FLUID MECHANICS I

SEMM 2313

BUOYANCY, FLOATATION AND STABILITY

ARCHIMEDES' PRINCIPLE

When a stationary body is completely submerged in a fluid, or floating so that it is only partially submerged, the resultant fluid force acting on the body is called the buoyant force.

Note that the forces F_1, F_2, F_3, and F_4 are simply the forces exerted on the plane surfaces, W(=mg) is the weight of the shaded fluid volume, and F_B is the force the body is exerting on the fluid.

The forces on the vertical surfaces, such as F_3 and F_4, are all equal and cancel, so the equilibrium equation of interest is in the *z* direction and can be expressed as

$$F_B = F_2 - F_1 - mg$$

If the specific weight of the fluid is constant, then ;

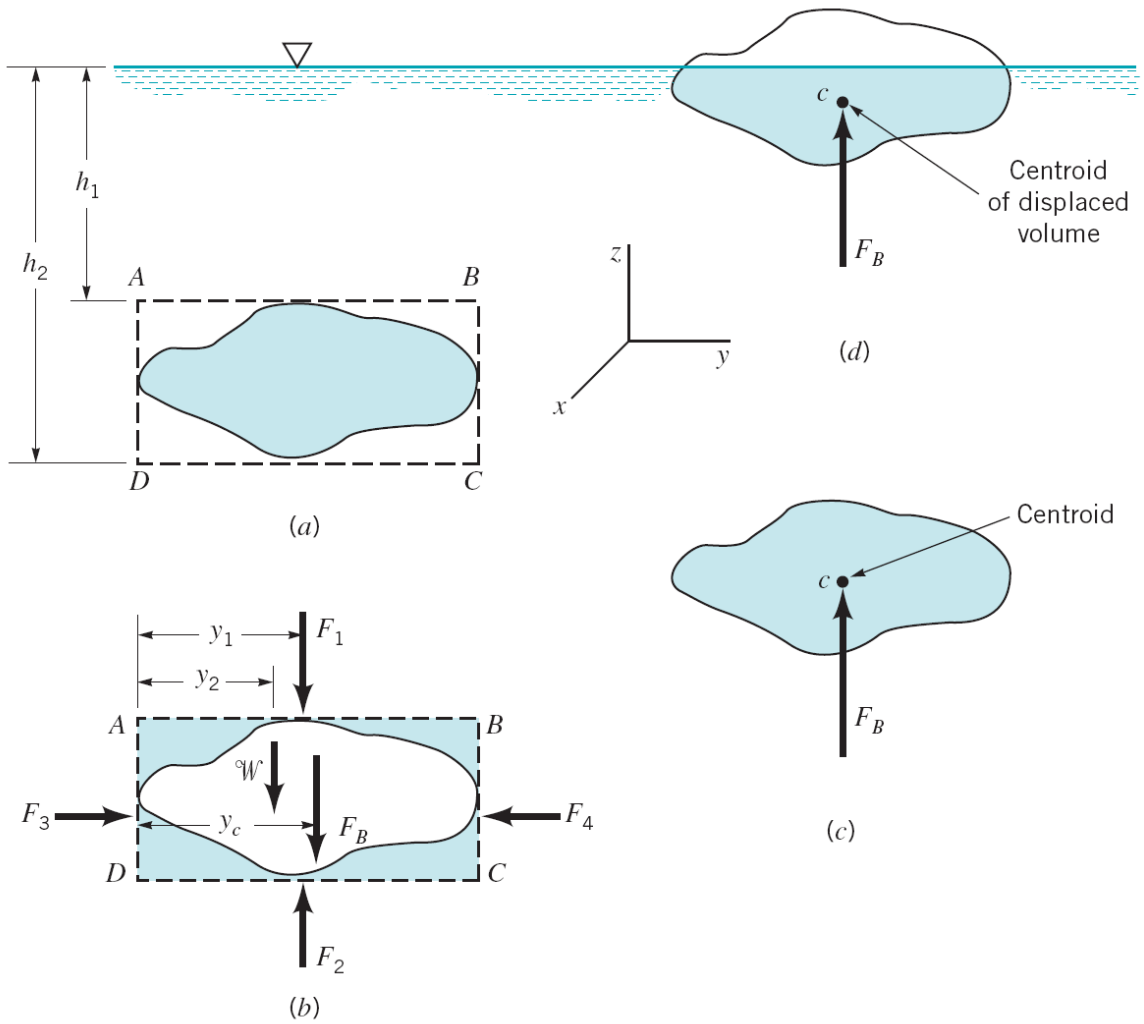
$$F_2 - F_1 = \rho g (h_2 - h_1) A \qquad (1)$$

where A is the horizontal area of the upper (or lower) surface, and Equation (1) can be written as ;

$$F_B = \rho g (h_2 - h_1) A - \rho g \left[(h_2 - h_1) A - V \right]$$

Simplifying, we arrive at the desired expression for the buoyant force ;

$$F_B = \rho g V$$



h_1
h_2
A
B
D
C
(a)
z
y
x
c
Centroid
of displaced
volume
F_B
(d)
y_1
F_1
y_2
A
B
$\mathscr{W}$
F_3
y_c
F_B
F_4
D
C
F_2
(b)
Centroid
c
F_B
(c)

Figure 1

Archimedes' principle states that the buoyant force has a magnitude equal to the weight of the fluid displaced by the body and is directed vertically upward.

Thus, we conclude that the buoyant force passes through the centroid of the displaced volume as shown in Figure 1(c).

The point through which the buoyant force acts is called the center of buoyancy.

Example

A spherical buoy has a diameter of 1.5 m, weighs 8.50 kN, and is anchored to the seafloor with a cable as is shown in Figure 2(a). Although the buoy normally floats on the surface, at certain times the water depth increases so that the buoy is completely immersed as illustrated. For this condition what is the tension of the cable?

