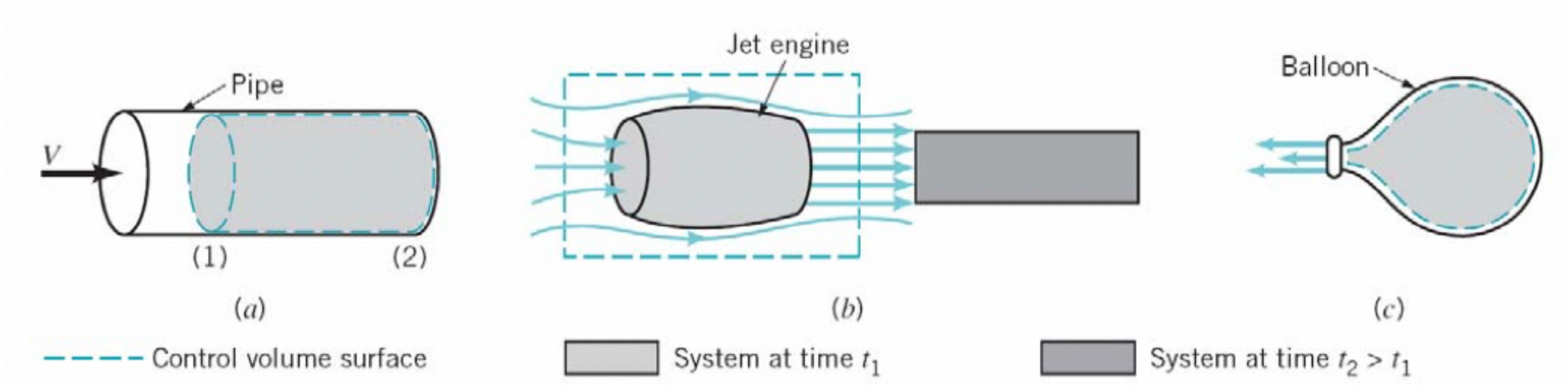
KINEMATICS OF FLUID MOTION

CONTROL VOLUME AND SYSTEM REPRESENTATIONS



A control volume, on the other hand, is a volume in space (a geometric entity, independent of mass) through which fluid may flow.

A control surface is the surface area that completely encloses the control volume.

A system is a specific, identifiable quantity of matter. It may consist of a relatively large amount of mass, or it may be an infinitesimal size.

The system may interact with its surroundings by various means. It may continually change size and shape, but it always contains the same mass.

