

MODELING AND SIMILITUDE

Models are widely used in fluid mechanics.

Major engineering projects involving structures, aircrafts, ships, rivers, harbor, and so on, frequently involve the use of models.

A **model** is a representation of a physical system that may be used to predict the behavior of the system in some desire respect.

The physical system for which the predictions are to be made is called the **prototype**.

Theory of Models

The theory of models can be readily developed by using the principles of dimensional analysis.

$$\pi_1 = f(\pi_2, \pi_3, \dots, \pi_n)$$

If above equation describes the behavior of a particular prototype, a similar relationship can be written for a model of this prototype, that is,

$$\pi_{1m} = f(\pi_{2m}, \pi_{3m}, \dots, \pi_{nm})$$

Pi terms, without a subscript will refer to the prototype.

The subscript **m** will be used to designate the model variables or pi terms.

The pi terms can be developed so that π_1 contains the variable that is to be predicted from observations made on the model. Therefore, if the model is designed and operated under the following conditions,

$$\pi_{2m} = \pi_2 \quad , \quad \pi_{3m} = \pi_3 \quad , \quad \pi_{nm} = \pi_n \quad \text{Eq.(1)}$$

Then with the presumption that the form of f is the same for model and prototype, it follows that,

$$\pi_{1m} = \pi_1 \quad \text{Eq.(2)}$$

Equation (2) is the desired prediction equation and indicates that the measured value of π_{1m} obtained with the model will be equal to the corresponding π_1 for the prototype as long as the other pi terms are equal.

The conditions specified by equation (1) provide the model design conditions, also called *similarity requirements* or *modeling laws*.

Model scales

We will take the ratio of the model value to the prototype value as the scale.

Length scales are often specified.

For example, as 1:10 or as a $\frac{1}{10}$ scale model.

The meaning of this specification is that the model is one-tenth the size of the prototype, and the tacit assumption is that all relevant lengths are scaled accordingly, so the model is geometrically similar to the prototype.

There are, however, other scales such as the ;

Velocity scale, $\frac{V_m}{V}$

Density scale, $\frac{\rho_m}{\rho}$

Viscosity scale, $\frac{\mu_m}{\mu}$

And so on.

Models for which one or more similarity requirements are not satisfied are called *distorted models*.

Models for which all similarity requirements are met are called *true models*.