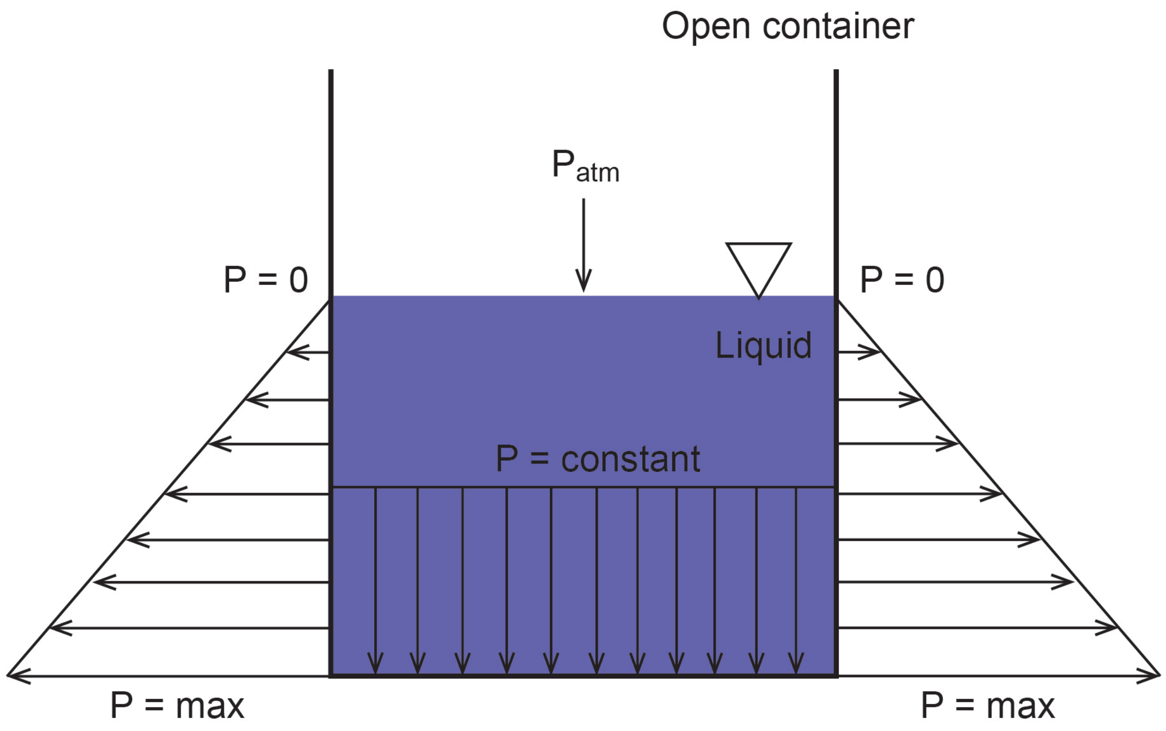
FLUID MECHANICS I

SEMM 2313

HYDROSTATIC FORCE ON PLANE SURFACE

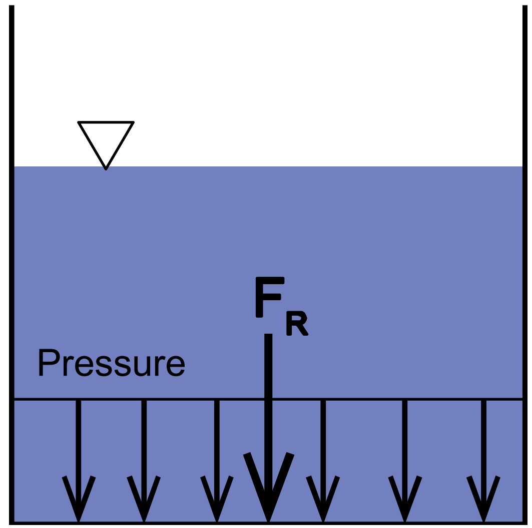
The existence of fluid cause pressure. This pressure act on surface to produce force. This force is known as hydrostatic force.



Hydrostatic force is a force that produce from static fluid (liquid and gas).

However, in gage pressure, atmospheric pressure is zero. Then, the force produce by the existence of atmosphere is zero too.

In this chapter, we are focusing the force that produce by the liquid (especially water).



Pressure that act on the bottom surface is constant because the depth of the liquid is constant. So, the force that act on the bottom surface can be calculate by using the following equation.

$$F_R = P \times A$$

The effective force, also known as resultant force, F_R can be indicated by using an arrow, as shown in the figure.

The effective location of the resultant force, F_R is at the centroid of the bottom surface.

This is valid only for constant pressure that act on the flat and horizontal surface.

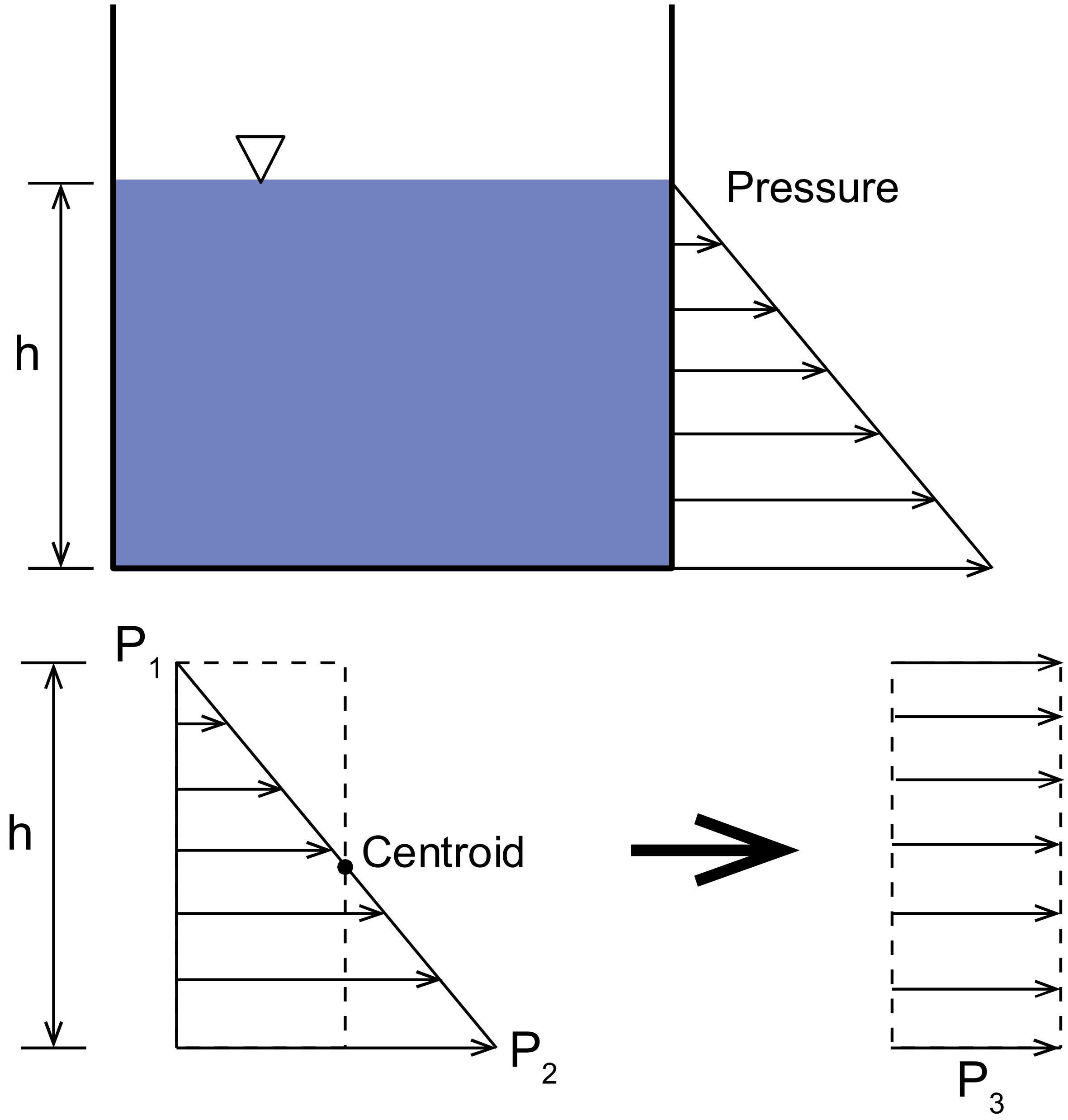
Pressure also act on the vertical surface of the tank. The pressure distribution is not constant.

Pressure on the liquid surface is zero, while pressure at the bottom is the highest one.

To calculate the resultant force, F_R , the equation is still the same but we need to determine the effective pressure that make effect on the vertical surface.

We cannot just pick one pressure value (from the pressure distribution) and multiply it with the area. It is totally wrong.

The simplest way to determine the effective pressure that occur on the vertical surface is by re-arrange the pressure distribution itself.



From figure, we could conclude that resultant force, F_R that act on the vertical surface can be determine by using this equation.

By using triangular pressure distribution;

$$F_R = \frac{1}{2}\rho g h \cdot hW$$

$$W = \text{width}$$

If we re-construct the triangular pressure distribution into rectangular pressure distribution (the conservation of area is satisfied), resultant force, F_R can be written as;

$$F_R = \rho g \left(\frac{1}{2}h\right) \cdot hW$$

$$F_R = \rho g h_c \cdot A$$

Thus, the magnitude of the hydrostatic force on plane surface is:

$$F_R = \rho g h_c \cdot A$$

h_c is the vertical distance (depth) from free surface to the centroid of the submerged plane surface.