REYNOLDS TRANSPORT THEOREM

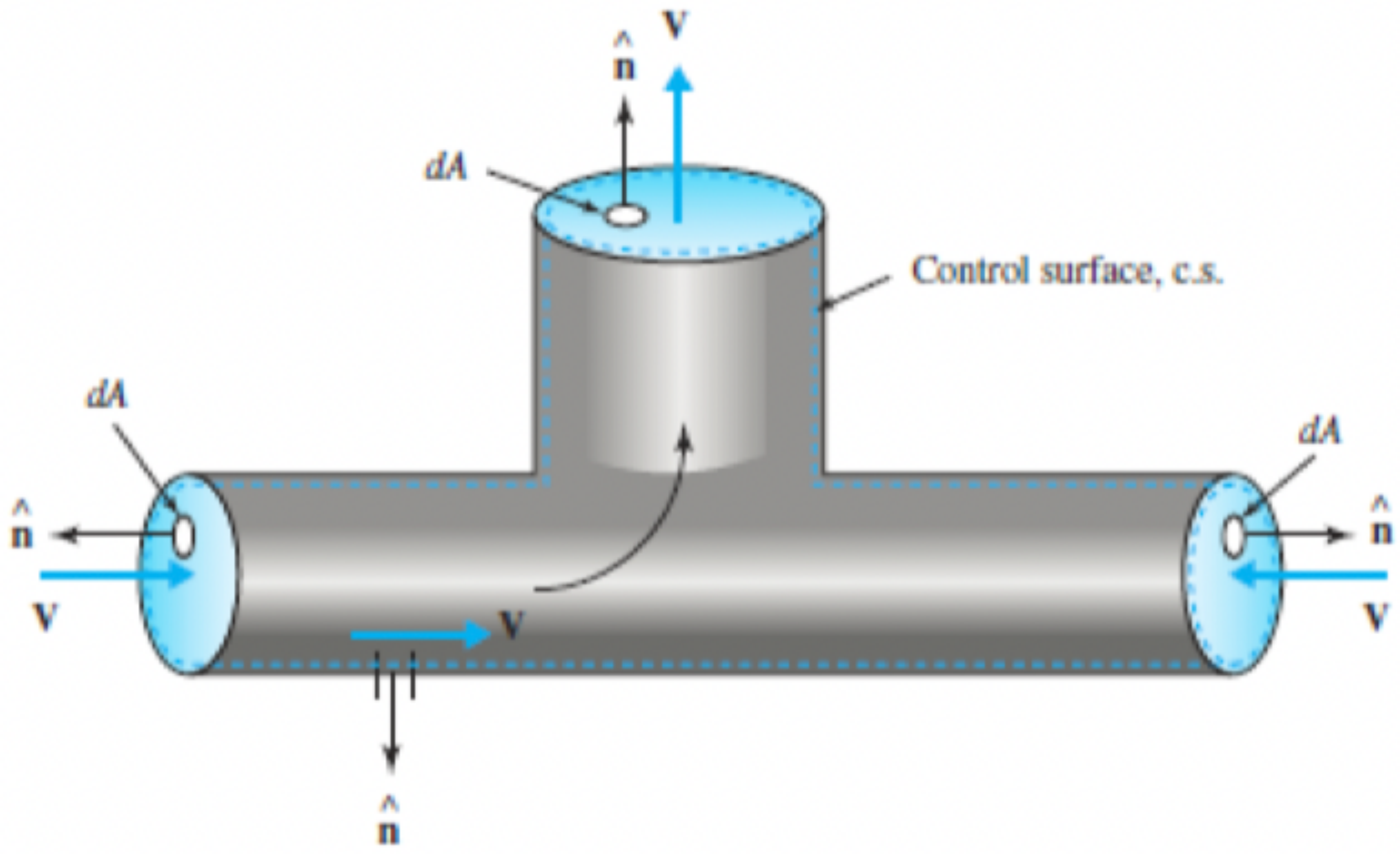
We are sometimes interested in what happens to a particular part of the fluid as it moves about.

Other times we may be interested in what effect the fluid has on a particular object or volume in space as fluid interact with it.

Thus, we need to describe the laws governing fluid motion using both system concept (consider a given mass of the fluid) and control volume concept (consider a given volume).

To do this we need an analytical tool to shift from one representation to the other.

The Reynolds Transport Theorem provides this tool.



General form of the Reynolds transport theorem for a fixed, non-deforming control volume is given as ;

$$\frac{DN_{sys}}{Dt} = \frac{\partial}{\partial t}\int_{cv} \rho b \cdot d\forall + \int_{cs} \rho b \cdot V \cdot \hat{n} \cdot dA$$

All physical laws are stated in terms of various physical parameters.

Mass is the common parameters.

Let N represent any other fluid parameters and b represent the amount of that parameter per unit mass. That is ;

$$N = m \cdot b$$

where m is the mass of the portion of fluid of interest.

The N is termed an extensive property and b is termed an intensive property.

If N is mass, it follows that b is equal to 1.

If N is kinetic energy;

$$N = \frac{1}{2} m V^2$$

Then,

$$b = \frac{V^2}{2}$$

Discussion:

$$\frac{DN_{sys}}{Dt} = \frac{\partial}{\partial t}\int_{cv} \rho b \cdot d\forall + \int_{cs} \rho b \cdot V \cdot \hat{n} \cdot dA$$

