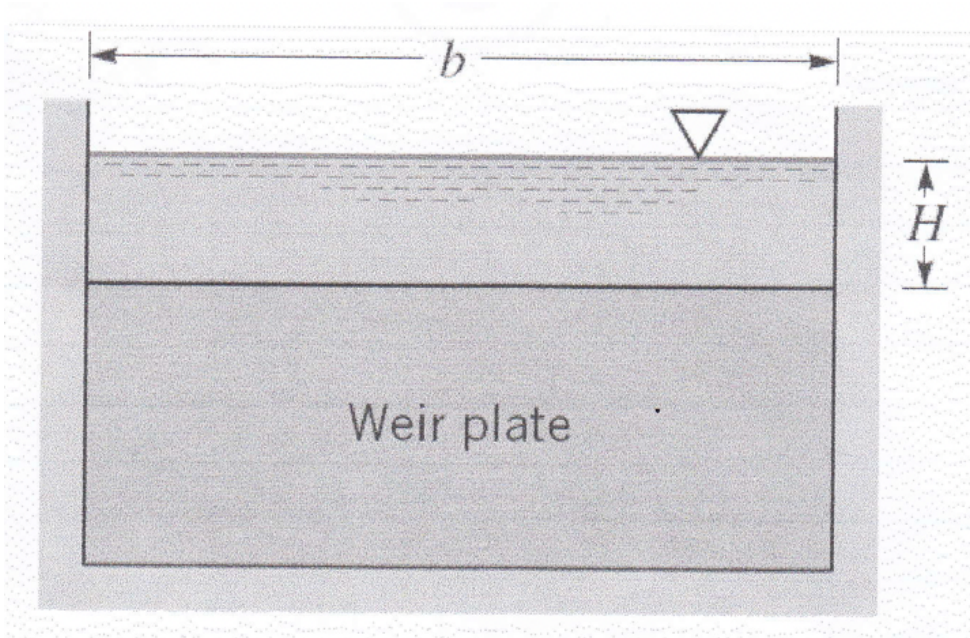
FLUID MECHANICS I

SEMM 2313

Question 1

The water flowrate, Q, in an open rectangular channel can be measured by placing a plate across the channel as shown below. This type of a device is called a weir. The height of water, H, above the weir crest is referred to as the head and can be used to determine the flowrate through the channel. Assume that Q is a function of the head, H, the channel width, b, and the acceleration of gravity, g. Determine a suitable set of dimensionless variables for this problem.

In some laboratory test, it was determined that if b = 0.9 m and H = 10 cm, then Q = 0.07 m^3/s. Based on these limited data, determine a general equation for the flowrate over this type of weir.



Answer 1

$$Q = f(H\ , b\ , g)$$

Parameter	MLT		Repeating parameter	Non-repeating parameter
Q	L^3T^{-1}	Main subject	$\times$	Q
H	L	Geometry	H	
b	L	Geometry		b
g	LT^{-2}	Kinematic	g	

$$pi\ term\ =\ 4 - 2 = 2\ pi\ term$$

$$\pi_1 = Q \times H^A g^B$$

$$\pi_2 = b \times H^A g^B$$

For π_1

$$\pi_1 = Q \times H^A g^B$$

$$M^0 L^0 T^0 = L^3 T^{-1} \times [L]^A g[LT^{-2}]^B$$

$$\pi_1 = \frac{Q}{H^{\frac{5}{2}} g^{\frac{1}{2}}}$$

For π_2

$$\pi_2 = b \times H^A g^B$$

$$M^0 L^0 T^0 = L \times [L]^A g [LT^{-2}]^B$$

$$\pi_2 = \frac{b}{H}$$

Final answer:

$$\pi_1 = f(\pi_2)$$

$$\frac{Q}{H^{\frac{5}{2}} g^{\frac{1}{2}}} = f\left(\frac{b}{H}\right)$$

$\frac{Q}{H^{\frac{5}{2}} g^{\frac{1}{2}}}$ is a function of $\left(\frac{b}{H}\right)$

From the mathematical expression, we could derive an equation.