A is the area of the plane surface that contact with liquid (water).

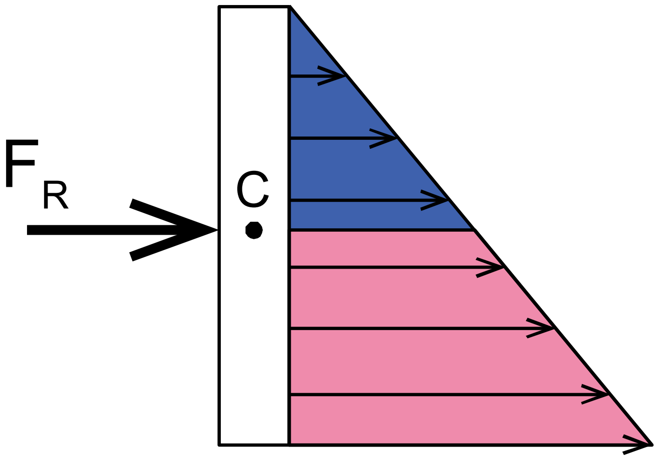
The hydrostatic force is act from liquid to the plane surface. (This force is pushing the wall outward)

The location where the resultant force, F_R act is not on the centroid of the submerged plane surface. It acts a little bit below the centroid.

If F_R acts on the centroid, it is not divided the force equally. This not a definition of the resultant force.

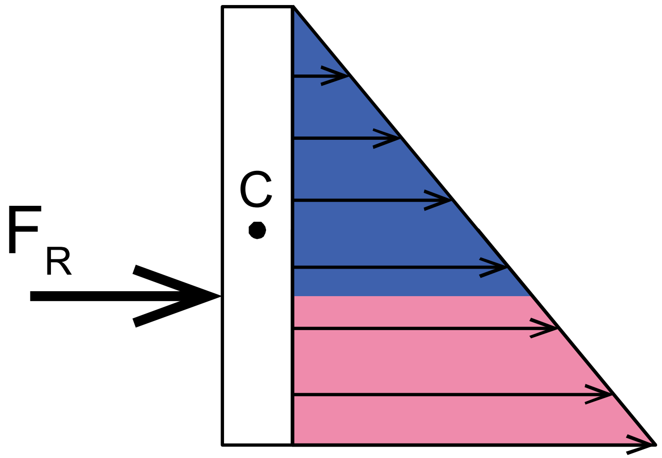
F_R needs to act below the centroid. Then, the force will be equally divided.

C = Centroid



(a)

C = Centroid



(b)

The location of F_R can be determine by using this equation.

$$y_R = \frac{I_x}{y_c \cdot A} + y_c$$

where;

y_R is the distance from free surface to the F_R along the y-axis. Y-axis is always parallel with the plane surface.

y_c is the distance from free surface to the centroid of the submerged plane surface along the y-axis.

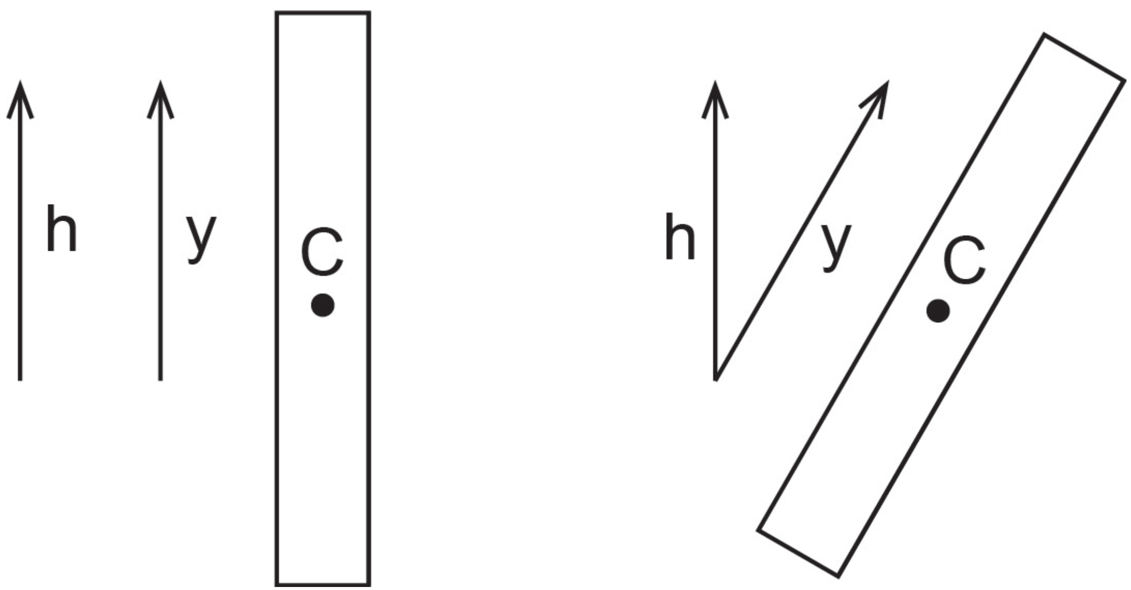
A is the submerged area of the plane surface.

Please be careful about the design of the plane surface, it may be vertical or inclined at certain angle.

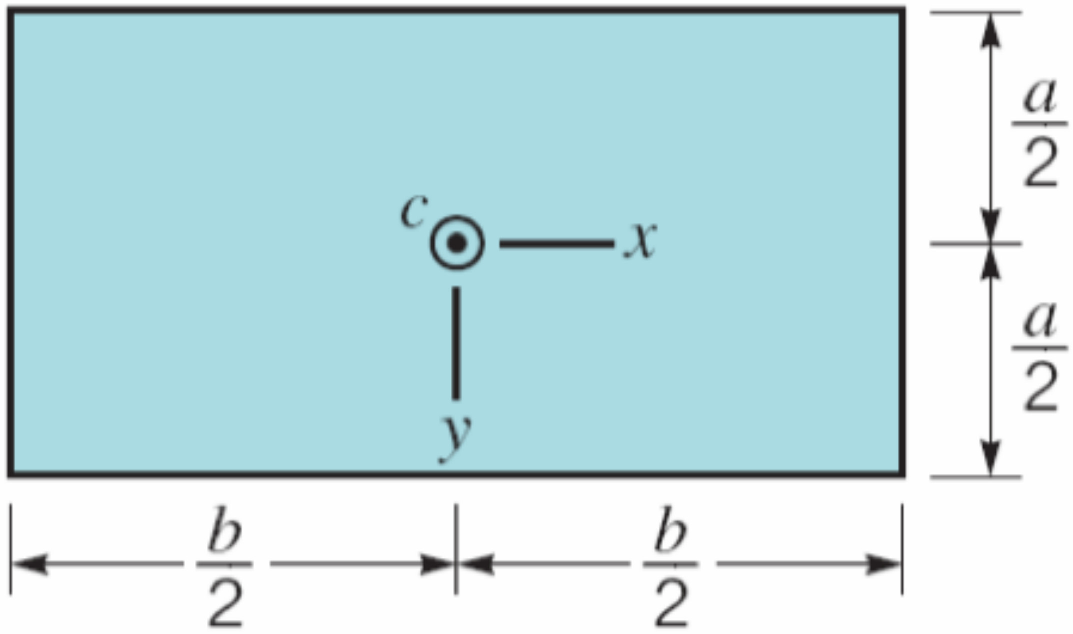
If the plane surface stays vertical, y-axis is parallel with h-axis (depth).

If the plane surface stays inclined, y-axis is parallel with the inclined plane surface while h-axis is remained vertical.