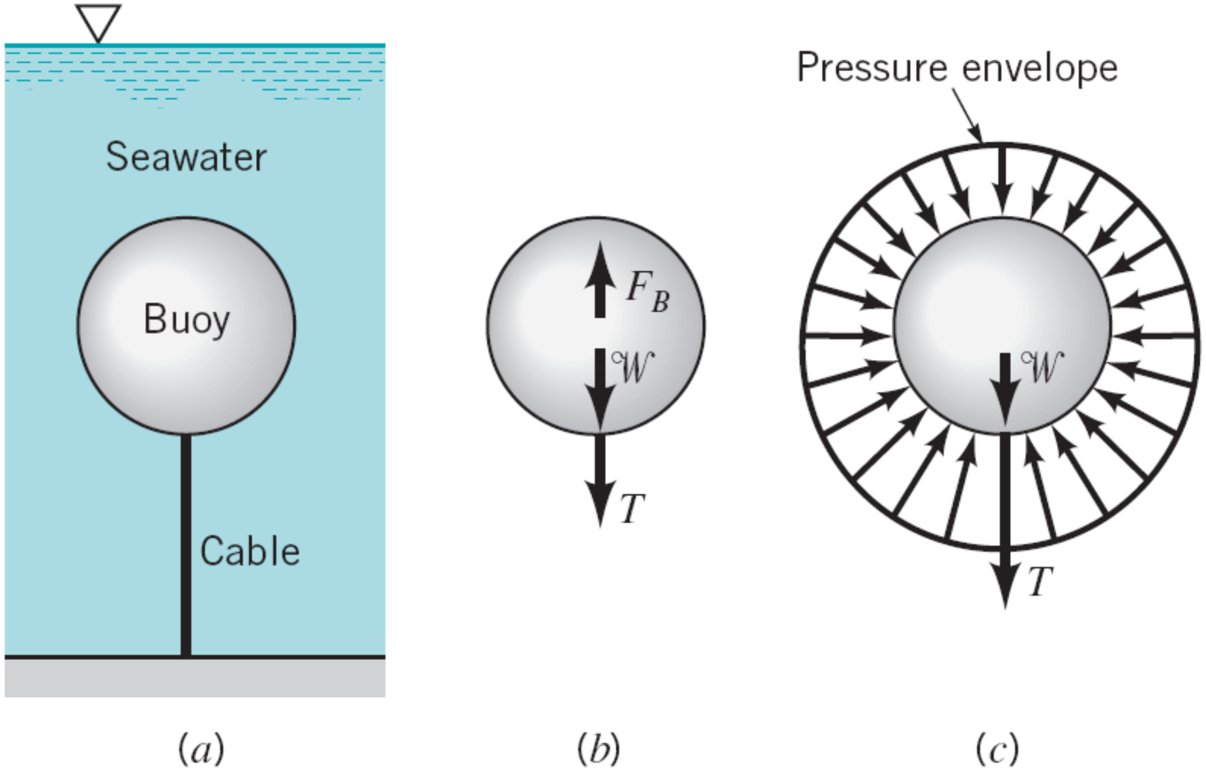

Seawater
Buoy
Cable
F_B
$\mathscr{W}$
T
Pressure envelope
$\mathscr{W}$
T
(a)
(b)
(c)

Figure 2

Force in equilibrium:

$$T + W = F_B$$

$$T = F_B - W$$

$$F_B = \rho g \forall = (1025)(9.81)\left(\frac{\pi D^3}{6}\right) = 17.77 \quad kN$$

$$T = 17.77 - 8.5 = 9.27 \quad kN$$

STABILITY

As illustrated by the Figure 3, a body is said to be in a stable equilibrium position if, when displaced, it returns to its equilibrium position.

Conversely, it is in an unstable equilibrium position if, when displaced (even slightly), it moves to a new equilibrium position.

Stability considerations are particularly important for submerged or floating bodies since the centers of buoyancy and gravity do not necessarily coincide.

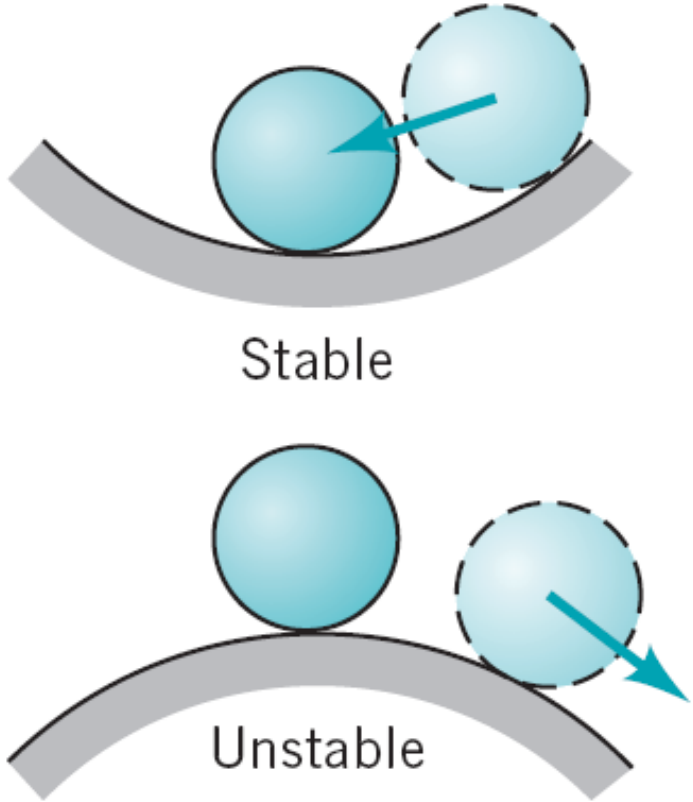


Figure 3

A small rotation can result in either a restoring or overturning couple.

For example, for the completely submerged body shown in Figure 4, which has a center of gravity below the center of buoyancy, a rotation from its equilibrium position will create a restoring couple formed by the weight, W, and the buoyant force, F_B, which causes the body to rotate back to its original position. Thus, for this configuration the body is stable. It is to be noted that as long as the center of gravity falls below the center of buoyancy, this will always be true; that is, the body is in a stable equilibrium position with respect to small rotations.