If N_{sys} is mass, then $\frac{DN_{sys}}{Dt} = \frac{Dm}{Dt} =$ mass flow rate

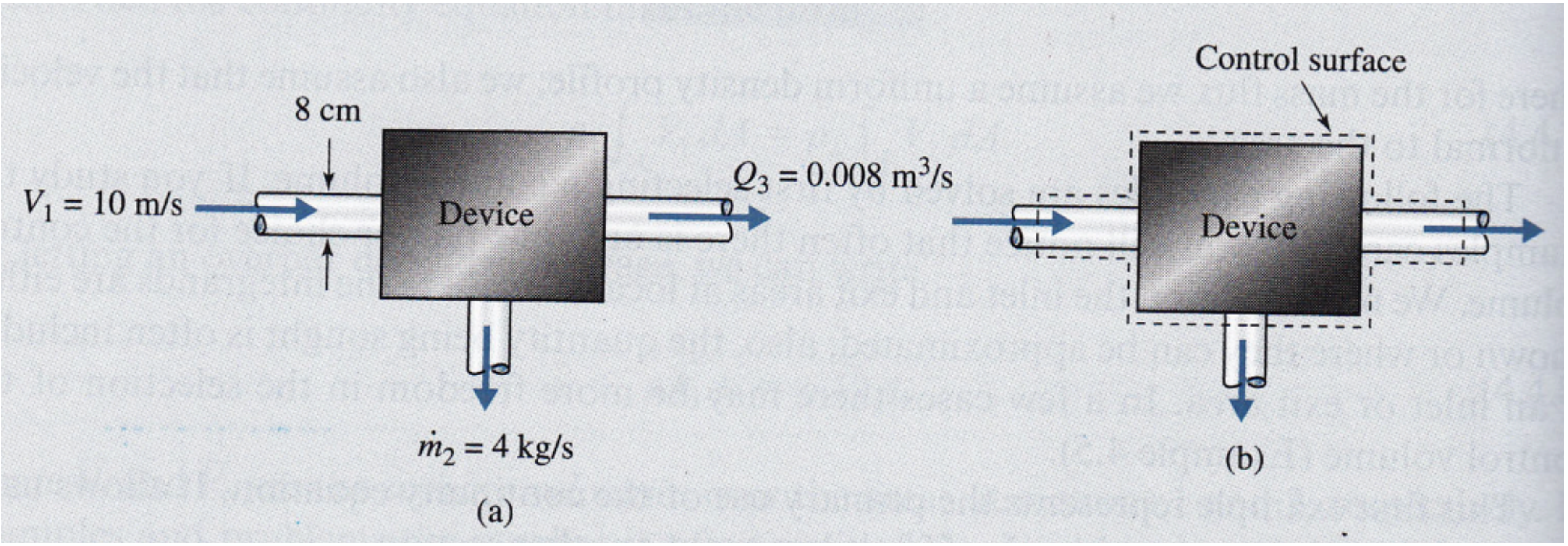
$$m = \rho\forall$$

$$\frac{\partial}{\partial t}\int_{cv} \rho b \cdot d\forall = \frac{\partial}{\partial t}\int_{cv} \rho(1) \cdot d\forall$$

$$\int_{cs} \rho b \cdot V \cdot \hat{n} \cdot dA = \int_{cs} \rho(1) \cdot V \cdot \hat{n} \cdot dA$$

Tutorial 1

Water flows in and out of a device as shown below. Calculate the rate of change of the mass of water in the device.



Determine the control volume and control surface.

Determine the system. In this case, the system is mass flow rate.

$$\frac{DN_{system}}{Dt} = \frac{d}{dt}\int_{CV} \rho \, d\forall + \int_{CS} \rho \, V_n \cdot dA .$$

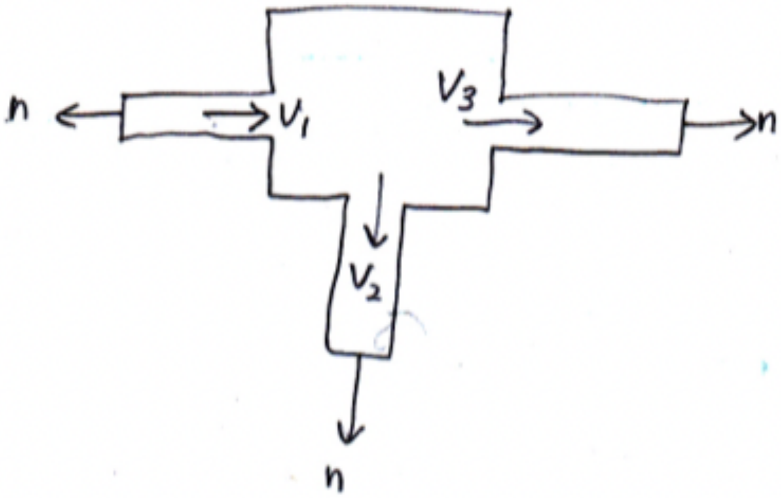
$$0 = \frac{d}{dt}\int_{CV} \rho \cdot d\forall + \int_{CS} \rho \, V_n \cdot dA$$

$$= \frac{dm}{dt} + \int_{CS} \rho \cdot V_n \cdot dA$$

$$\rho = \frac{m}{\forall}$$

$$m = \rho \forall .$$

Draw the free body diagram of the system and determine the equation base on the Reynolds Transport Theorem.