C = Centroid



CENTROID FOR RECTANGULAR



$$A = ba$$

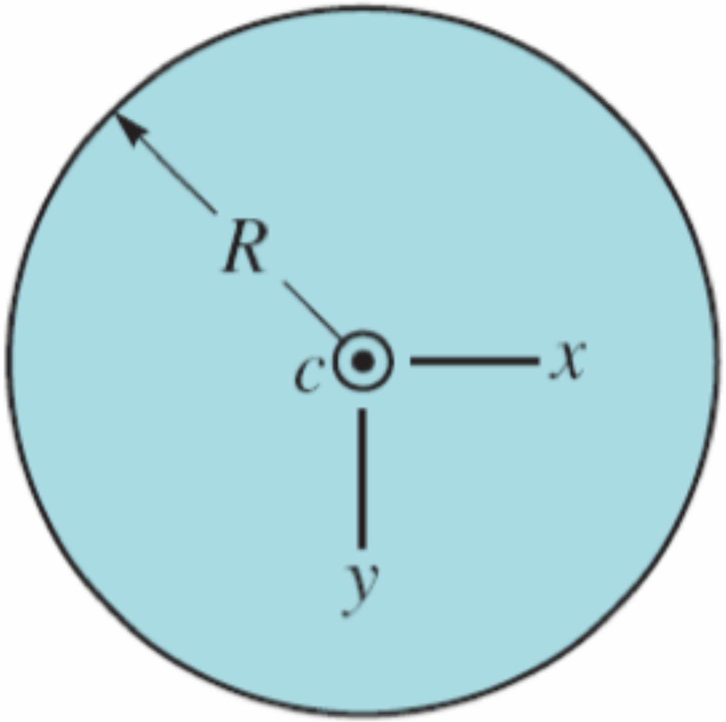
$$I_{xc} = \frac{1}{12} ba^3$$

$$I_{yc} = \frac{1}{12} ab^3$$

$$I_{xyc} = 0$$

(a) Rectangle

CENTROID FOR CIRCLE



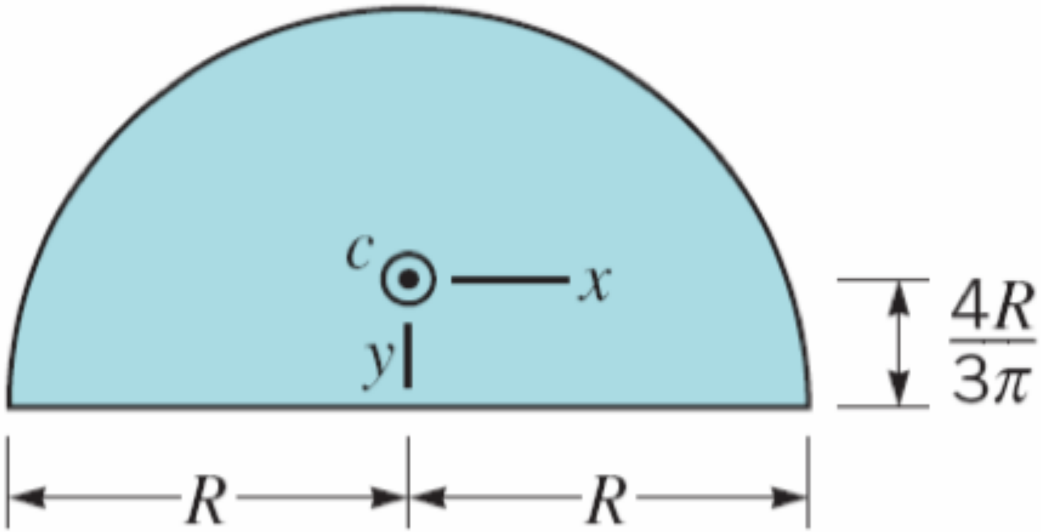
$$A = \pi R^2$$

$$I_{xc} = I_{yc} = \frac{\pi R^4}{4}$$

$$I_{xyc} = 0$$

(*b*) Circle

CENTROID FOR SEMICIRCLE



$$A = \frac{\pi R^2}{2}$$

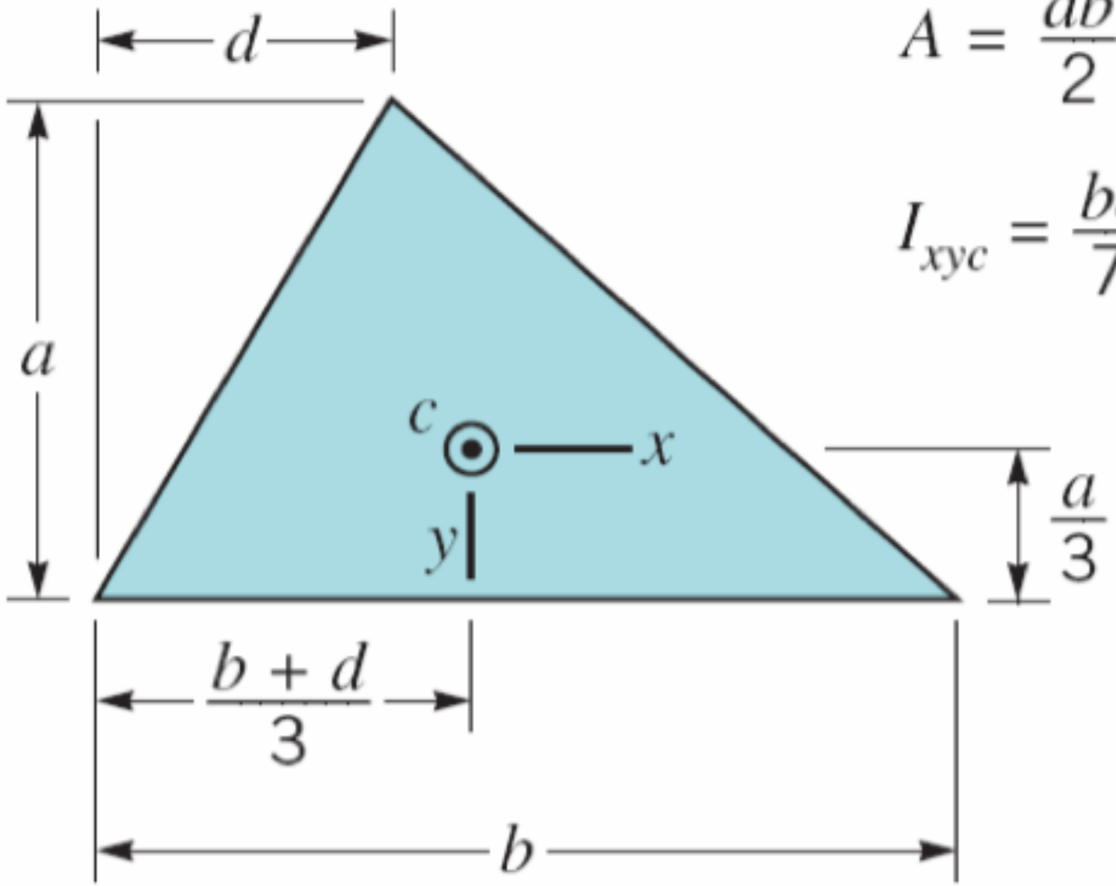
$$I_{xc} = 0.1098R^4$$

$$I_{yc} = 0.3927R^4$$

$$I_{xyc} = 0$$

(*c*) Semicircle

CENTROID FOR TRIANGLE

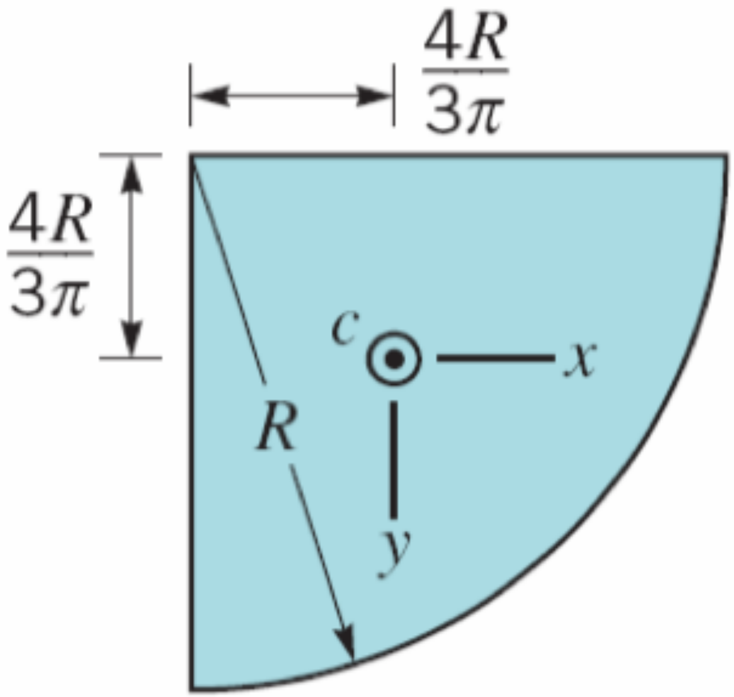


$$A = \frac{ab}{2} \qquad I_{xc} = \frac{ba^3}{36}$$

$$I_{xyc} = \frac{ba^2}{72}(b - 2d)$$

(*d*) Triangle

CENTROID FOR QUARTER CIRCLE

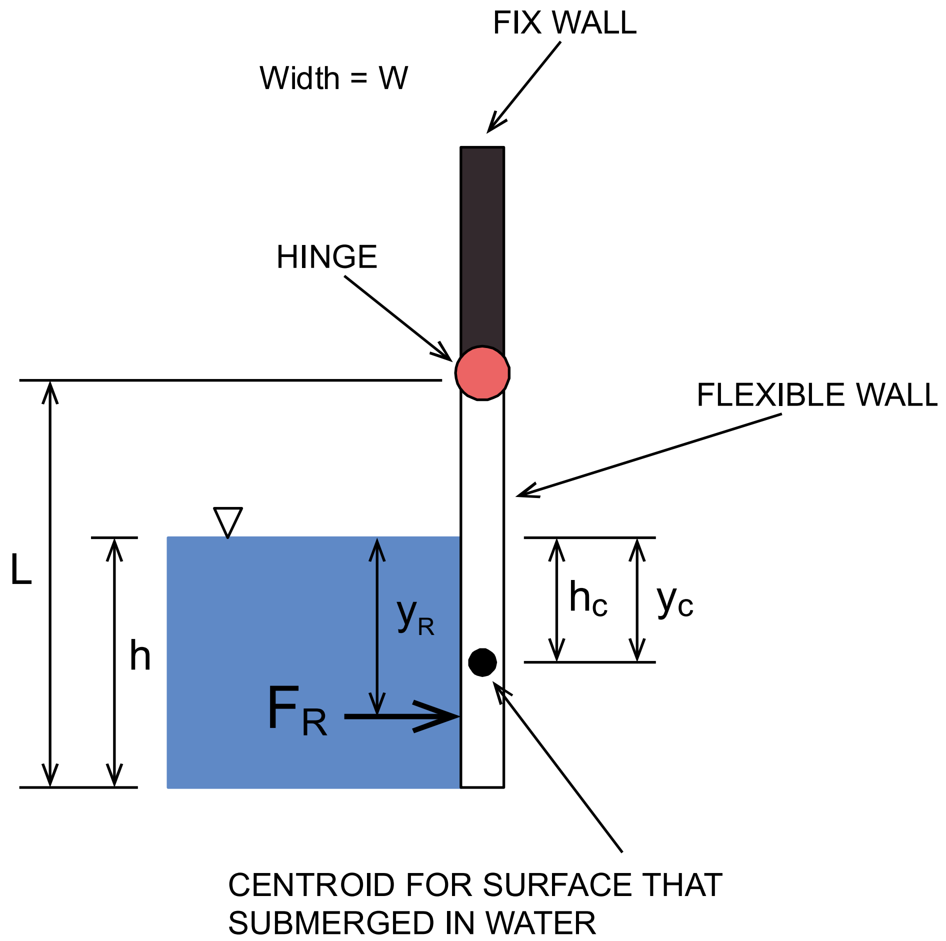


(e) Quarter circle

$$A = \frac{\pi R^2}{4}$$

$$I_{xc} = I_{yc} = 0.05488 R^4$$

$$I_{xyc} = -0.01647 R^4$$



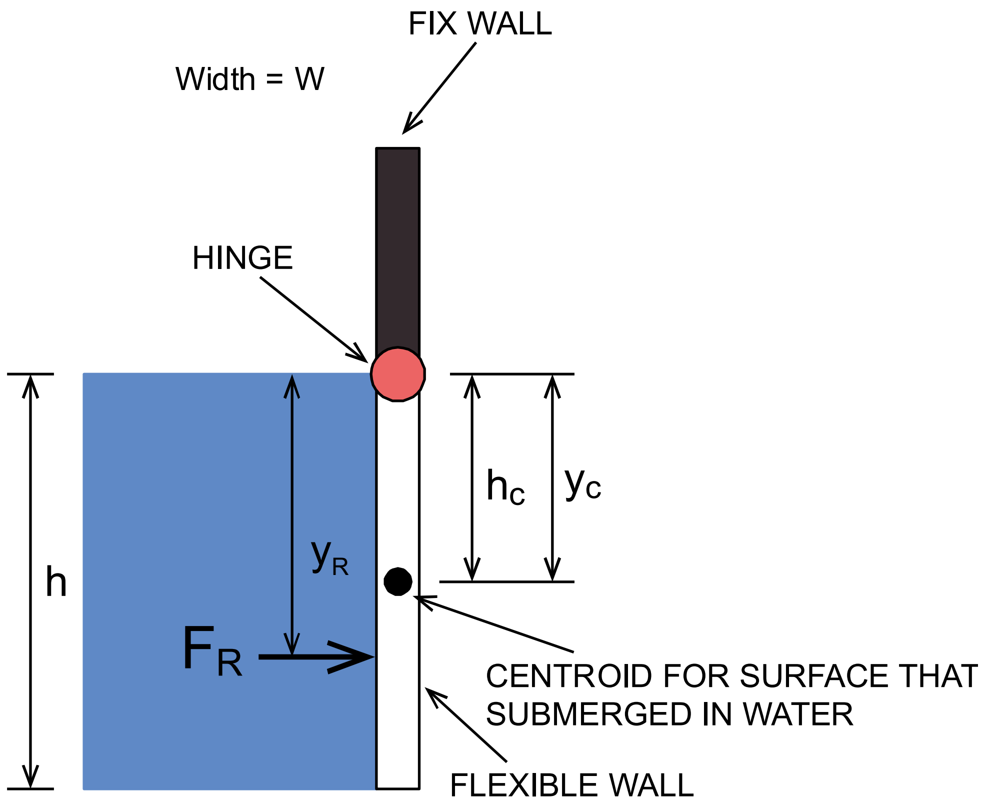