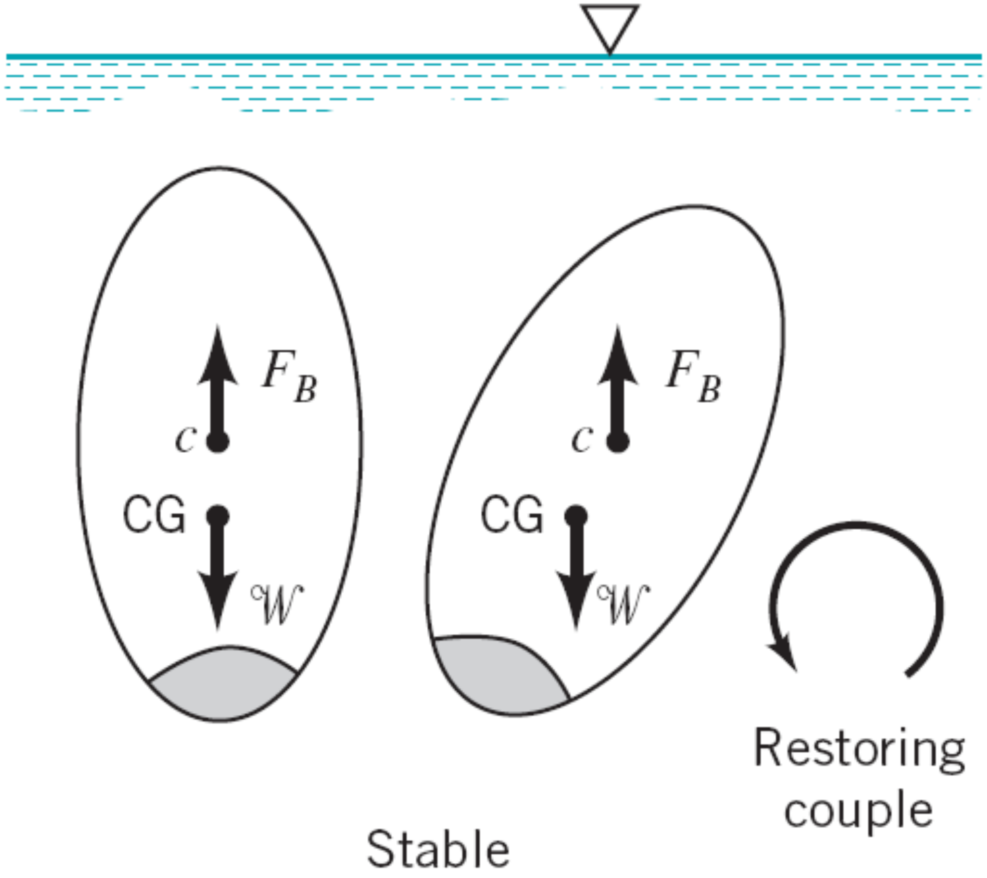
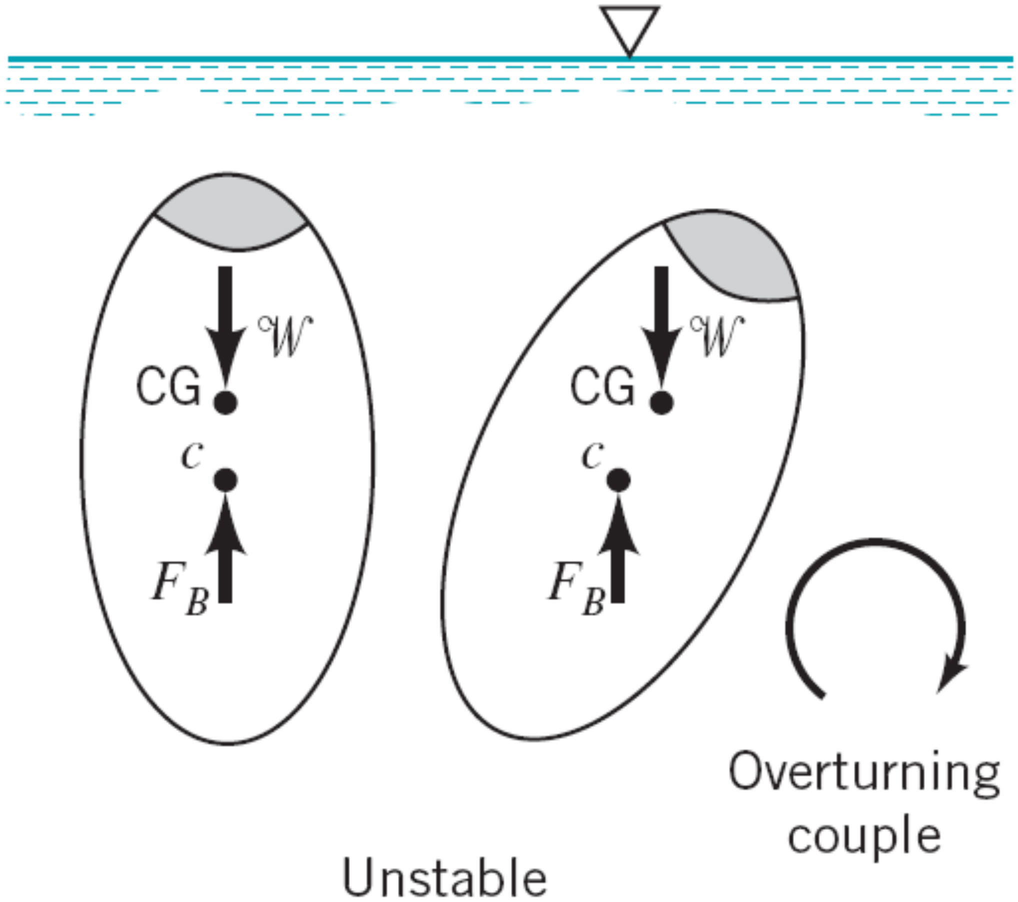


Figure 4

However, as is illustrated in Figure 5, if the center of gravity of the completely submerged body is above the center of buoyancy, the resulting couple formed by the weight and the buoyant force will cause the body to overturn and move to a new equilibrium position.

Thus, a completely submerged body with its center of gravity above its center of buoyancy is in an unstable equilibrium position.



CG
c
$\mathscr{W}$
F_B
CG
c
$\mathscr{W}$
F_B
Overturning
couple
Unstable

Figure 5

For floating bodies the stability problem is more complicated, since as the body rotates the location of the center of buoyancy (which passes through the centroid of the displaced volume) may change.

As is shown in Figure 6, a floating body such as a barge that rides low in the water can be stable even though the center of gravity lies above the center of buoyancy.

This is true since as the body rotates the buoyant force, F_B, shifts to pass through the centroid of the newly formed displaced volume and, as illustrated, combines with the weight, W, to form a couple which will cause the body to return to its original equilibrium position.

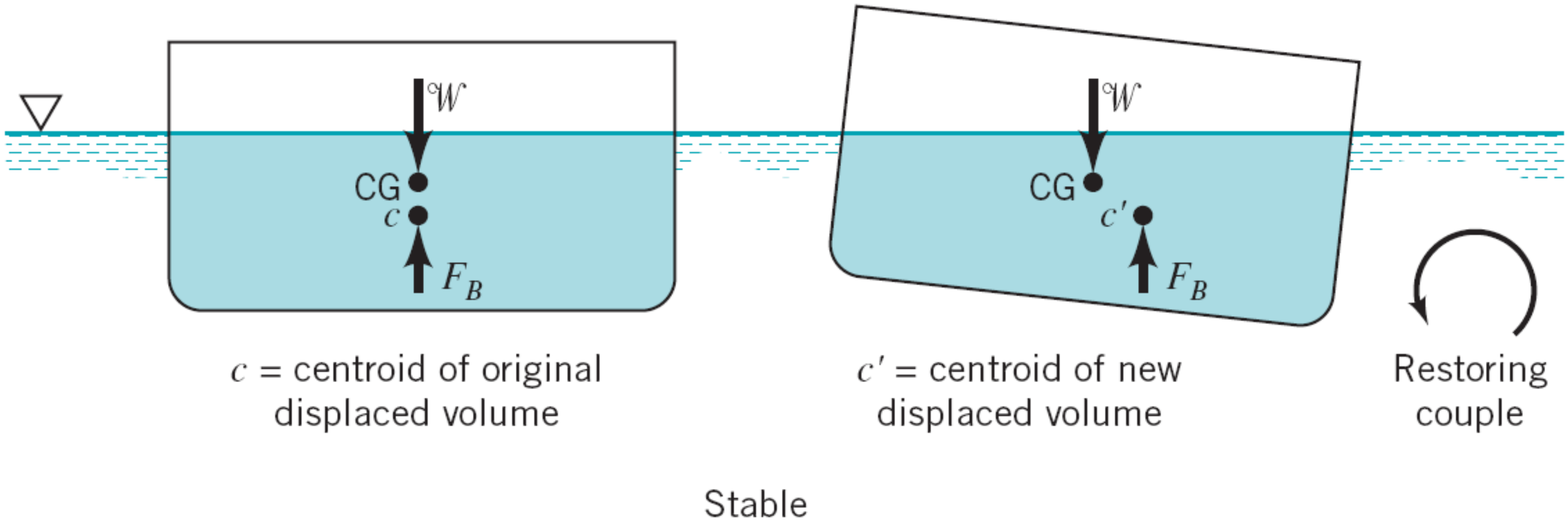

𝒲
CG
c
F_B
c = centroid of original displaced volume
𝒲
CG
c′
F_B
c' = centroid of new displaced volume
Restoring couple
Stable

Figure 6

However, for the relatively tall, slender body shown in Figure 7, a small rotational displacement can cause the buoyant force and the weight to form an overturning couple as illustrated.