$$\therefore \quad 0 = \frac{dm}{dt} + \rho(-V_1)(A_1) + \rho(V_2)A_2 + \rho(V_3)A_3$$

$$\therefore \quad 0 = \frac{dm}{dt} + \rho(-V_1)(A_1) + \rho(V_2)A_2 + \rho(V_3)A_3$$

$$\frac{dm}{dt} = \rho A_1 V_1 - \rho A_2 V_2 - \rho A_3 V_3$$

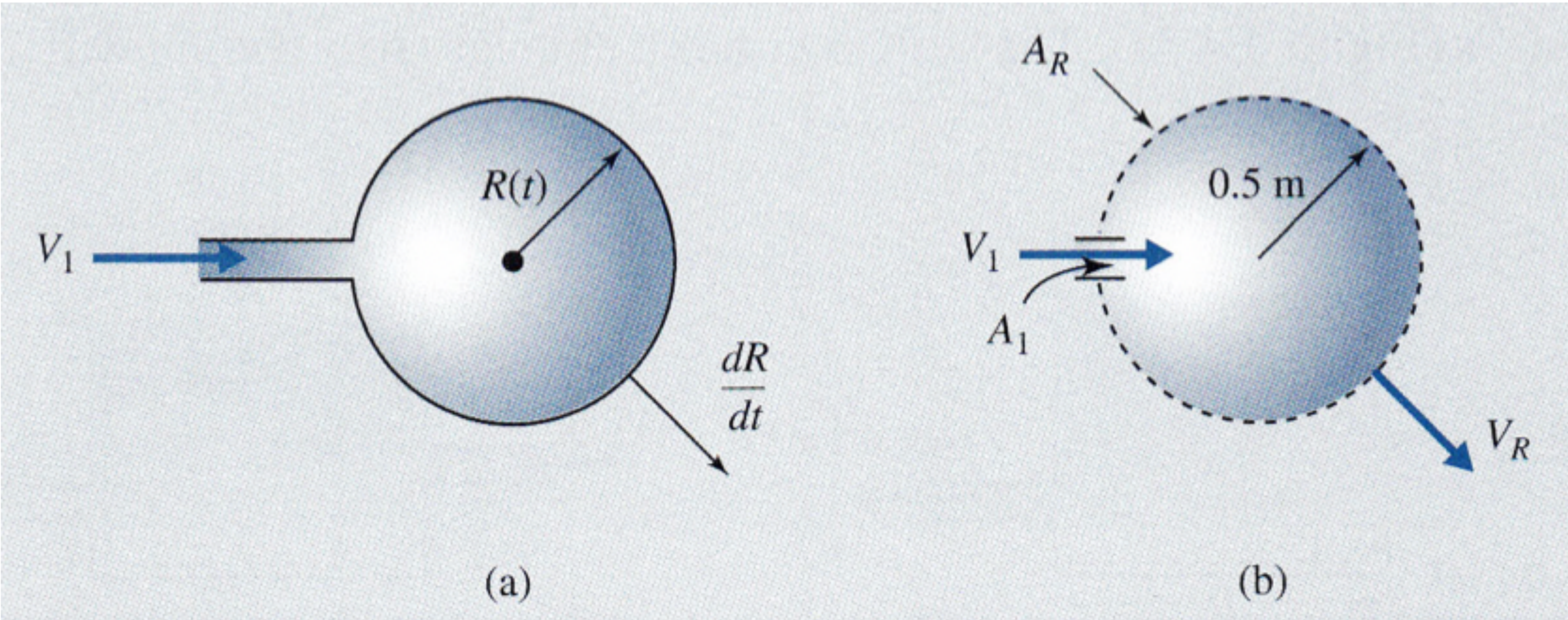
$$= 1000\left(\frac{\pi}{4}\right)(0.08)^2(10) - 4 - (1000)(0.008)$$

$$= 50.272 - 4 - 8$$

$$= 38.272 \text{ kg/s}$$

Tutorial 2

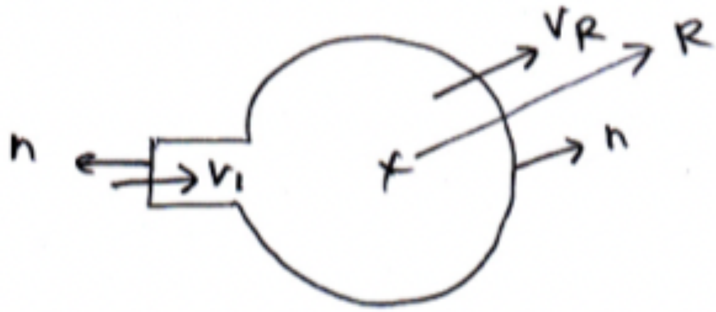
A balloon is being inflated with a water supply of 0.6 m^3/s. Find the rate of growth of the radius at the instant when R = 0.5 m.