$$F_R = \rho g h_c \cdot A$$

$$h_c = \frac{h}{2}$$

$$A = \text{Surface area that contact with water} = h \times W$$

$$y_R = \frac{I_x}{y_c \cdot A} + y_c$$

$$y_c = h_c = \frac{h}{2}$$



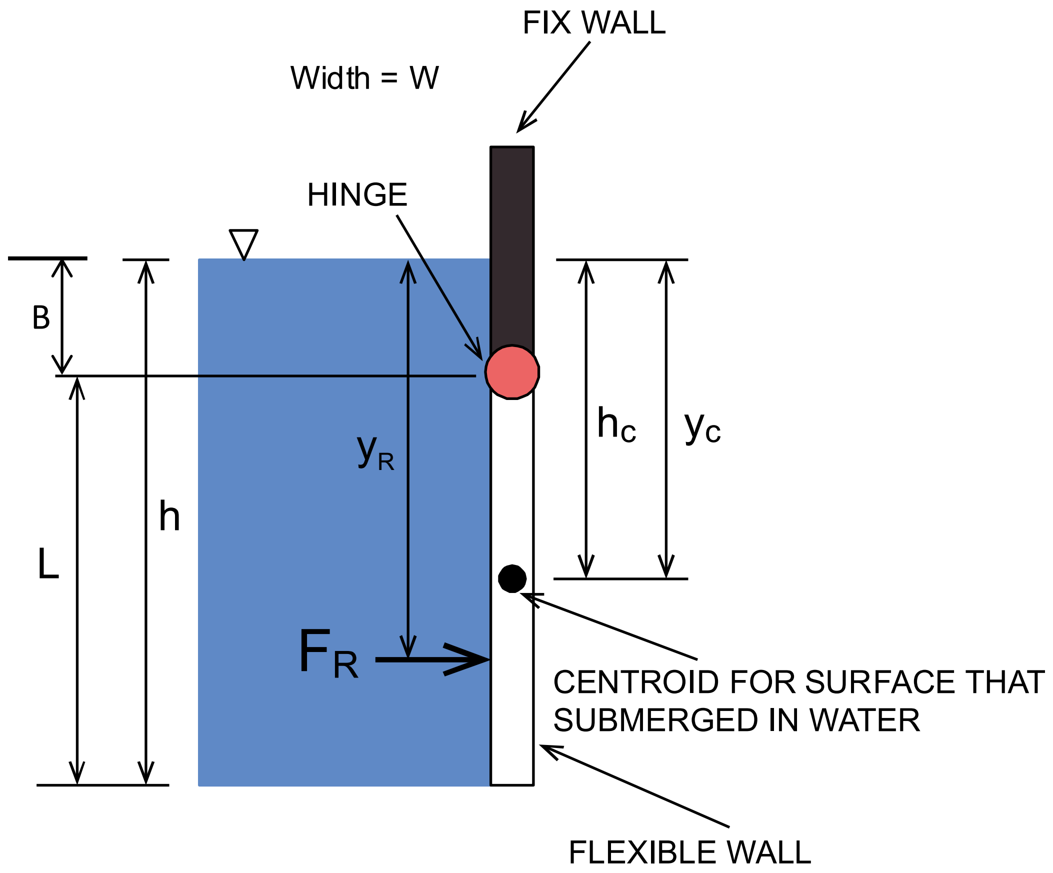
$$F_R = \rho g h_c \cdot A$$

$$h_c = \frac{h}{2} \quad \text{where} \quad h = L$$

$$A = \text{Surface area that contact with water} = h \times W$$

$$y_R = \frac{I_x}{y_c \cdot A} + y_c$$

$$y_c = h_c = \frac{h}{2}$$



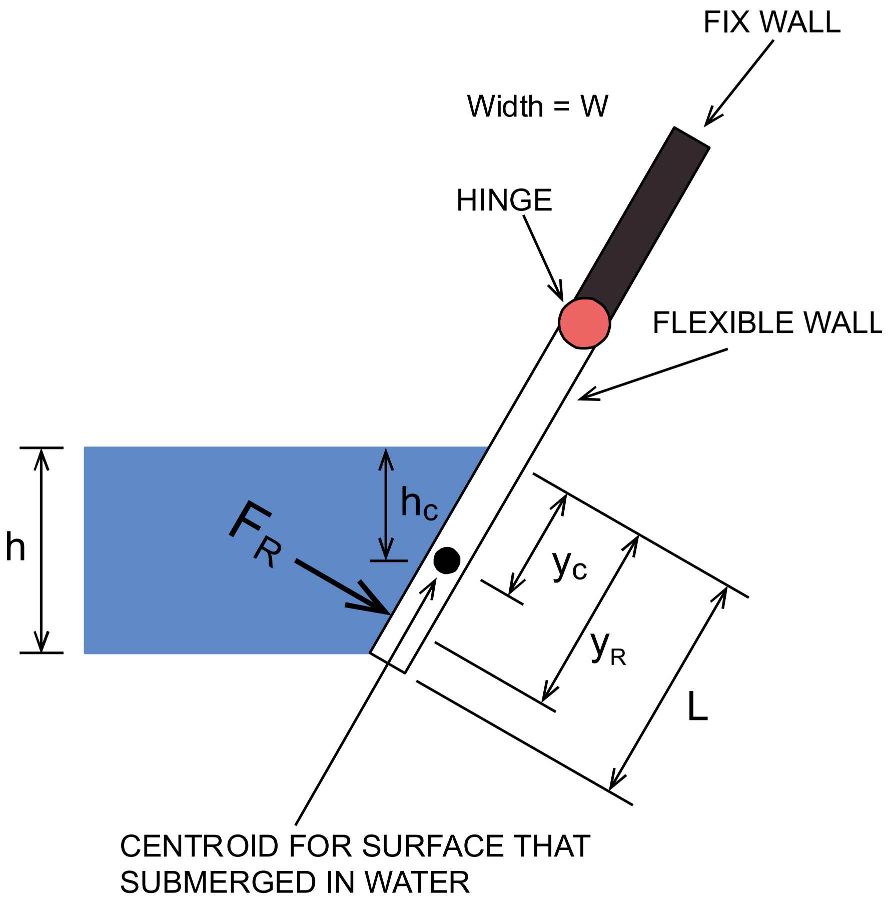
$$F_R = \rho g h_c \cdot A$$

$$h_c \neq \frac{h}{2} = (h - L) + \frac{L}{2} = B + \frac{L}{2}$$

$$A = \text{Surface area that contact with water} = L \times W$$

$$y_R = \frac{I_x}{y_c \cdot A} + y_c$$

$$y_c = h_c$$



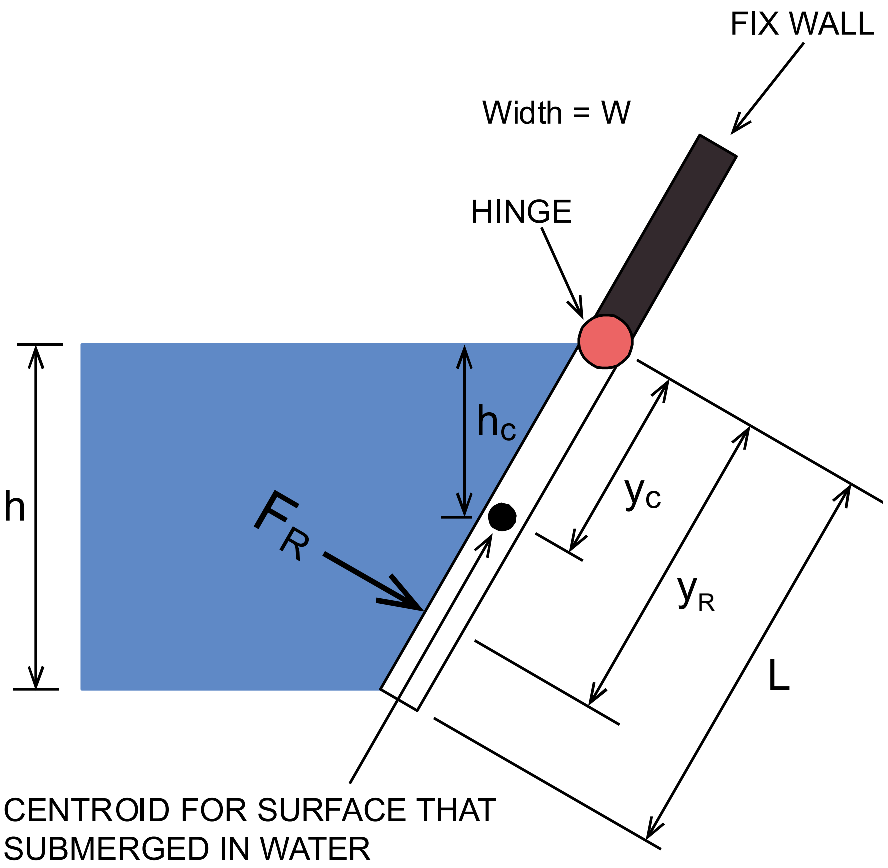
$$F_R = \rho g h_c \cdot A$$

$$h_c = \frac{h}{2}$$

$$h_c \neq y_c$$

$$A = \text{Surface area that contact with water} = L \times W$$

$$y_R = \frac{I_x}{y_c \cdot A} + y_c$$