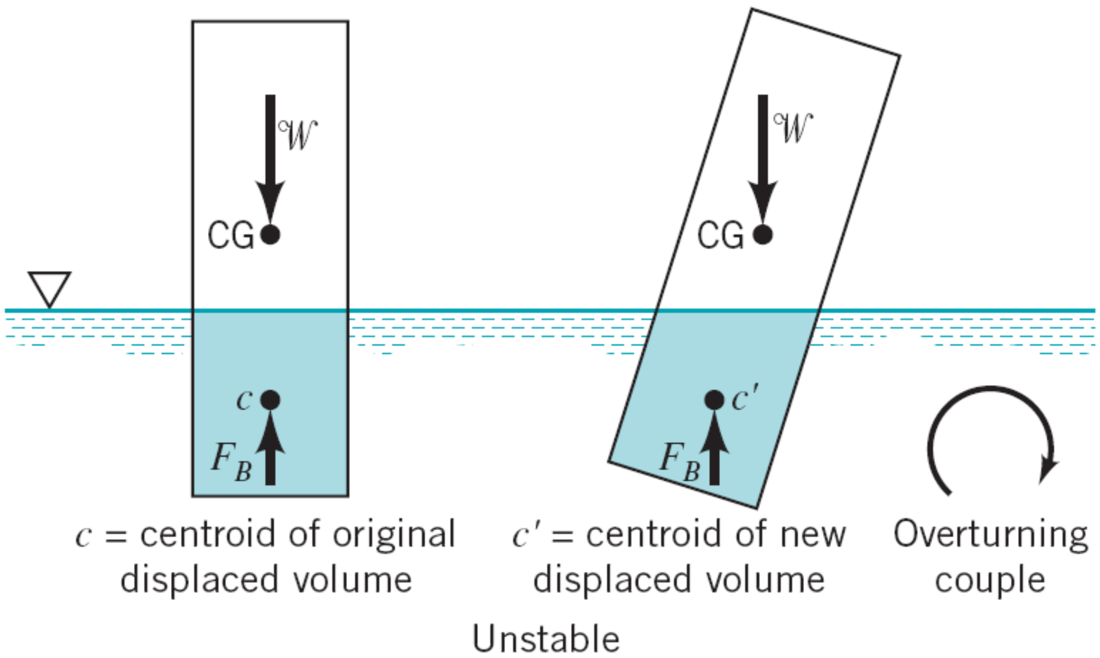


Figure 7

STABILITY OF A SUBMERGED BODY

For a body totally immersed in a fluid, the weight $W = mg$ acts through the center of gravity, G, of the body. While the force of buoyancy, F_B acts through the centroid of the body B, which is the center of buoyancy.

If the center of gravity, G, is below the center of buoyancy, B, this will be a righting moment and the body will tend to return to its equilibrium position. The body is stable.

If the center of gravity, G, is above the center of buoyancy, an overturning moment is produced, and the body is unstable.

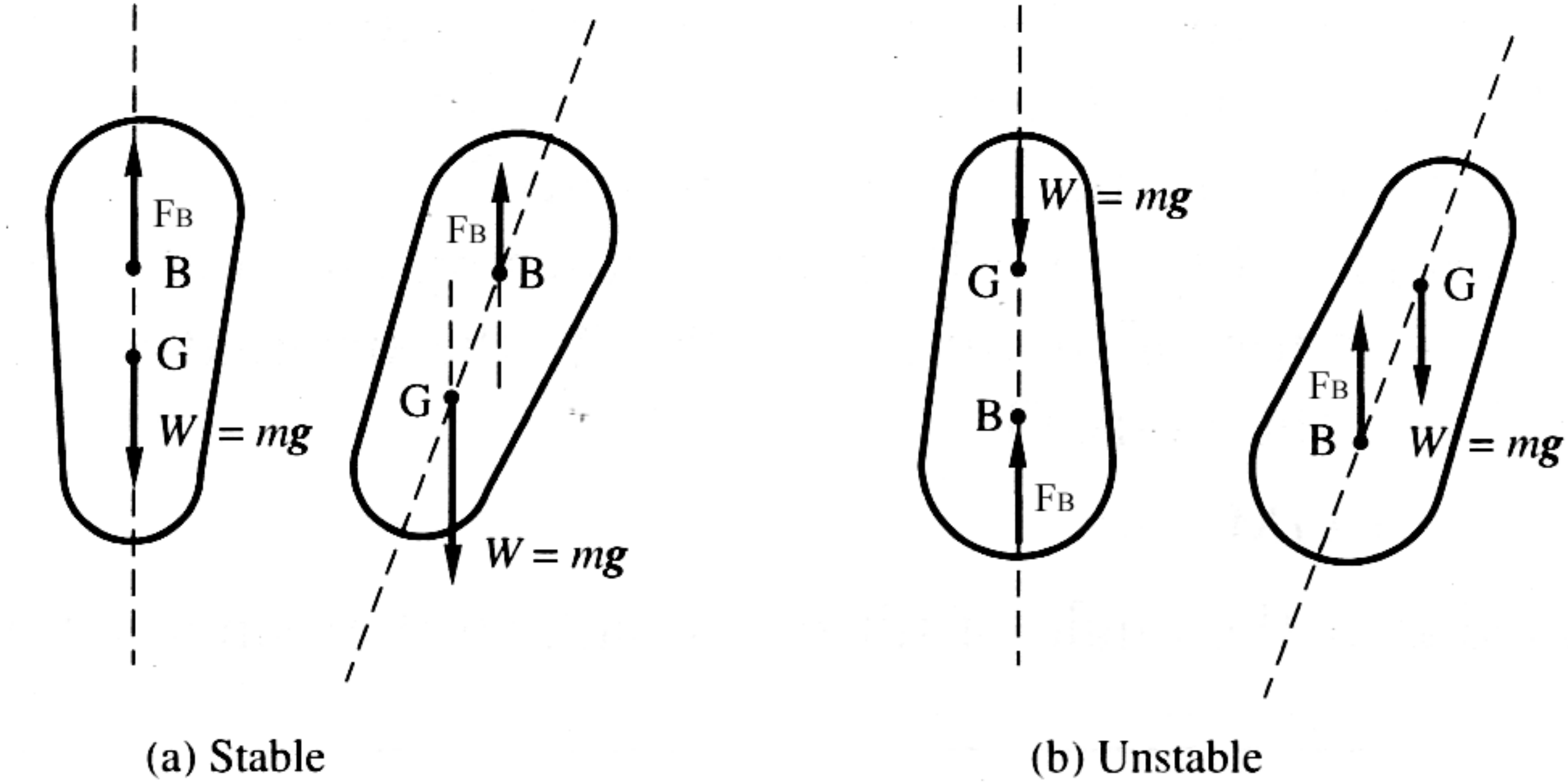


Figure 8

STABILITY OF FLOATING BODIES

Figure 9(a) shows a body floating in equilibrium. The weight, W, acts through the center of gravity, G, and the force of buoyancy F_B acts through the center of buoyancy, B, of the displaced fluid in the same straight line as weight.