$$\frac{Q}{H^{\frac{5}{2}}g^{\frac{1}{2}}} = f\left(\frac{b}{H}\right)$$

We could write as :

$$\frac{Q}{H^{\frac{5}{2}}g^{\frac{1}{2}}} = C\left(\frac{b}{H}\right)$$

C is a constant

Therefore;

$$C = \frac{Q}{H^{\frac{3}{2}}g^{\frac{1}{2}}b}$$

Substitute with the value given in the question,

b = 0.9 m and H = 10 cm, Q = 0.07 m^3/s

$$C = 0.785$$

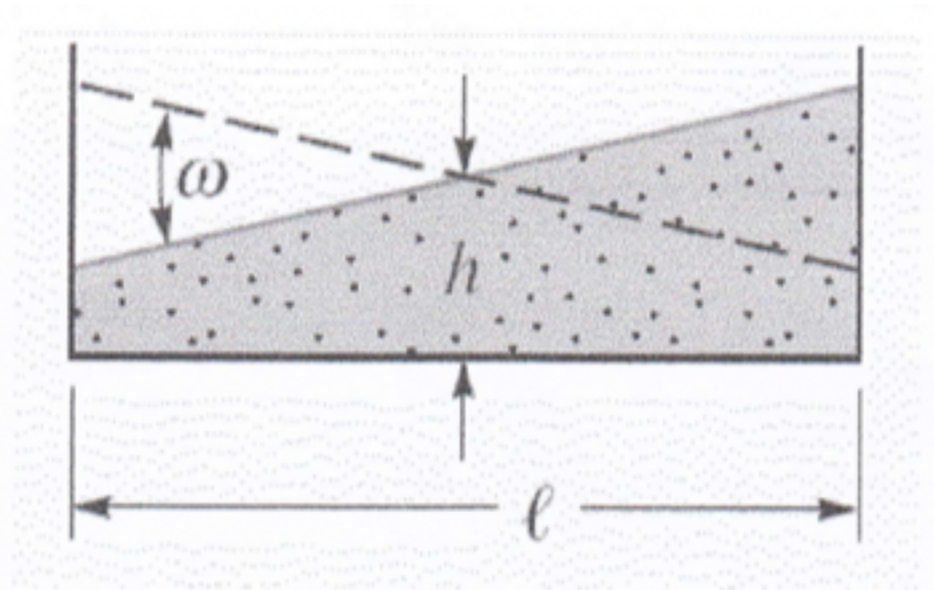
General equation to find the flowrate for that rectangular weir is

$$Q = (0.785)b\sqrt{gH^3}$$

You may compare this equation with the equation of rectangular weir in "Introduction to fluid dynamic" section.

Question 2

Water sloshes back and forth in a tank as shown below. The frequency of sloshing, ω, is assumed to be a function of the acceleration of gravity, g, the average depth of the water, h, and the length of the tank, L. Develop a suitable set of dimensionless parameters for this problem using g and L as repeating parameter.



Answer 2

$$\omega = f(g\,, h\,, L)$$

Parameter	MLT		Repeating parameter	Non-repeating parameter
ω	T^{-1}	Main subject	$\times$	ω
g	LT^{-2}	Kinematic	g	
h	L	Geometry		h
L	L	Geometry	L	

$$pi\ term\ = \ 4 - 2 = 2\ pi\ term$$

$$\pi_1 = \omega \times g^A L^B$$

$$\pi_2 = h \times g^A L^B$$

For π_1

$$\pi_1 = \omega \times g^A L^B$$

$$\pi_1 = \omega\sqrt{\frac{L}{g}}$$

For π_2

$$\pi_2 = h \times g^A L^B$$

$$\pi_2 = \frac{h}{L}$$

Final answer:

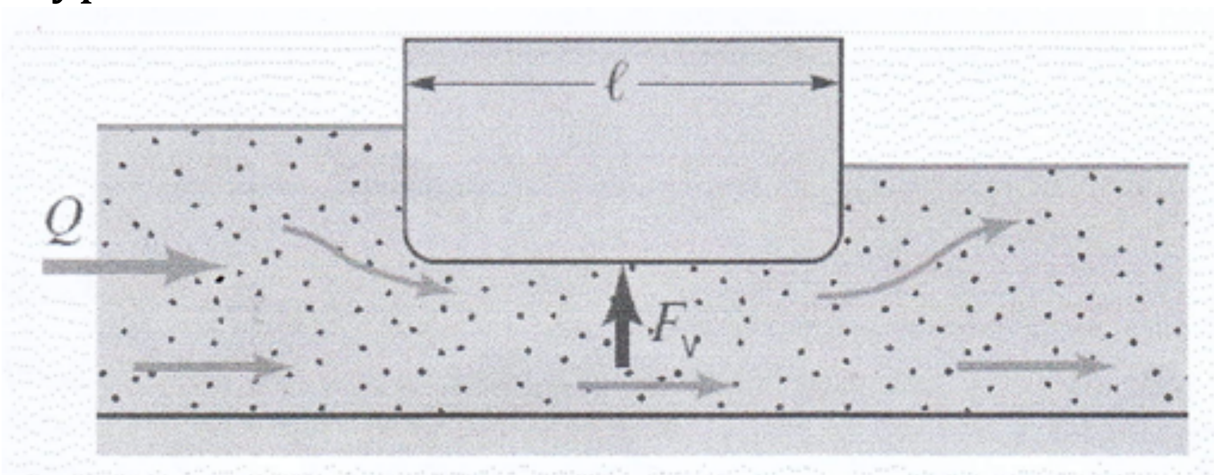
$$\pi_1 = f(\pi_2)$$

$$\omega\sqrt{\frac{L}{g}} = f\left(\frac{h}{L}\right)$$

Question 3

Water flowing under the obstacle puts a vertical force, Fv on the obstacle. This force is assume to be a function of the flowrate, Q, the density of water, ρ, the acceleration of gravity, g, and a length, L, that characterized the size of obstacle. A 1/20 scale model is to be used to predict the vertical force on the prototype.

(a) If the prototype flowrate is 30 m^3/s, determine the water flowrate for the model if the flows are to be similar.
(b) If the model force is measured as 80 N, predict the corresponding force on the prototype.



Answer 3

$$F_V = f(Q\,,\rho\,,g\,,L)$$

Parameter	MLT		Repeating parameter	Non-repeating parameter
F_V	MLT^{-2}	Main subject	$\times$	F_V
Q	L^3T^{-1}	Kinematic		Q
ρ	ML^{-3}	Dynamic	ρ	
g	LT^{-2}	Kinematic	g	
L	L	Geometry	L	

$$pi\ term\ = 5 - 3 = 2\ pi\ term$$

$$\pi_1 = F_V \times L^A g^B \rho^C$$

$$\pi_2 = Q \times L^A g^B \rho^C$$

For π_1

$$\pi_1 = F_V \times L^A g^B \rho^C$$

$$\pi_1 = \frac{F_V}{L^3 g \rho}$$

For π_2

$$\pi_2 = Q \times L^A g^B \rho^C$$

$$\pi_2 = \frac{Q}{\sqrt{L^5 g}}$$

Final answer

$$\frac{F_V}{L^3 g\rho} = f\left(\frac{Q}{\sqrt{L^5 g}}\right)$$

We can develop the similirity form as:

$$\left(\frac{F_V}{L^3 g\rho}\right)_{model} = \left(\frac{F_V}{L^3 g\rho}\right)_{prototype}$$

$$\left(\frac{Q}{\sqrt{L^5 g}}\right)_{model} = \left(\frac{Q}{\sqrt{L^5 g}}\right)_{prototype}$$

From question,

$Q_{prototype} = 30 \quad m^3/s$

$L_{model} : L_{prototype} \quad = \quad 1{:}20$

$$\frac{L_{model}}{L_{prototype}} = \frac{1}{20}$$

$$\left(\frac{Q}{\sqrt{L^5 g}}\right)_{model} = \left(\frac{Q}{\sqrt{L^5 g}}\right)_{prototype}$$

$$\frac{Q_m}{\sqrt{L_m^5}} = \frac{Q_p}{\sqrt{L_p^5}} \qquad \Longrightarrow \qquad Q_m = 0.0168 \quad m^3/s$$

The effect of the gravity can be neglected.

$$\left(\frac{F_V}{L^3 g\rho}\right)_{model} = \left(\frac{F_V}{L^3 g\rho}\right)_{prototype}$$

$$\frac{F_{Vm}}{L_m^3 \rho_m} = \frac{F_{Vp}}{L_p^3 \rho_p}$$

Experimental conducted with water, therefore, $\rho_m = \rho_p$

$$F_{Vp} = 640 \quad kN$$

Question 4

The pressure rise across a nuclear blast wave is assumed to be a function of the amount of energy release in the explosion, the air desity, the speed of sound and the distance from the blast. Consider two blast, the prototype blast and a model blast. Model blast has 0.1% energy compare to the prototype. At what distance from the model blast will the pressure rise be the same as that at a distance of 2 km from the prototype blast?