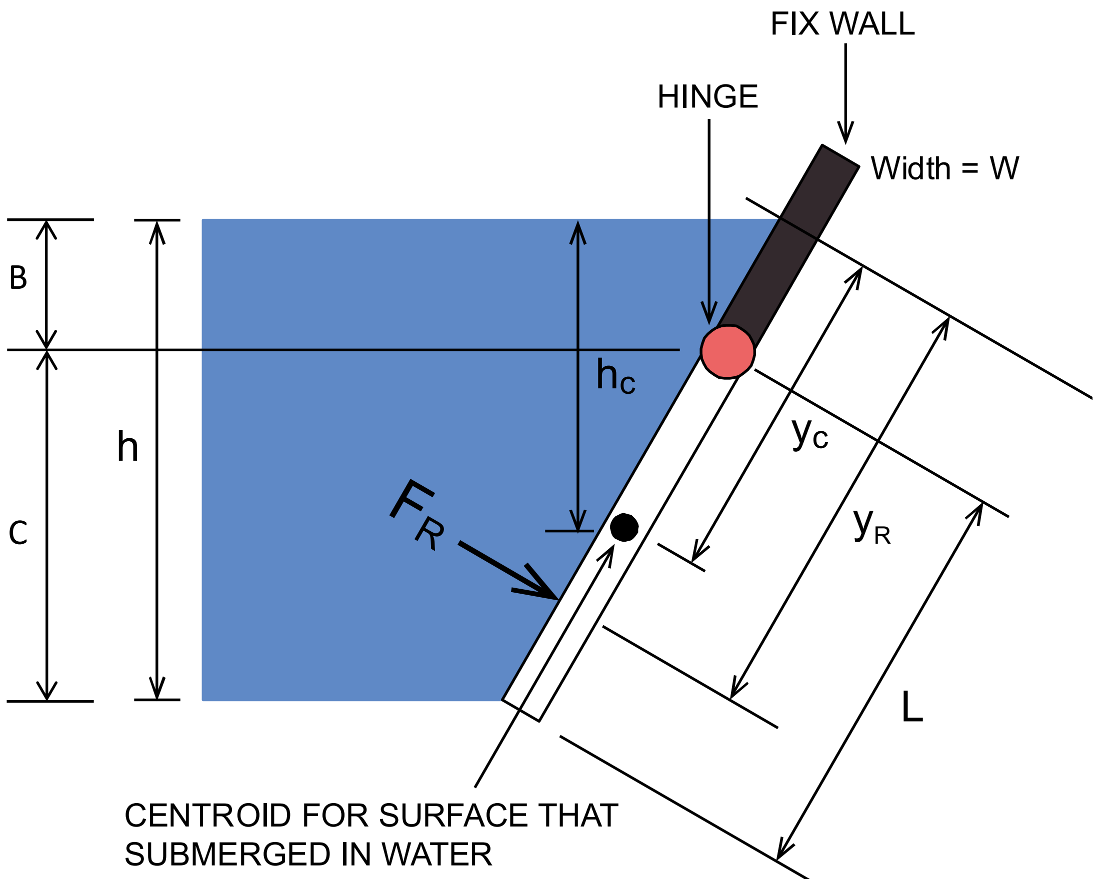
$$F_R = \rho g h_c \cdot A$$

$$h_c = \frac{h}{2}$$

$$h_c \neq y_c$$

$$A = \text{Surface area that contact with water} = L \times W$$

$$y_R = \frac{I_x}{y_c \cdot A} + y_c$$



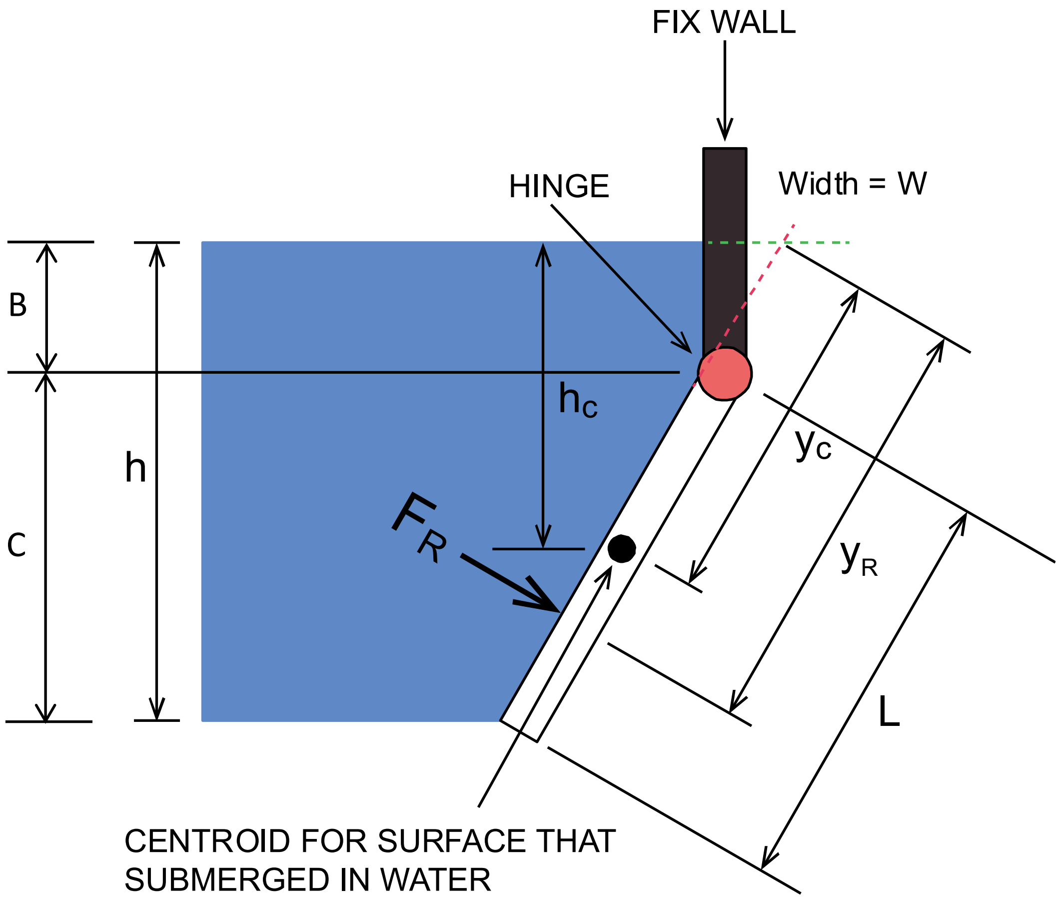
$$F_R = \rho g h_c \cdot A$$

$$h_c = B + \frac{C}{2} \neq \frac{h}{2}$$

$$h_c \neq y_c$$

$$A = \text{Surface area that contact with water} = L \times W$$

$$y_R = \frac{I_x}{y_c \cdot A} + y_c$$



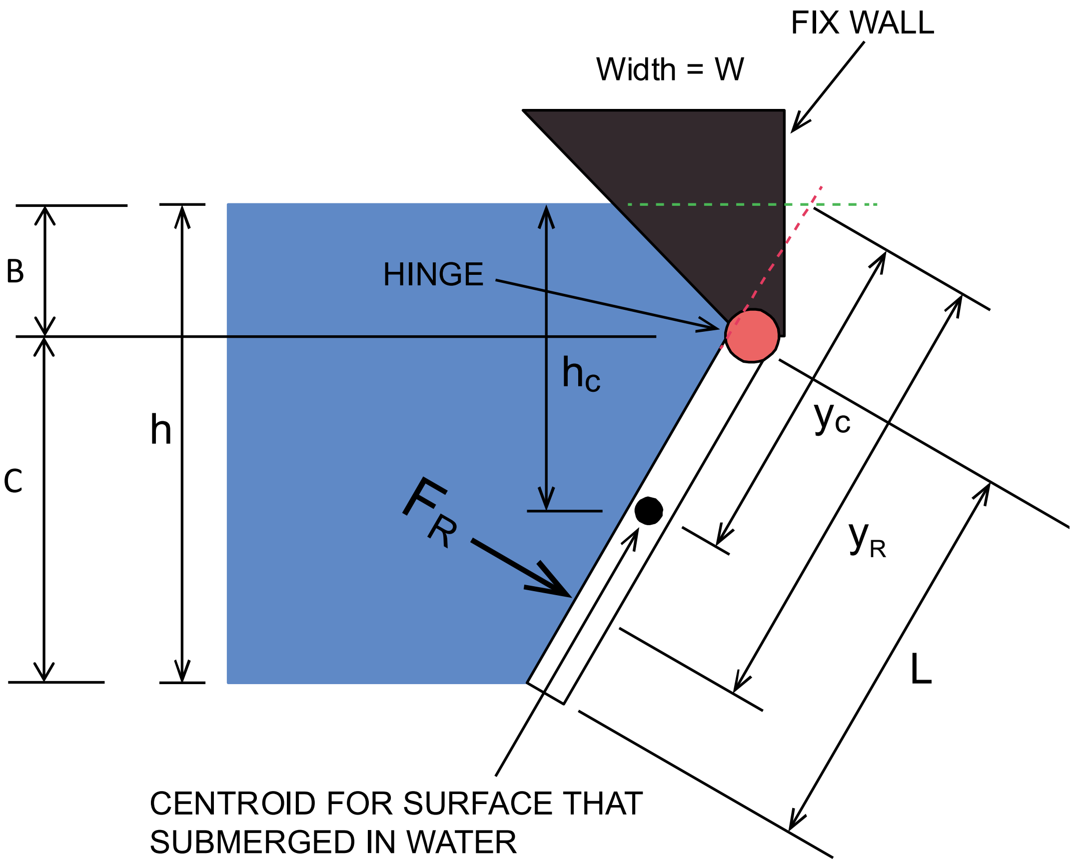
$$F_R = \rho g h_c \cdot A$$

$$h_c = B + \frac{C}{2} \neq \frac{h}{2}$$

$$h_c \neq y_c$$

$$A = \text{Surface area that contact with water} = L \times W$$

$$y_R = \frac{I_x}{y_c \cdot A} + y_c$$



$$F_R = \rho g h_c \cdot A$$

$$h_c = B + \frac{C}{2} \neq \frac{h}{2}$$

$$h_c \neq y_c$$

$$A = \text{Surface area that contact with water} = L \times W$$

$$y_R = \frac{I_x}{y_c \cdot A} + y_c$$

Question 1