When the body is displaced through an angle θ, as shown in Figure 9(b), W continues to act through G. The volume of liquid remains unchanged, but the shape of this volume changes and its center of buoyancy moves relative to the body from B to B_1.

Since the force of buoyancy, F_B and weight, W, are no longer in the same straight line, a turning moment is produced.

Figure 9(b) shows the righting moment.

Figure 9(d) shows the overturning moment.

Point M is the point at which the line of action of the force of buoyancy, F_B, cuts the original vertical line through the center of gravity, G.

The point M is called the metacenter.

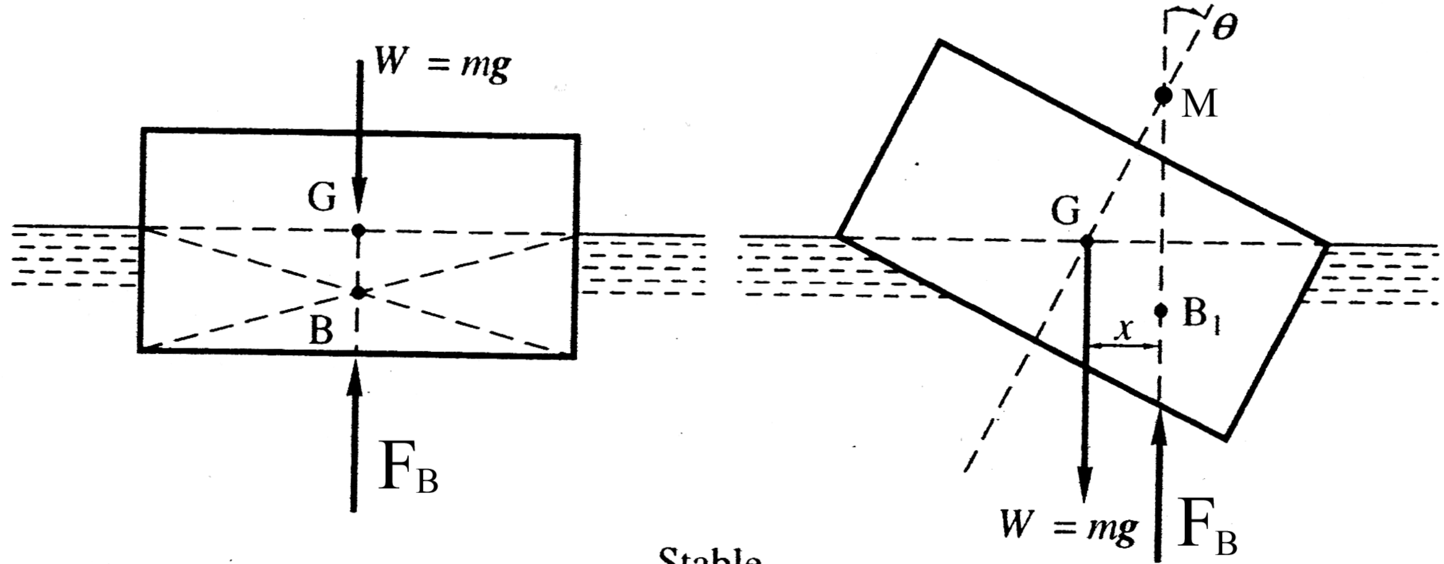
The distance GM is the metacentric height.

From Figure 9(b) and (d), it can be seen that ;

1. If point M lies above G, a righting moment is produced, equilibrium is stable and GM is regarded as positive.

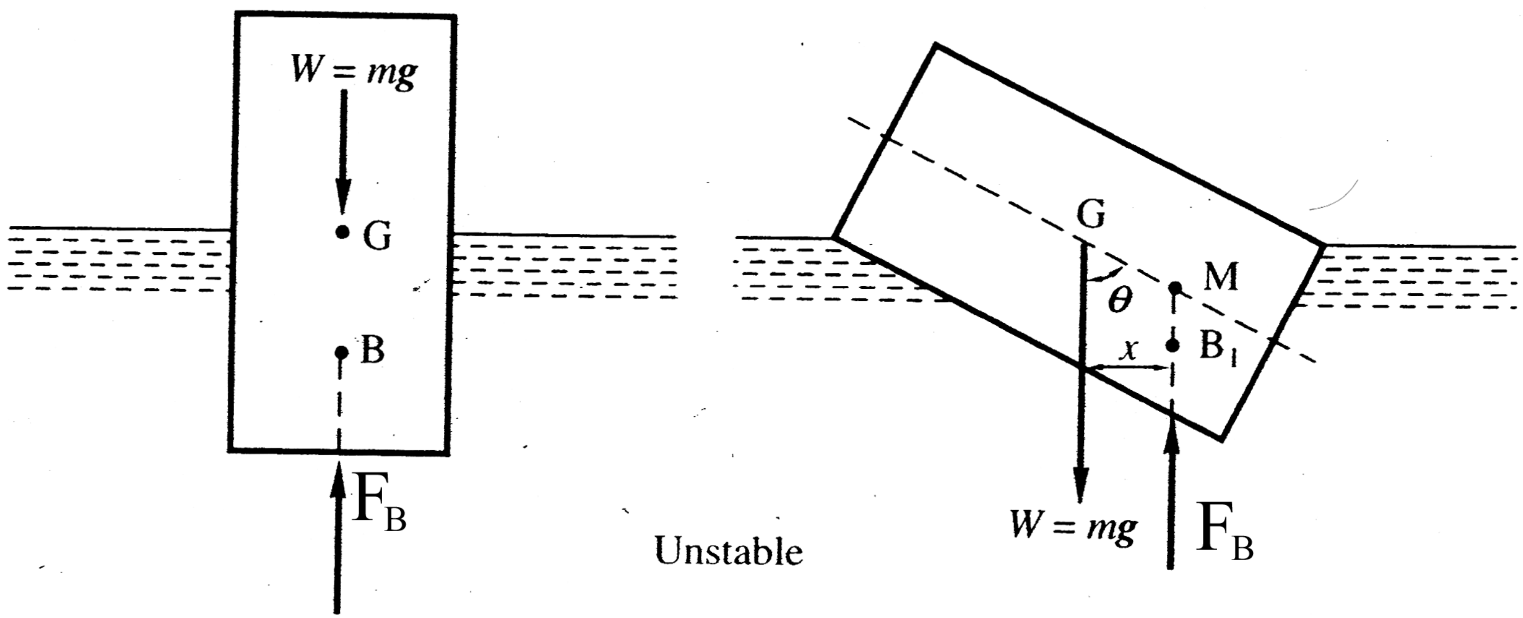
2. If point M lies below G, an overturning moment is produced, equilibrium is unstable and GM is regarded as negative.