$$\frac{dN_{sys}}{dt} = \underbrace{\frac{d}{dt}\int_{cv} \rho \cdot d\forall}_{\text{zero.}} + \int_{cs} \rho \cdot V_n \cdot dA.$$

at instant $R = 0.5$ m.

Soalan : $\frac{dR}{dt}$.



The first term is zero because the density of water inside the control volume does not change in time.

$$0 = \rho(-V_1)(A_1) + \rho(V_R)(A_R)$$

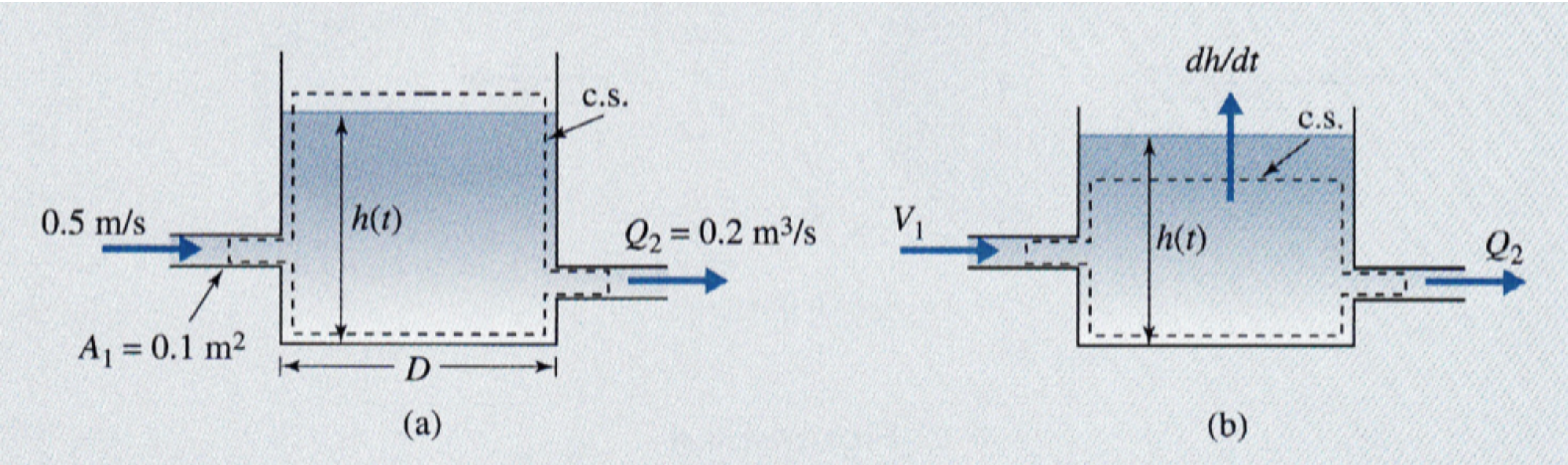
$$= \rho(-V_1)(A_1) + \rho\left(\frac{dR}{dt}\right)(4\pi R^2)$$

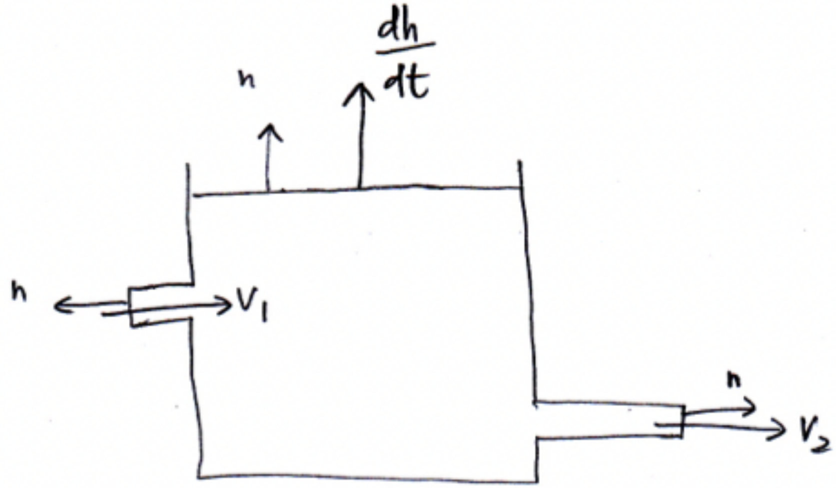
$$= (1.23)(-0.6) + (1.23)\left(\frac{dR}{dt}\right)(4\pi)(0.5)^2$$