A 3-m-wide, 8-m-high rectangular gate is located at the end of a rectangular passage that is connected to a large open tank filled with water as shown in Figure 1. The gate is hinged at its bottom and held closed by a horizontal force, FH , located at the center of the gate. The maximum value for FH is 3500 kN.
(a) Determine the maximum water depth, h , above the center of the gate that can exist without the gate opening.
(b) Is the answer the same if the gate is hinged at the top ? Explain your answer.

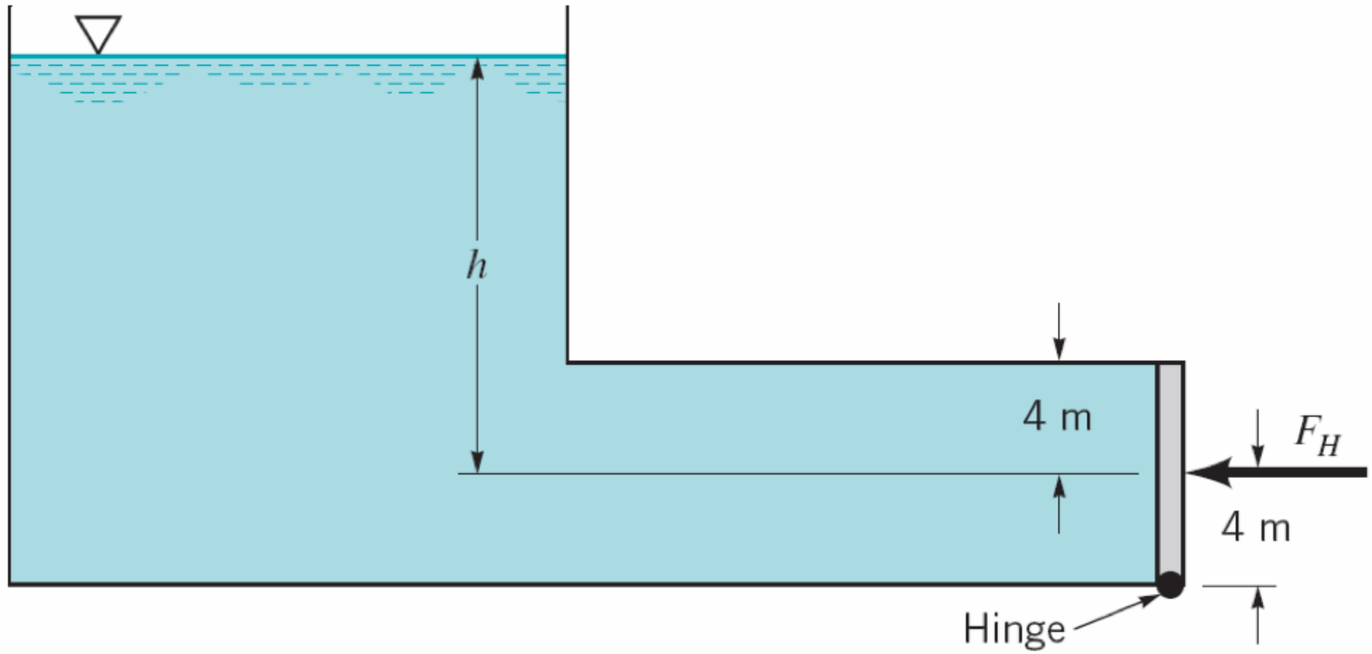


Figure 1

Question 2

An open tank has a vertical partition and on one side contains gasoline with a density $\rho = 700\ kg/m^3$ at a depth of 4 m, as shown in Figure 2. A rectangular gate that is 4 m high and 2 m wide and hinged at one end is located in the partition. Water is slowly added to the empty side of the tank. At what depth, h, will the gate start to open ?