3. If point M coincides with G, the body is in neutral equilibrium (stable equilibrium).



Stable

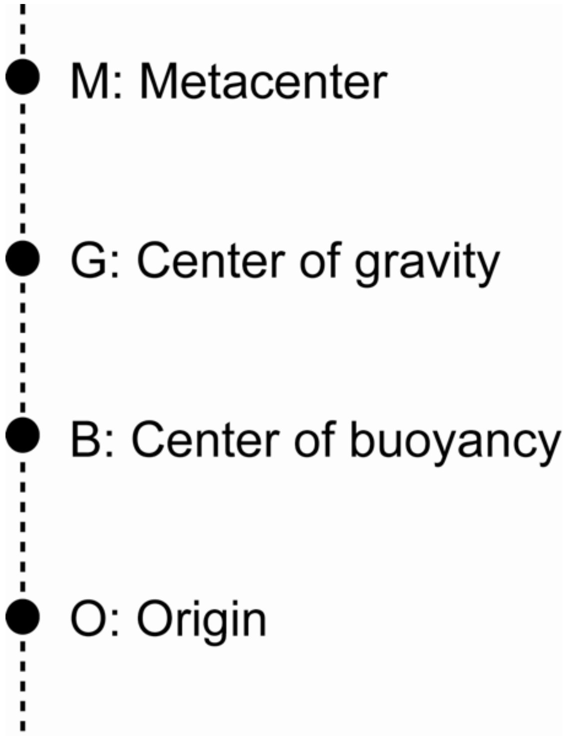
(a) (b)



Unstable

(c) (d)

Determination of the position of the metacenter relative to the center of buoyancy.



The distance BM is determine as ;

$$BM = \frac{I}{\forall}$$

$$BM = BG + GM$$

$$GM = \frac{I}{\forall} - BG$$

The distance BM is known as the metacentric radius.

I is second moment of area.

Example 1

A river barge, whose cross section is approximately rectangular, carries a load of grain. The barge is 9-m wide and 30-m long. When unloaded its draft (depth of submergence) is 3-m, and with the load of grain the draft is 4-m. Determine :

(a) The unloaded weight of the barge

(b) The weight of the grain

Example 2

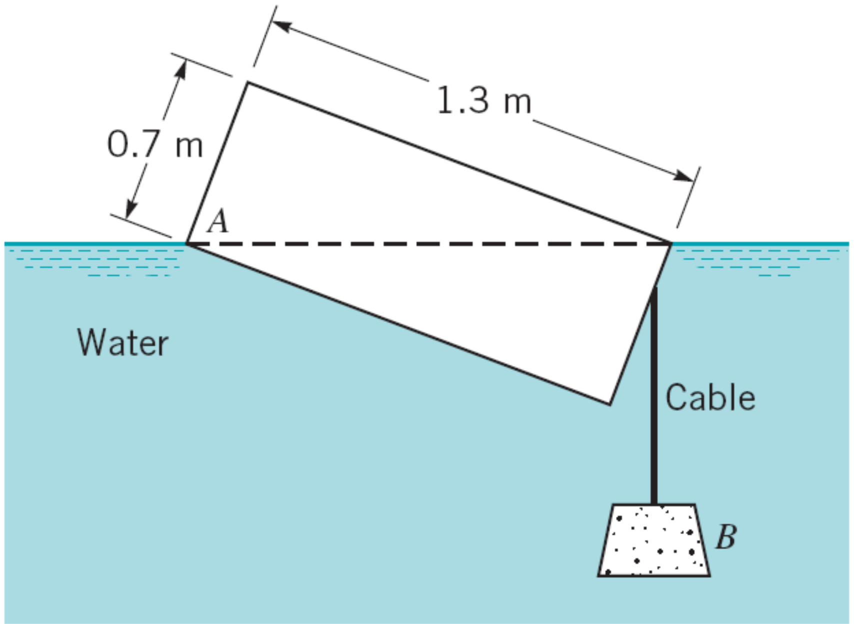


Figure 8

The homogeneous wooden block *A* of Figure 8 is 0.7-m by 0.7-m by 1.3-m and weighs 2.4 kN. The concrete block *B* (specific weight = 23.6 kN/m^3) is suspended from *A* by means of the slender cable causing *A* to float in the position indicated. Determine the volume of *B.*

Example 3

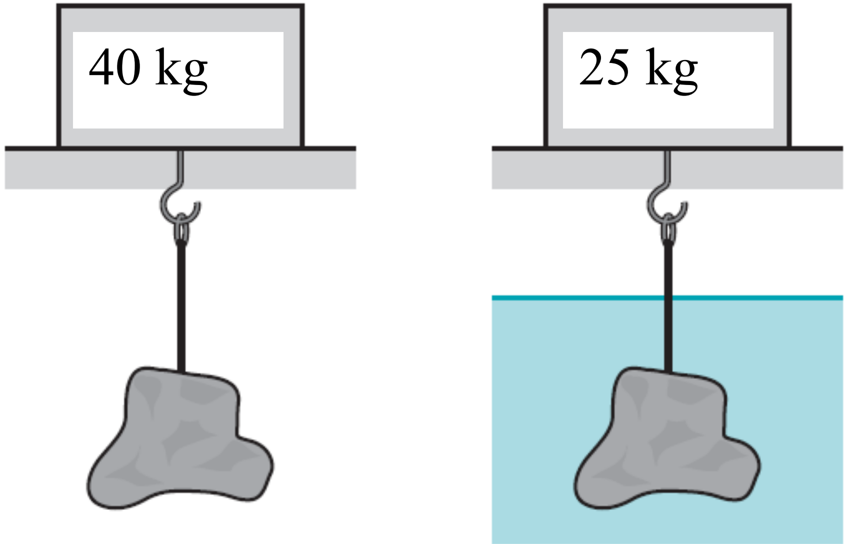
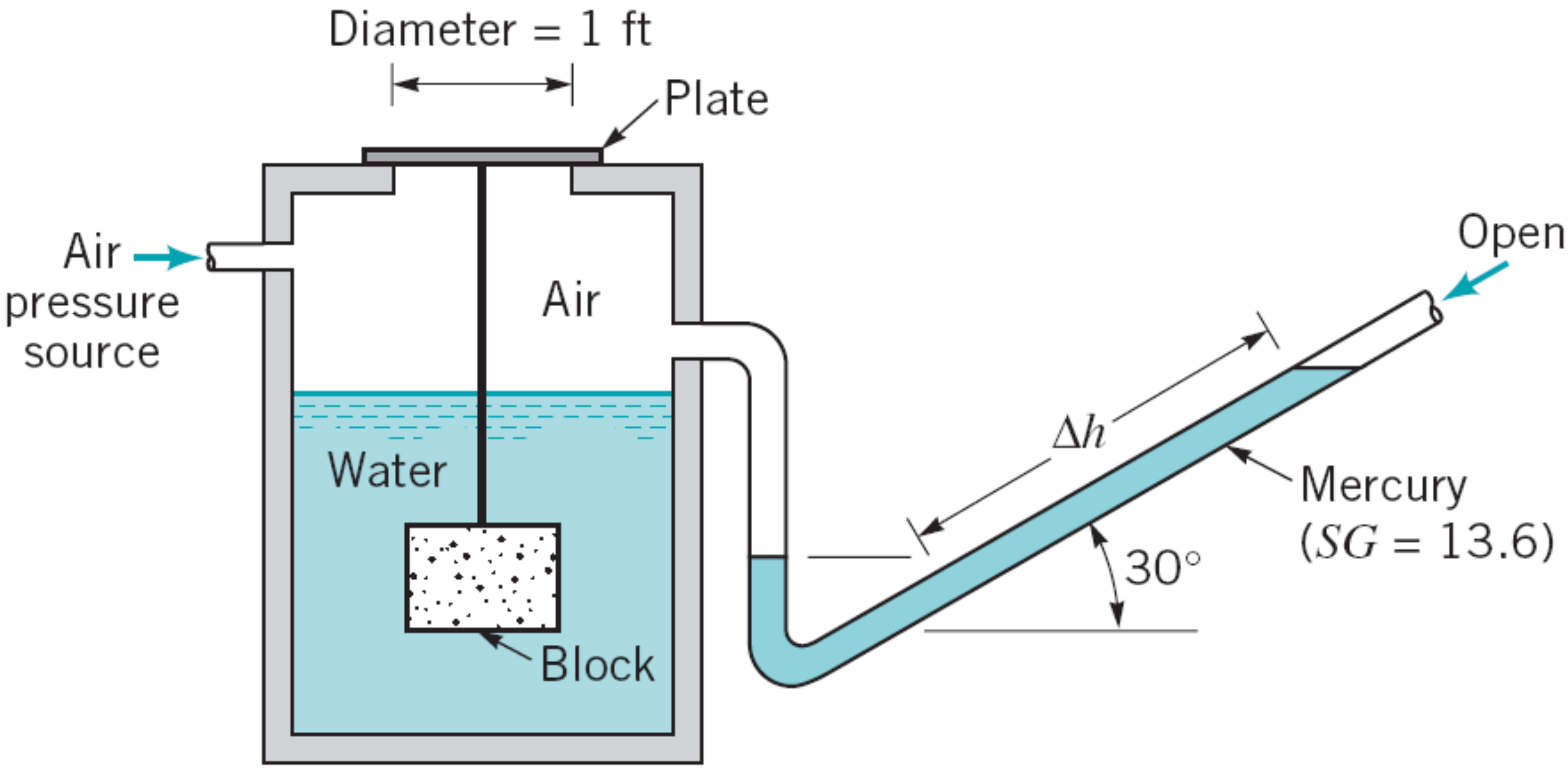


