimensional analysis to any given problem is the selection of the variables that are involved.

For most engineering problems (including areas outside fluid mechanics), pertinent variables can be classified into three groups – geometry, material properties and external effects.

Geometry :

The geometry characteristics can be usually be described by a series of lengths and angles.

Example: length [L]

Material properties / kinematic :

More relates to the kinematic properties of fluid particles.

Example: velocity [LT^{-1}]

External effects / dynamic :

This terminology is used to denote any variable that produces, or tends to produce, a change in the system. For fluid mechanics, variables in this class would be related to pressure, velocities, or gravity. (combination of geometry and material properties)

Example: force [MLT^{-2}]

Common Dimensionless Groups in Fluid Mechanics

■ **TABLE 7.1**

Some Common Variables and Dimensionless Groups in Fluid Mechanics

Variables: Acceleration of gravity, g; Bulk modulus, E_v; Characteristic length, ℓ; Density, ρ; Frequency of oscillating flow, ω; Pressure, p (or Δp); Speed of sound, c; Surface tension, σ; Velocity, V; Viscosity, μ

Dimensionless Groups	Name	Interpretation (Index of Force Ratio Indicated)	Types of Applications
$\frac{\rho V \ell}{\mu}$	Reynolds number, Re	$\frac{\text{inertia force}}{\text{viscous force}}$	Generally of importance in all types of fluid dynamics problems
$\frac{V}{\sqrt{g\ell}}$	Froude number, Fr	$\frac{\text{inertia force}}{\text{gravitational force}}$	Flow with a free surface
$\frac{p}{\rho V^2}$	Euler number, Eu	$\frac{\text{pressure force}}{\text{inertia force}}$	Problems in which pressure, or pressure differences, are of interest
$\frac{\rho V^2}{E_v}$	Cauchy number,[a] Ca	$\frac{\text{inertia force}}{\text{compressibility force}}$	Flows in which the compressibility of the fluid is important
$\frac{V}{c}$	Mach number,[a] Ma	$\frac{\text{inertia force}}{\text{compressibility force}}$	Flows in which the compressibility of the fluid is important
$\frac{\omega \ell}{V}$	Strouhal number, St	$\frac{\text{inertia (local) force}}{\text{inertia (convective) force}}$	Unsteady flow with a characteristic frequency of oscillation
$\frac{\rho V^2 \ell}{\sigma}$	Weber number, We	$\frac{\text{inertia force}}{\text{surface tension force}}$	Problems in which surface tension is important

[a]The Cauchy number and the Mach number are related and either can be used as an index of the relative effects of inertia and compressibility. See accompanying discussion.

Items	Problems
Reynolds number	Flow in pipe.
Froude number	Flow of water around ship. Flow through rivers or open conduits.
Euler number	Pressure problems. Pressure difference between two points.
Cauchy number	Fluid compressibility.
Mach number	Fluid compressibility.
Strouhal number	Unsteady, oscillating flow.
Weber number	Interface between two fluid. Surface tension problems.

Modeling and Similitude

Models are widely used in fluid mechanics.

Major engineering projects involving structures, aircraft, ships, rivers, harbor, and so on, frequently involve the used of models.

A model (engineering model) is a representation of a physical system that may be used to predict the behavior of the system in some desired respect.

The physical system for which the predictions are to be made is called the prototype.

Usually a model is smaller than the prototype. Therefore, it is more easily handled in the laboratory and less expensive to construct and operate than a large prototype.

However, if the prototype is very small, it may be advantageous to have a model that is larger than a prototype so that it can be more easily studied.

TABLE 1.1

Dimensions Associated with Common Physical Quantities

	FLT System	*MLT* System
Acceleration	LT^{-2}	LT^{-2}
Angle	$F^0L^0T^0$	$M^0L^0T^0$
Angular acceleration	T^{-2}	T^{-2}
Angular velocity	T^{-1}	T^{-1}
Area	L^2	L^2
Density	$FL^{-4}T^2$	ML^{-3}
Energy	FL	ML^2T^{-2}
Force	F	MLT^{-2}
Frequency	T^{-1}	T^{-1}
Heat	FL	ML^2T^{-2}
Length	L	L
Mass	$FL^{-1}T^2$	M
Modulus of elasticity	FL^{-2}	$ML^{-1}T^{-2}$
Moment of a force	FL	ML^2T^{-2}
Moment of inertia (area)	L^4	L^4

	FLT System	*MLT* System
Moment of inertia (mass)	FLT^2	ML^2
Momentum	FT	MLT^{-1}
Power	FLT^{-1}	ML^2T^{-3}
Pressure	FL^{-2}	$ML^{-1}T^{-2}$
Specific heat	$L^2T^{-2}\Theta^{-1}$	$L^2T^{-2}\Theta^{-1}$
Specific weight	FL^{-3}	$ML^{-2}T^{-2}$
Strain	$F^0L^0T^0$	$M^0L^0T^0$
Stress	FL^{-2}	$ML^{-1}T^{-2}$
Surface tension	FL^{-1}	MT^{-2}
Temperature	Θ	Θ
Time	T	T
Torque	FL	ML^2T^{-2}
Velocity	LT^{-1}	LT^{-1}
Viscosity (dynamic)	$FL^{-2}T$	$ML^{-1}T^{-1}$
Viscosity (kinematic)	L^2T^{-1}	L^2T^{-1}
Volume	L^3	L^3
Work	FL	ML^2T^{-2}