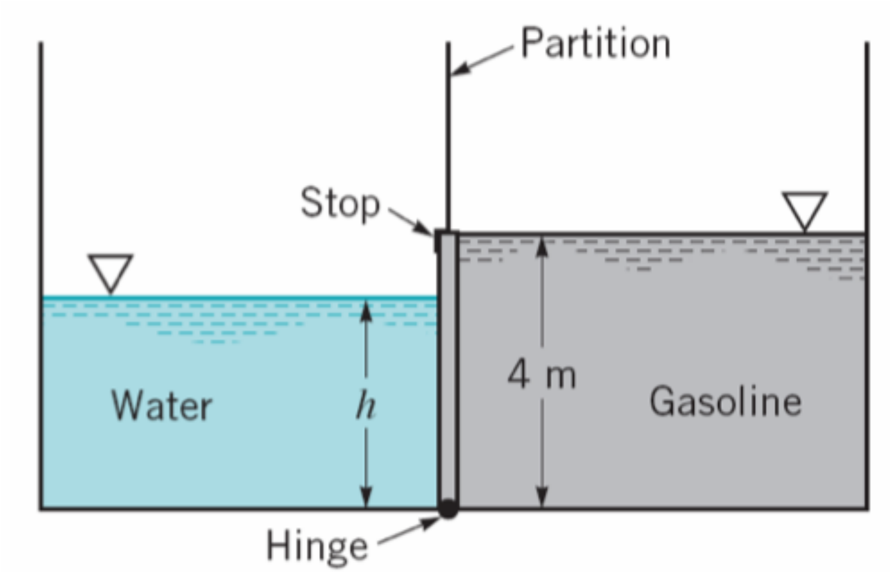


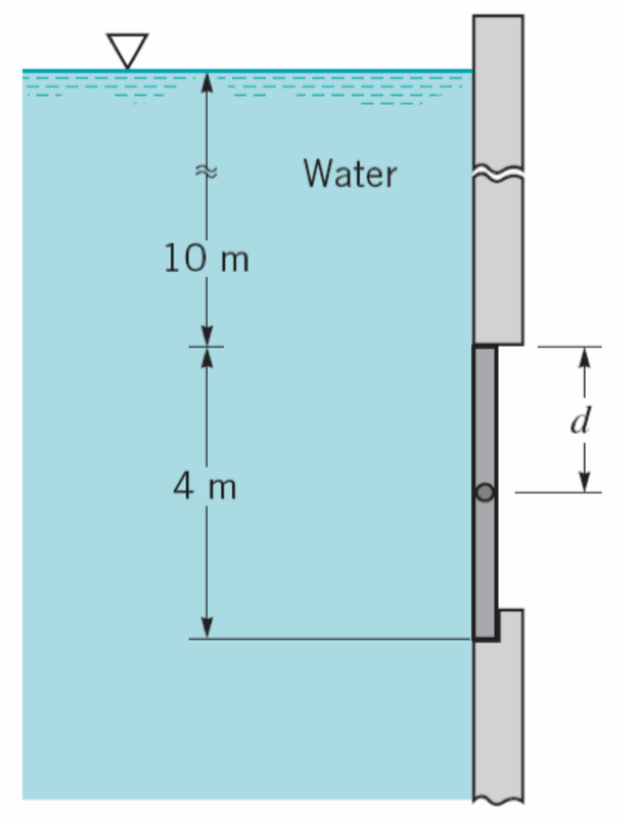
Figure 2

Question 3

A rectangular gate that is 2 m wide is located in the vertical wall of a tank containing water as shown below. It is desired to have the gate open automatically when the depth of water above the top of the gate reaches 10 m.

(a) At what distance, d, should the frictionless horizontal shaft be located ?

(b) What is the magnitude of the force on the gate when it opens ?



Question 4

A rectangular gate having a width of 5 m is located in the sloping side of a tank as shown in Figure 4. The gate is hinged along its top edge and is held in position by the force P. Friction at the hinge and the weight of the gate can be neglected. Determine the required value of P.

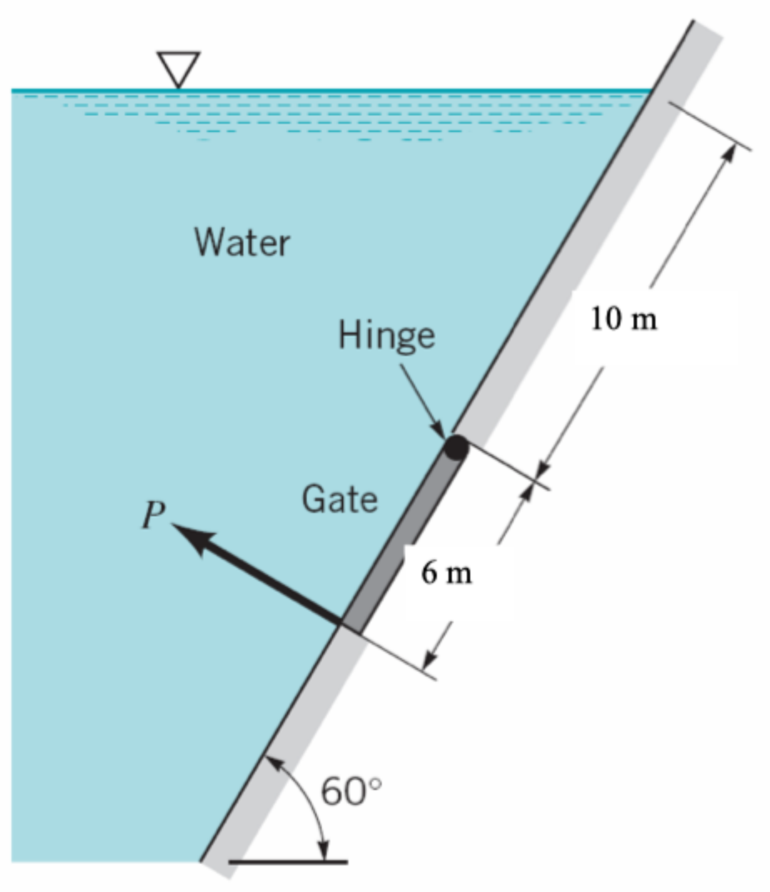


Figure 4

Question 5

A homogeneous, 4-m-wide, 8-m-long rectangular gate weighing 300 kg is held in place by a horizontal flexible cable as shown in Figure 5. Water acts against the gate which is hinged at point A. Friction in the hinge is negligible. Determine the tension in the cable.

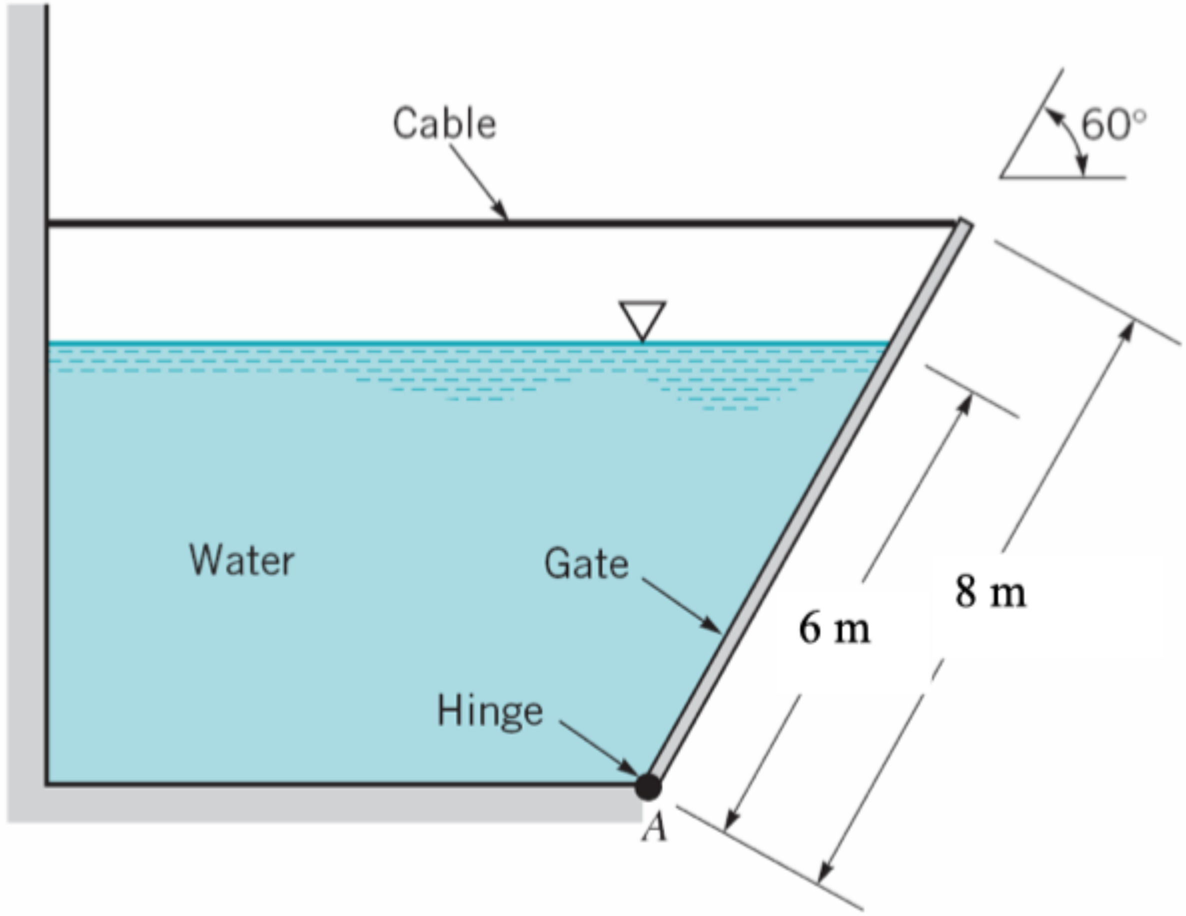
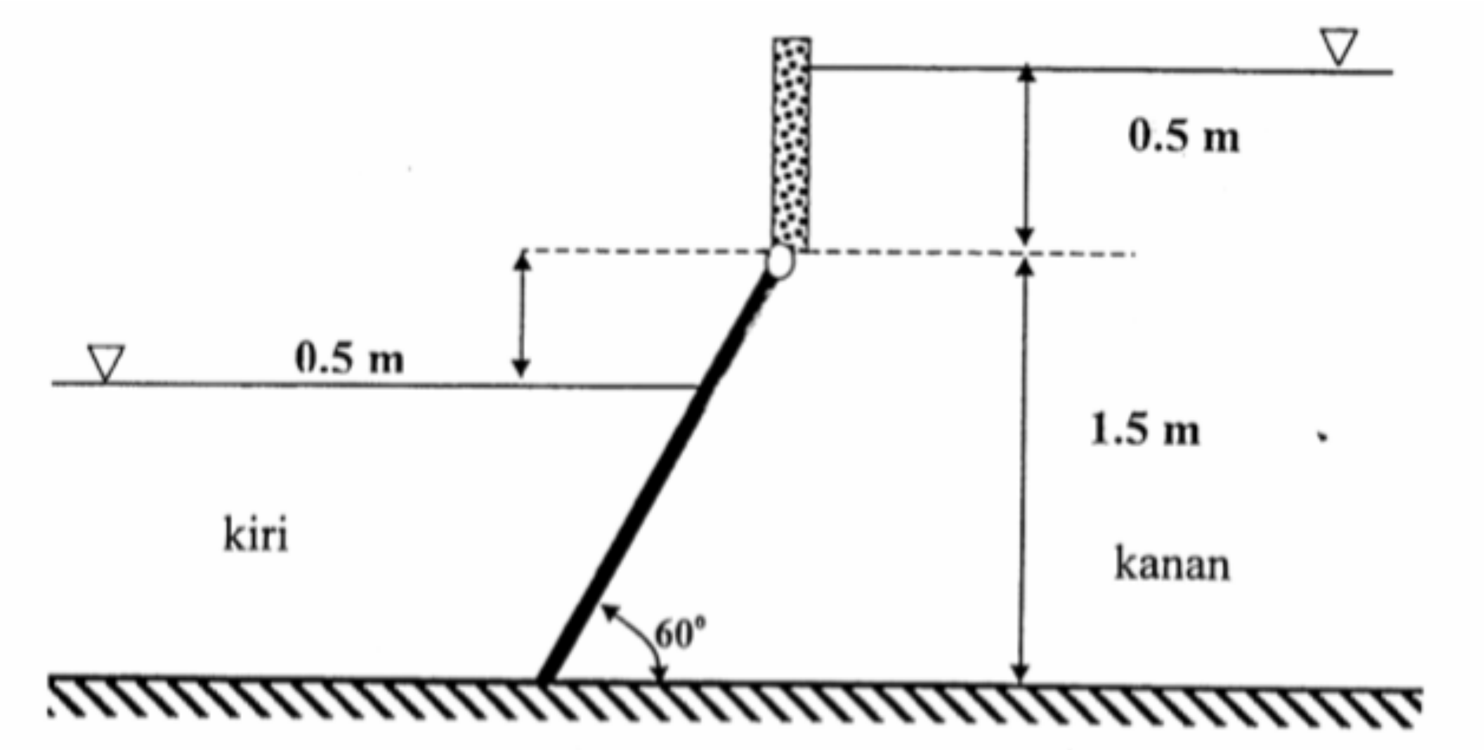