Figure 9

As shown in Figure 9, an irregularly shaped object weighs 40 kg in air and 25 kg when fully submerged in water. Determine the volume and specific gravity of the object.

Example 4

A plate of negligible weight closes a 0.3-m-diameter hole in a tank containing air and water as shown in Figure 10. A block of concrete (specific weight = 15000 N/m^3), having a volume of 0.5-m^3, is suspended from the plate and is completely immersed in the water. As the air pressure is increased the differential reading, Δh, on the inclined-tube mercury manometer increases. Determine Δh just before the plate starts to lift off the hole. The weight of the air has a negligible effect on the manometer reading.



Diameter = 0.3 m

Example 5

When a hydrometer in Figure 11 having a stem diameter of 1-cm is placed in water, the stem protrudes 80-cm above the water surface. If the water is replaced with a liquid having a specific gravity of 1.10, how much of the stem would protrude above the liquid surface? The hydrometer weighs 20 grams.

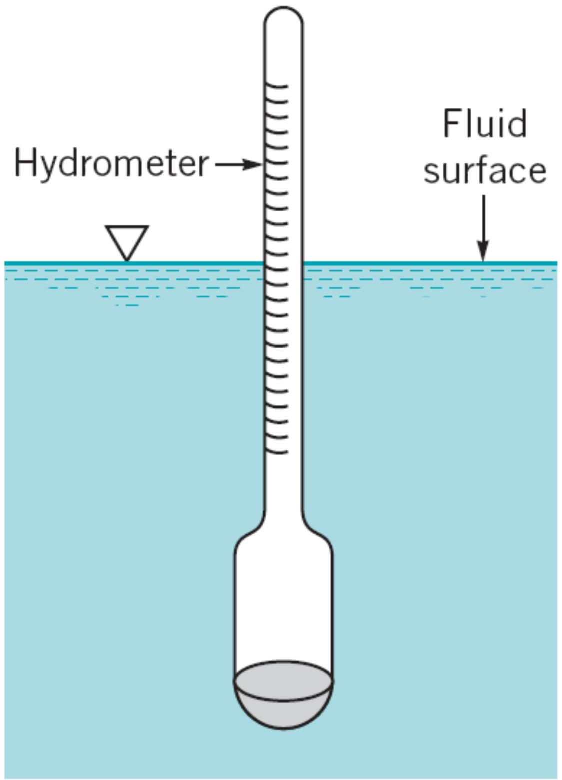


Figure 11

QUESTION 1

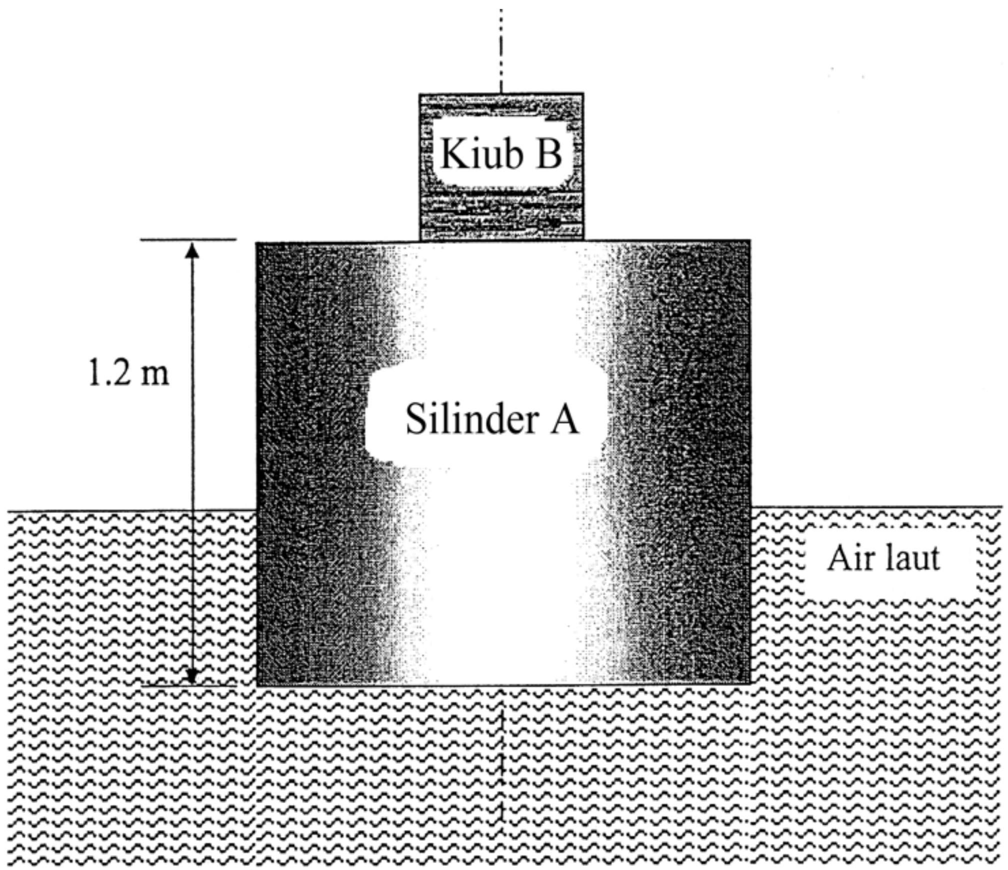
a) Terangkan prinsip Archimedes

b) Sebuah silinder A dengan ketinggian 1.2m dan diameter 2.4m, terendam di dalam air laut (ρ=1025kg/m^3) lalu menyesarkan 1050 kg air laut, seperti yang ditunjukkan pada Rajah 1. Kemudian kiub B yang mempunyai sisi-sisi yang sama panjang, seberat 3.5kN, diletakkan ditengah-tengah silinder tersebut dan kedalaman silinder A bertambah. Pusat gravity keseluruhan system bertambah 0.3m dari pusat gravity silinder A sebelum kiub B diletakkan. Tentukan