Answer 4

$$\Delta P = f(E\ , \rho\ , c\ , D)$$

Parameter	MLT		Repeating parameter	Non-repeating parameter
ΔP	$ML^{-1}T^{-2}$	Main subject	$\times$	ΔP
E	ML^2T^{-2}	Dynamic		E
ρ	ML^{-3}	Dynamic	ρ	
c	LT^{-1}	Kinematic	c	
D	L	Geometry	D	

$$pi\ term\ =\ 5 - 3 = 2\ pi\ term$$

$$\pi_1 = \Delta P \times D^A c^B \rho^C$$

$$\pi_2 = E \times D^A c^B \rho^C$$

$$\frac{\Delta P}{\rho c^2} = f\left(\frac{E}{\rho c^2 D^3}\right)$$

Similarity term;

$$\left(\frac{\Delta P}{\rho c^2}\right)_{model} = \left(\frac{\Delta P}{\rho c^2}\right)_{prototype}$$

$$\left(\frac{E}{\rho c^2 D^3}\right)_{model} = \left(\frac{E}{\rho c^2 D^3}\right)_{prototype}$$

From question;

$$E_{model} = 0.001 E_p$$

$$\frac{E_m}{\rho_m c_m^2 D_m^3} = \frac{E_p}{\rho_p c_p^2 D_p^3}$$

$$\rho_m = \rho_p \qquad , \qquad c_m = c_p$$

$$D_m^3 = \frac{E_m}{E_p} D_p^3$$

$$D = 1.25992 \quad m$$

Question 5

Wind blowing past a flag causes it to flutter in the breeze. The frequency of this fluttering is assumed to be a function of the wind speed, the air density, the acceleration of gravity, the length of the flag and the area density (unit = kg/m2). It is desired to predict the flutter frequency of a large 12 m flag in a 9 m/s wind speed. To do this, a model flag with length of 1.5 m is to be tested in a wind tunnel.

(a) Determine the required area density of the model flag material if the large flag has an area density of 0.07 kg/m^2.

(b) What wind tunnel velocity is required for testing the model?

(c) If the model flag flutters at 6 Hz, predict the frequency for the large flag.

Answer 5

$$\omega = f(V, \rho, g, L, \rho_A)$$

Parameter	MLT		Repeating parameter	Non-repeating parameter
ω	T^{-1}	Main subject	$\times$	ΔP
V	LT^{-1}	Kinematic		E
ρ	ML^{-3}	Dynamic	ρ	
g	LT^{-2}	Kinematic	g	
L	L	Geometry	L	
ρ_A	ML^{-2}	Dynamic		

$$pi\ term\ =\ 6-3=3\ pi\ term$$

$$\omega\sqrt{\frac{L}{g}}=f\left(\frac{V}{\sqrt{gL}},\frac{\rho_A}{\rho L}\right)$$

Similarity form;

$$\omega_m\sqrt{\frac{L_m}{g}}=\omega_p\sqrt{\frac{L_p}{g}}$$

$$\frac{V_m}{\sqrt{gL_m}}=\frac{V_p}{\sqrt{gL_p}}$$

$$\frac{\rho_{A_m}}{\rho_m L_m}=\frac{\rho_{A_p}}{\rho_p L_p}$$

(a)

$$\rho_{A_m} = \frac{\rho_{A_p}}{\rho_p L_p} \rho_m L_m$$

$$\rho_p = \rho_m$$

$$\rho_{A_m} = \frac{1.5}{12}(0.07) = 8.75 \times 10^{-3} \quad kg/m^2$$

(b)

$$\frac{V_m}{\sqrt{gL_m}} = \frac{V_p}{\sqrt{gL_p}}$$

$$V_m = V_p\sqrt{\frac{L_m}{L_p}} = (9)\sqrt{\frac{1.5}{12}} = 1.125 \quad m/s$$

(c)

$$\omega_m \sqrt{\frac{L_m}{g}} = \omega_p \sqrt{\frac{L_p}{g}}$$

$$\omega_p = \omega_m \sqrt{\frac{L_m}{L_p}} = 6\sqrt{\frac{1.5}{12}} = 0.75 \quad Hz$$

