BUCKINGHAM PI THEOREM

The friction factor f depends on the various quantities that affect the flow such as the density of fluid, dynamic viscosity of fluid, fluid velocity, pipe diameter and pipe inner wall roughness.

Determine the function

$$f = f(\rho\ , \mu\ , V\ , D\ , e)$$

In some textbook, it can be written as :

$$f = \emptyset(\rho\ , \mu\ , V\ , D\ , e)$$

We read as;
friction factor, f is the function of $\rho\ , \mu\ , V\ , D\ \ and\ \ e$

There are 6 pi terms

List the MLT term for all the pi terms

Term	MLT
f	$M^0L^0T^0$
ρ	ML^{-3}
μ	$ML^{-1}T^{-1}$
V	LT^{-1}
D	L
e	L

Divide all pi terms into geometry, kinematic and dynamic groups, except for friction factor, f . Friction factor is the main subject for this function.

Then, choose one most influence parameter for each group.

Group	parameter	Most influence parameter as known as "repeating parameter"	Non-repeating parameter
Geometry (L)	D , e	D	e
Kinematic (LT^{-1})	V	V	
Dynamic	ρ , μ	ρ	μ

There are 3 repeating parameter here.
(maximum repeating parameter is 3, it can be 2 or 1, but cannot more than 3)

Develop the pi term

$$number\ of\ pi\ term = (total\ parameter) - (total\ repeating\ parameter)$$

$$number\ of\ pi\ term = (6) - (3) = 3\ pi\ term$$

List the pi term;

$$\pi_n = (non - repeating\ parameter) \times (repeating\ parameter)^A$$

$$\pi_1 = f \times \rho^A V^B D^C$$

$$\pi_2 = e \times \rho^A V^B D^C$$

$$\pi_3 = \mu \times \rho^A V^B D^C$$

Determine the π_1

$$\pi_1 = f \times \rho^A V^B D^C$$

$$M^0 L^0 T^0 = M^0 L^0 T^0 \times [ML^{-3}]^A [LT^{-1}]^B [L]^C$$

$$A = 0 \ , \qquad B = 0 \ , \qquad C = 0$$

$$\pi_1 = f$$

Determine the π_2

$$\pi_2 = e \times \rho^A V^B D^C$$

$$M^0 L^0 T^0 = L \times [ML^{-3}]^A [LT^{-1}]^B [L]^C$$

$$A = 0 \ , \qquad B = 0 \ , \qquad C = -1$$

$$\pi_2 = e \times D^{-1} = \frac{e}{D}$$

Determine the π_3

$$\pi_3 = \mu \times \rho^A V^B D^C$$

$$M^0 L^0 T^0 = ML^{-1}T^{-1} \times [ML^{-3}]^A [LT^{-1}]^B [L]^C$$

$$A = -1 \ , \qquad B = -1 \ , \qquad C = -1$$

$$\pi_3 = \mu \times \rho^{-1} V^{-1} D^{-1} = \frac{\mu}{\rho VD} = \frac{\rho VD}{\mu}$$

Note:

$\frac{\mu}{\rho VD}$ can be reversed because this term is dimensionless.

List all the pi term ;

$$\pi_1 = \emptyset(\pi_2, \pi_3)$$

$$f = \emptyset\left(\frac{e}{D}, \frac{\rho V D}{\mu}\right)$$

That is why in the Moody chart, Reynolds number and relative roughness being selected to determine the friction factor.

Those parameters are not randomly selected. It is selected based on the scientific method.