(i) Kestabilan system

(ii) Isipadu kiub B

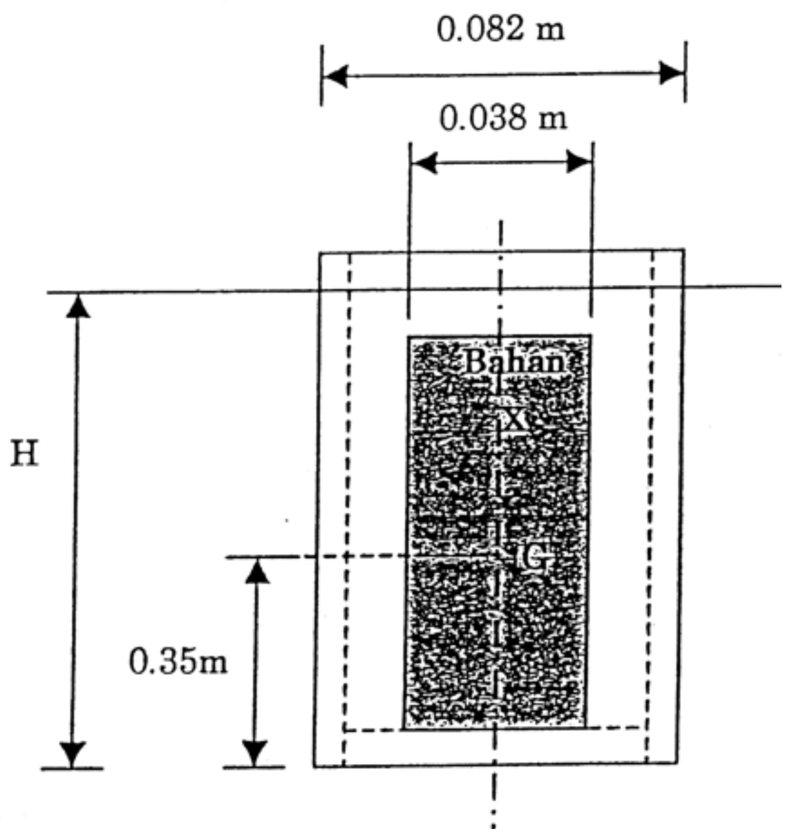
Diberi I_{cc} bulatan ialah $\frac{\pi D^4}{64}$ dan I_{cc} segiempat ialah $\frac{bh^3}{12}$



Rajah 1

QUESTION 2

Berat sebuah silinder berongga ialah 0.05N. Apabila sebatang bahan X dengan isipada (0.038m x 0.038m x 0.6m) diletekkan di dalamnya, silinder berongga itu akan tenggelam di dalam air dengan ketinggian H, seperti yang ditunjukkan dalam Rajah 2. Oleh kerana bahan X adalah berkali ganda lebih berat daripada silinder berongga ($W_{bahan\ X} >> W_{silinder}$), pusat gravity system (silinder berongga dan bahan X) adalah 0.35m dari dasar permukaan tenggelam. Tentukan berat tentu bahan X, jika keseluruhan system terapung pada keseimbangan neutral pada paksi tegaknya.



Rajah 2

QUESTION 3

A uniform wooden cylinder of diameter, D and length, L has a relative density, $s = 0.6$. It floats upright in water. Show that the ratio of diameter, D with length, L (i.e. D/L) equals to $\sqrt{8s - 8s^2}$ so that the cylinder just floats upright in water. Calculate D/L.

QUESTION 4

a) What is buoyant force? Are the buoyant forces acting on a submerged 7cm in diameter spherical ball made of aluminum and a submerged spherical ball made of iron the same? Explain your answer.

b) A hollow wooden cylinder of specific gravity, SG=0.55 as shown in Figure 4 has outside diameter, d0 of 0.6m, inner diameter, di of 0.3m and has its end open. The cylinder is required to float in oil of specific gravity, SG=0.84. Calculate the maximum height, hmax of the cylinder so that it shall be stable when floating with its axis vertical.

c) If the cylinder as in question 4(b), with the same height but its upper end is closed, what will happen to its stability if the cylinder is inserted in the oil vertically? Give your reason.

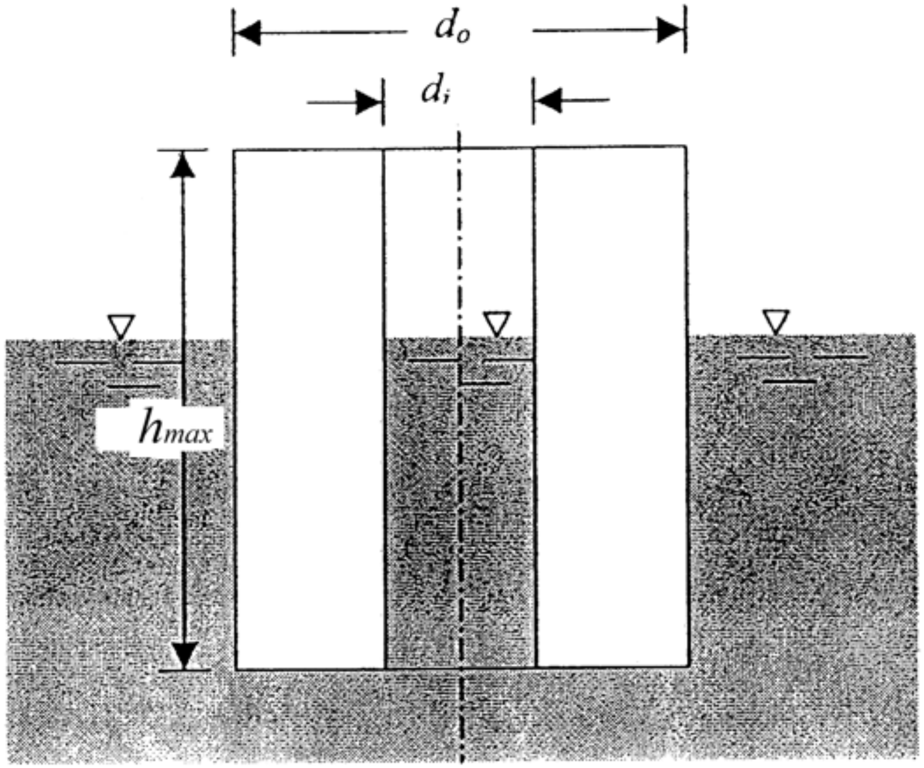


Figure 4