$$\frac{dR}{dt} = \frac{(1.23)(0.6)}{(1.23)(4\pi)(0.5)^2} = 0.19096 \text{ m/s}.$$

Tutorial 3

Determine the rate at which the water level rises in an open container if the water coming in through a 0.10 m^2 pipe has a velocity of 0.5 m/s and the flow rate going out is 0.2 m^3/s. The container has a circular cross section with a diameter of 0.5 m.





$$\frac{DN_{sys}}{Dt} = \frac{d}{dt}\int_{cv} \rho \cdot d\forall + \int_{cs} \rho \cdot V_n \cdot dA.$$

tidak sifar kerana volume (isipadu) berubah dengan masa.

$$0 = \frac{d}{dt}\int_{cv} \rho \cdot d\forall + \int_{cs} \rho \cdot V_n \cdot dA.$$

$$= \frac{d}{dt}\left(\rho \cdot \frac{\pi D^2}{4} \cdot h\right) + \rho(-V_1)A_1 + \rho(V_2)(A_2)$$

$$= \rho \cdot \frac{\pi D^2}{4} \cdot \frac{dh}{dt} + \rho(-V_1)A_1 + \rho(V_2)A_2$$

$$\rho \frac{\pi D^2}{4} \cdot \frac{dh}{dt} = \rho V_1 A_1 - \rho V_2 A_2$$

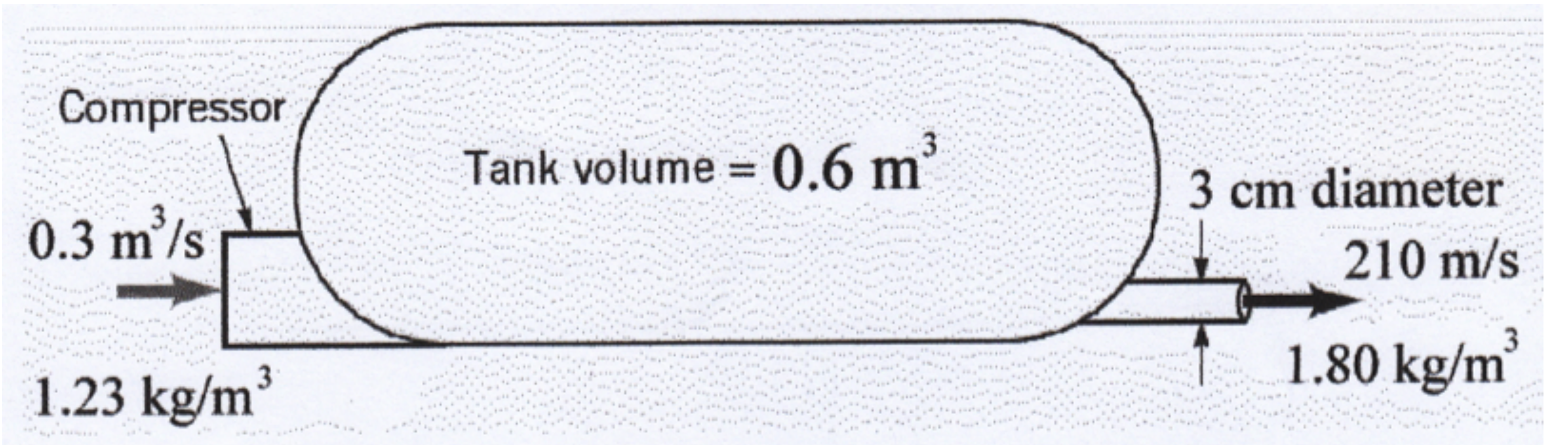
$$\frac{dh}{dt} = \frac{(0.5)(0.1) - (0.2)}{\left(\frac{\pi D^2}{4}\right)} = \frac{0.05 - 0.2}{0.196375}$$