Question 6

Consider a relatively general relationship between the pressure drop, ΔP, a characteristic length, L, a characteristic velocity, V, the viscosity, μ, the gravity, g, the surface tension, σ, the speed of sound, c, and an angular frequency, ω, written as:

$$\Delta P = f(L, V, \rho, \mu, g, c, \omega, \sigma)$$

The pi-theorem applied to this problem with L, V, ρ as repeating parameters, result in;

$$\frac{\Delta P}{\rho V^2} = f\left(\frac{V\rho L}{\mu}, \frac{V^2}{Lg}, \frac{V}{c}, \frac{L\omega}{V}, \frac{V^2 \rho L}{\sigma}\right)$$

Each of the pi terms in this expression is a common dimensionless parameter that appears in numerous fluid flow situations. They are identified as follows:

$$\text{Euler number, Eu} = \frac{\Delta p}{\rho V^2}$$

$$\text{Reynolds number, Re} = \frac{V\rho l}{\mu}$$

$$\text{Froude number}^2\text{, Fr} = \frac{V}{\sqrt{lg}}$$

$$\text{Mach number, M} = \frac{V}{c}$$

$$\text{Strouhal number}^2\text{, St} = \frac{l\omega}{V}$$

$$\text{Weber number}^2\text{, We} = \frac{V^2 l\rho}{\sigma}$$

For specific application, you may use the common dimensionless parameter to solve problems.

Parameter	*Expression*	*Flow situations where parameter is important*
Euler number	$\frac{\Delta p}{\rho V^2}$	Flows in which pressure drop is significant: most flow situations
Reynolds number	$\frac{\rho l V}{\mu}$	Flows that are influenced by viscous effects: internal flows, boundary layer flows
Froude number	$\frac{V}{\sqrt{lg}}$	Flows that are influenced by gravity: primarily free surface flows
Mach number	$\frac{V}{c}$	Compressibility is important in these flows, usually if $V > 0.3\ c$
Strouhal number	$\frac{l\omega}{V}$	Flow with an unsteady component that repeats itself periodically
Weber number	$\frac{V^2 l \rho}{\sigma}$	Surface tension influences the flow; flow with an interface may be such a flow

Question 7

Carbon tetrachloride flows with a velocity of 0.30 m/s through a 30 mm diameter tube. A model of this system is to be developed using standard air tube as the model fluid. The air velocity is to be 2 m/s. What tube diameter is required for the model if dynamic similarity is to be maintain between model and prototype?

Answer 7

This is flow in pipe problem. We can use Reynolds number to do the similarity.

$$Re = \frac{\rho V D}{\mu} = \frac{V D}{\upsilon}$$

$$D_{air} = \frac{\upsilon_{air}}{\upsilon_{tetra}} \times \frac{V_{tetra}}{V_{air}} \times D_{tetra}$$

$$D_{air} = \frac{1.46 \times 10^{-5}}{6.03 \times 10^{-7}} \times \frac{0.3}{2} \times 0.03 = 0.109 \quad m$$

Question 8

The flowrate over the spillway of a dam is 1000 m^3/min. Determine the required flowrate for a 1:25 scale model that is operated.

Answer 8

This is an open channel problem. We can use Froude number.

$$Fr = \frac{V}{\sqrt{gL}}$$

$$\frac{V_m}{\sqrt{gL_m}} = \frac{V_p}{\sqrt{gL_p}}$$

$$1:25 \quad \Longrightarrow \quad L_m : L_p$$

$$\frac{V_m}{V_p} = \sqrt{\frac{L_m}{L_p}}$$

In this question, flowrate is one of the characteristic, so we could use;

$$\frac{Q_m}{Q_p} = \frac{A_m V_m}{A_p V_p} = \frac{L_m^2}{L_p^2} \times \sqrt{\frac{L_m}{L_p}} = \left(\frac{L_m}{L_p}\right)^{\frac{5}{2}}$$

$$Q_m = \left(\frac{L_m}{L_p}\right)^{\frac{5}{2}} \times Q_p = \left(\frac{1}{25}\right)^{\frac{5}{2}} \times 1000 = 0.32 \quad m^3/min$$

No need to convert m^3/min to m^3/s because similarity is about ratio between two items.