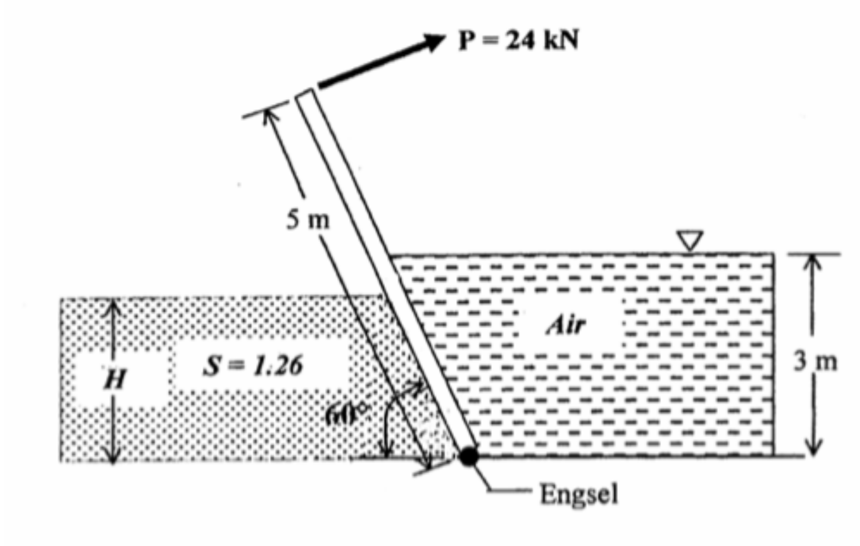
Figure 5

Question 6

A rectangular water gate, width is 0.5 m, used to separate two water tank. Water gate start to open when water level in the left tank drop 0.5 m from the hinge. Water level in the right tank remains at a height of 2 m. Calculate the weight of the water gate.



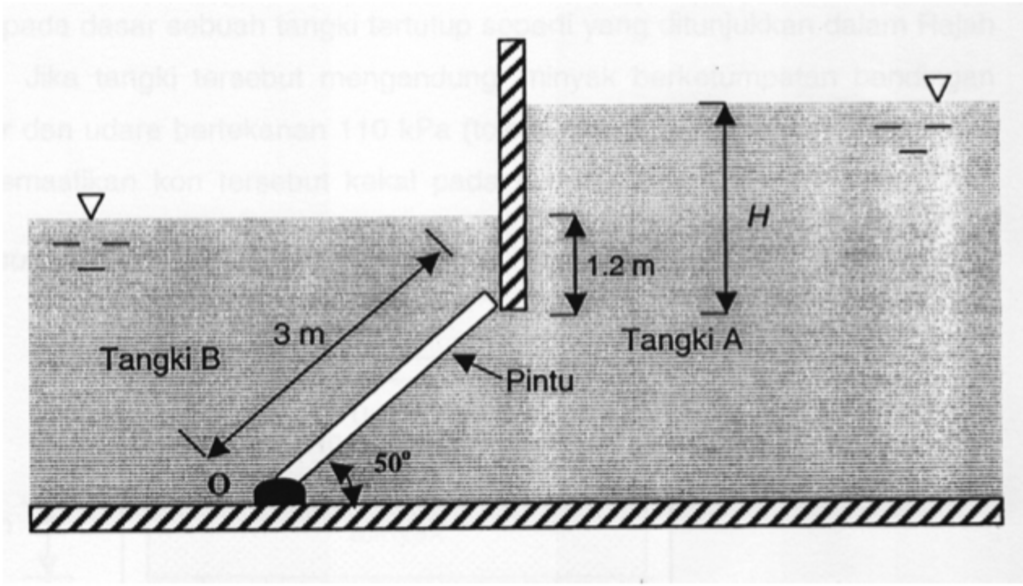
Question 7



Rajah 2

Pintu segi empat tepat nipis (berat 10kN) dengan ketinggian 5m dan lebar 2m digunakan untuk memisahkan dua jenis bendalir. Pintu sepatutnya sentiasa berada di dalam keadaan tegak. Walau bagaimanapun, system kawalan bendalir rosak menyebabkan air melebihi ketinggian yang sepatutnya. Akibat daripada kerosakkan itu, pintu telah condong seperti yang dilakarkan dalam Rajah 2. Untuk memastikan pintu tidak jatuh, daya dengan magnitude 24kN dikenakan dihujung pintu. Dengan data yang diberikan, tentukan kedalaman *H* bendalir *B* yang berjaya menampung pintu supaya ia tidak terus jatuh.

Question 8



Rajah 3

a. Dengan bantuan gambarajah, takrifkan pusat tekanan (CP: Center of Pressure) dan pusat sentroid, C bagi sebuah plat rata tegak yang tenggelam di dalam air static.

b. Dua buah tangki air dipisahkan dengan sebuah pintu segiempat tepat yang mempunyai panjang 3m dan lebar 1.7m seperti yang ditunjukkan dalam Rajah 3. Pintu dengan jisim 900kg itu di engsel pada titik *O* dengan geseran pada engsel itu boleh diabaikan. Jika aras air adalah seperti yang ditunjukkan dalam rajah, tentukan ketinggian *H* yang maksimum bagi memastikan pintu itu masih tertutup.

Question 9

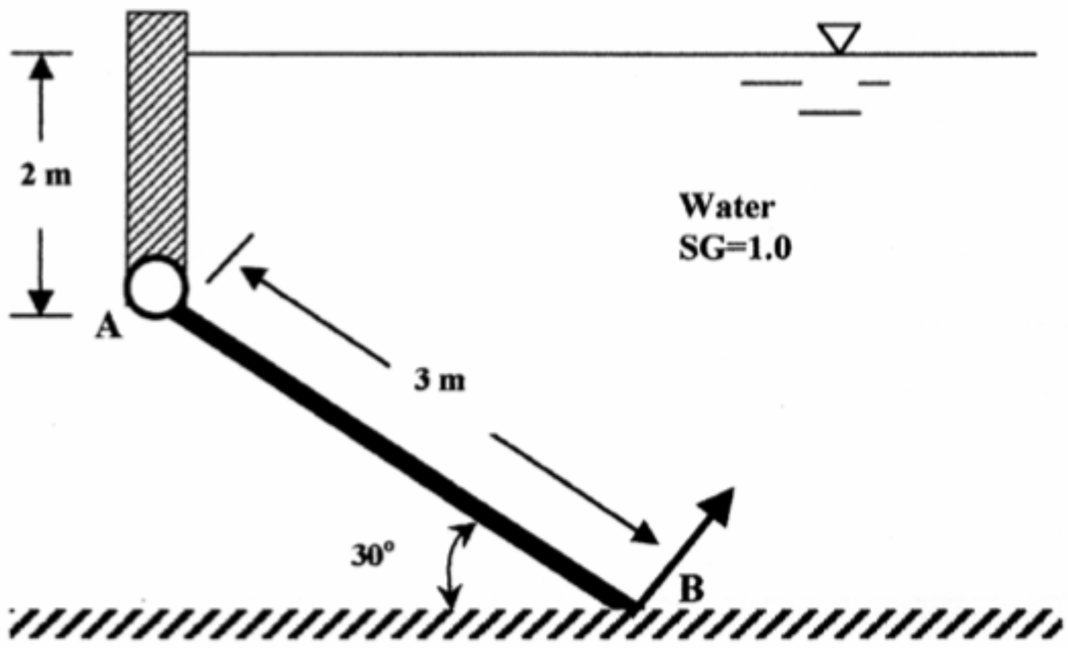


Figure 4

The gate shown in Figure 4 is 2m wide and hinged at point *A*. Calculate the force required at point *B* to open the gate if the mass of the gate is 10kg.