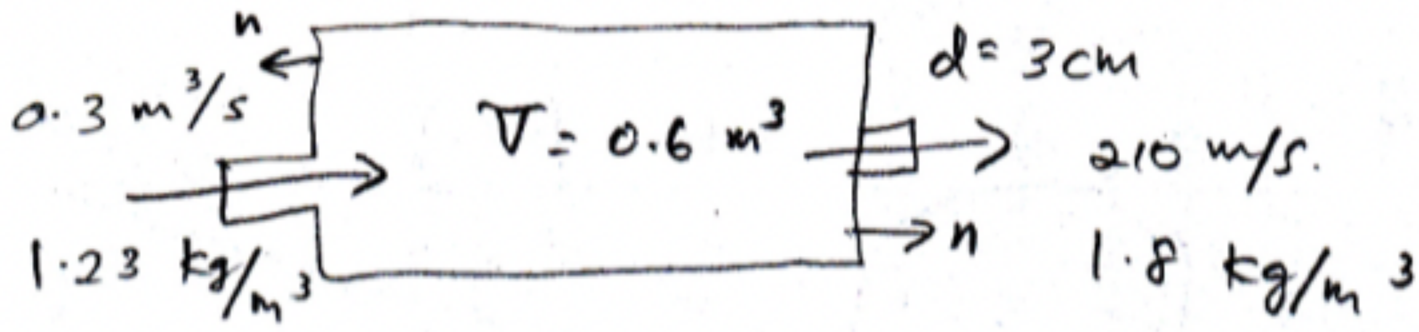
$$= -0.76384 \quad m/s.$$

Tutorial 4

Air at standard conditions enters the compressor at rate of 0.3 m^3/s. It leaves the tank through a 3 cm diameter pipe with a density of 1.80 kg/m^3 and a uniform speed of 210 m/s.

a) Determine the rate at which the mass of air in the tank in increasing or decreasing.
b) Determine the average time rate of change of air density within the tank.





n
0.3 m³/s
1.23 kg/m³
V = 0.6 m³
d = 3cm
210 m/s.
n
1.8 kg/m³

(a) conservation of mass flow rate.

$$\dot{m}_{in} = \dot{m}_{out}$$

$$\rho_1 A_1 V_1 = \rho_2 A_2 V_2$$

$$(1.23)(0.3) = (1.8)(\pi)(0.015)^2(210)$$

$$0.369 \text{ kg/s} = 0.2672 \text{ kg/s}$$

tak sama.

oleh itu, mass of air in the tank is increasing by 0.1018 kg/s

$$\frac{DN_{sys}}{Dt} = \frac{Dm}{Dt} = 0 = \frac{\partial}{\partial t}\int_{cv} \rho b \cdot d\forall + \int_{cs} \rho b \cdot V \cdot \hat{n} \cdot dA$$

$$0 = \frac{dm}{dt} + \rho_{in}(-Q_{in}) + \rho_{out} \cdot V \cdot A$$

$$0 = \frac{dm}{dt} + (1.23)(-0.3) + (1.80)(210)\left(\frac{\pi}{4}\right)(0.03)^2$$

$$\frac{dm}{dt} = 0.1018 \quad \frac{kg}{s}$$

(b)

$$\frac{DN_{sys}}{Dt} = \frac{\partial}{\partial t}\int_{cv} \rho \cdot d\forall + \int_{cs} \rho \cdot v_n \cdot dA$$

oleh kerana, mass dalam tank bertambah,

persamaan di atas menjadi

$$\frac{dN_{sys}}{Dt} = 0 = \frac{\partial}{\partial t}\int_{CV} \rho \cdot d\forall + \left[\rho_1 (-V_1)(A_1) + \rho_2 (V_2)(A_2) \right]$$

$$0 = \frac{\partial}{\partial t}\int_{CV} \rho \cdot d\forall + \left[-\rho Q_1 + \rho_2 V_2 A_2 \right]$$

$$= \frac{\partial}{\partial t}\int_{CV} \rho \cdot d\forall - 0.1018$$

$$0.1018 = \frac{\partial \rho}{\partial t} V$$

volume tank = constant

soalan berkehendakan time rate of change of air density.

$$\frac{\partial \rho}{\partial t} = \frac{0.1018 \ kg/s}{0.6 \ m^3} = 0.1697 \ kg/m^3 \cdot s$$