FLUID MECHANICS I

SEMM 2313

REYNOLDS TRANSPORT THEOREM FOR MOMENTUM EQUATION

$$\frac{DN_{sys}}{Dt} = \frac{\partial}{\partial t}\int_{cv} \rho b \cdot d\forall + \int_{cs} \rho b \cdot (V \cdot \hat{n}) \cdot dA$$

If $\frac{DN_{sys}}{Dt}$ is a force; N_{sys} must be a momentum.

$$\frac{DN_{sys}}{Dt} = F = ma = \frac{D(mV)}{Dt}$$

$$N_{sys} = mV$$

$$b = V$$

Where V is velocity of moving fluid.

From the Reynolds Transport Theorem general equation;

$$\frac{DN_{sys}}{Dt} = F = ma = \frac{D(mV)}{Dt} = \frac{\partial}{\partial t}\int_{cv} \rho b \cdot d\forall = \frac{\partial}{\partial t}\int_{cv} \rho(V) \cdot d\forall$$

$$F = \frac{D(mV)}{Dt} = \frac{\partial}{\partial t}\int_{cv} \rho(V) \cdot d\forall + \int_{cs} \rho(V) \cdot (V \cdot \hat{n}) \cdot dA$$

Where V is velocity and $\forall$ is volume.

In general, Reynolds Transport Theorem for momentum equation can be written as:

$$\sum F = \frac{\partial}{\partial t}\int_{cv} \rho(V) \cdot d\forall + \int_{cs} \rho(V) \cdot (V \cdot \hat{n}) \cdot dA$$

$$(V) \neq (V \cdot \hat{n})$$

$$V = \text{Velocity of moving fluid}$$

$$V \cdot \hat{n} = \text{Velocity of fluid compare with vector } \hat{n}$$

For incompressible and steady flow, velocity of fluid did not change with time, it is means that;

$$\frac{\partial}{\partial t}\int_{cv} \rho(V) \cdot d\forall = 0$$

Momentum equation can be simplified as:

$$\sum F = \int_{cs} \rho(V) \cdot (V \cdot \hat{n}) \cdot dA$$

In some textbook, it is written as:

$$\sum F = \sum \rho AV \cdot (V \cdot \hat{n}) \sum \rho Q \cdot (V \cdot \hat{n})$$

Please remember this:

$$(V) \neq (V \cdot \hat{n})$$

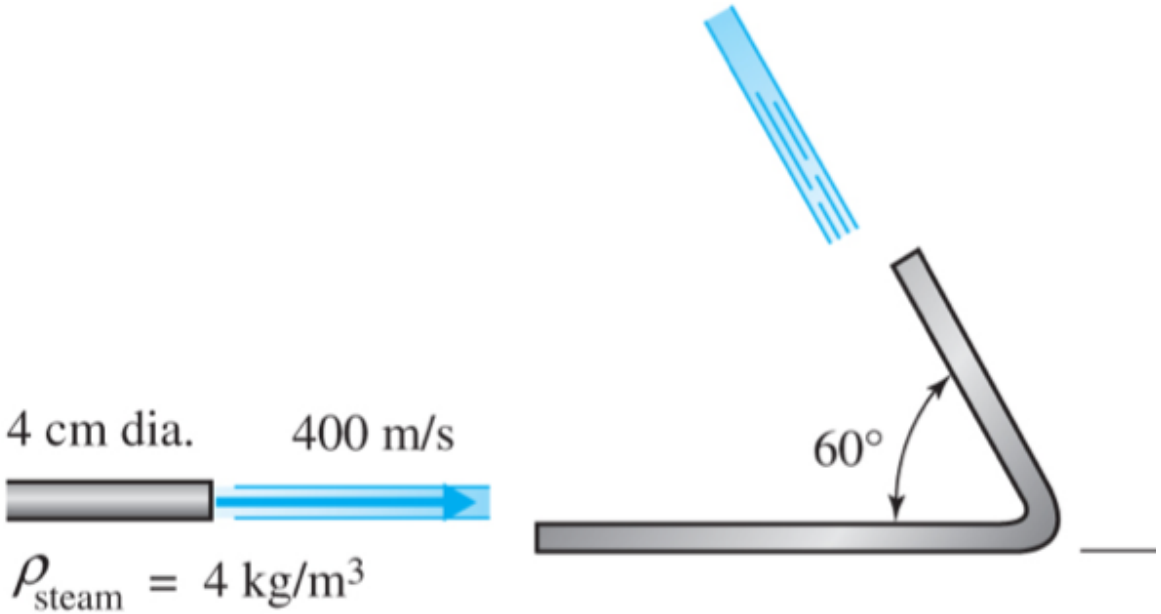
$$V = \text{Velocity of moving fluid}$$

$$A = \text{Cross-sectional area of moving fluid}$$

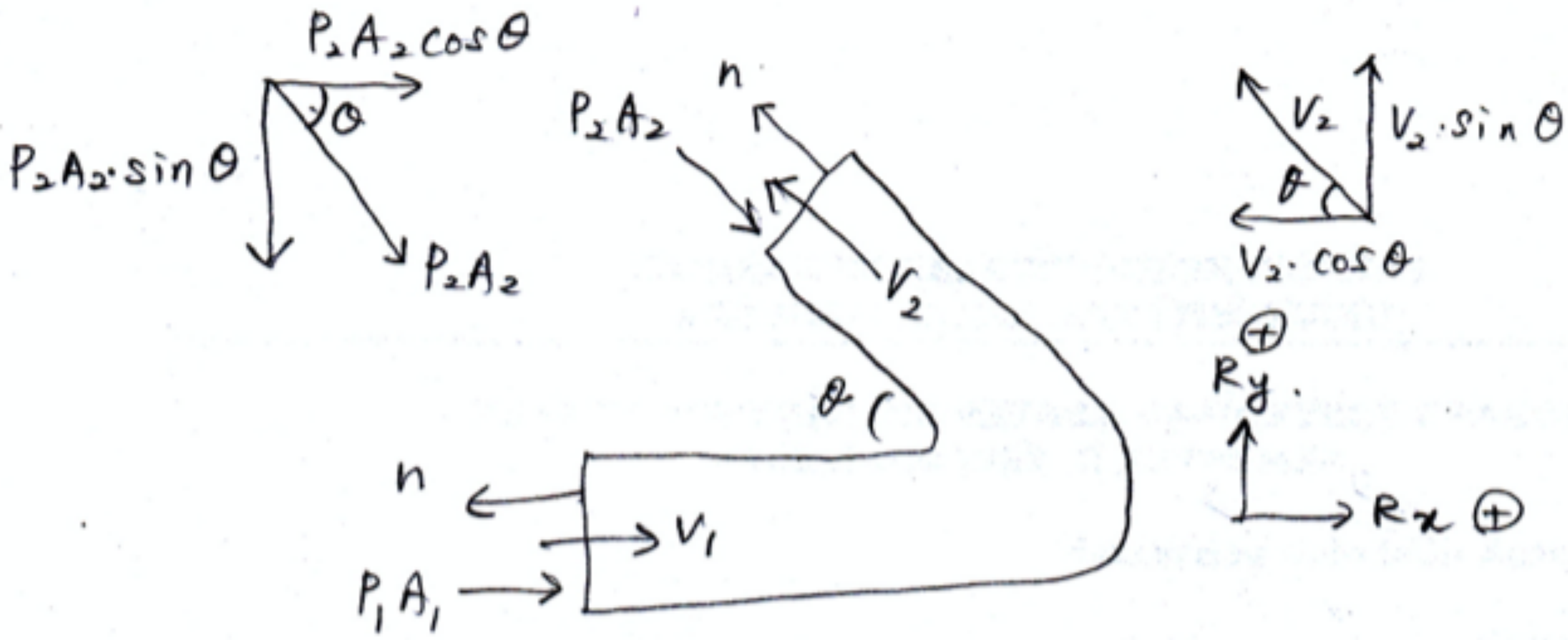
$$V \cdot \hat{n} = \text{Velocity of fluid compare with vector } \hat{n}$$

Tutorial 1

Determine the force component of superheated steam acting on the blade.
a) The blade is stationary
b) The blade moves to the right at 100 m/s
c) The blade moves to the left at 100 m/s



Sketch the control volume and its important information.



$$\sum F = \sum \rho . V_n . A . V$$

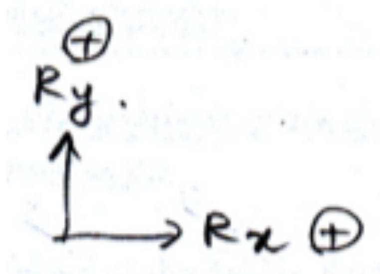
Here,

$$\sum F = \text{Total force that occur due to pressure and reaction force}$$

So that, in the control volume diagram, we need to add force due to the pressure and the reaction force that occur in the system.

Force due to the pressure must act perpendicular to the control surface.

Reaction force (normally) predicted as this. Our final calculation could determine whether its direction correct or not.



$$\sum F = \sum \rho \cdot V_n \cdot A \cdot V$$

$$R_x + P_1 A_1 + P_2 A_2 \cos\theta = \rho(-V_1) A_1 (V_1) + \rho (V_2) A_2 (-V_2 \cdot \cos\theta)$$

$$R_x = (4)(-400)\left(\frac{\pi}{4}\right)(0.04)^2(400) + (4)(400)\left(\frac{\pi}{4}\right)(0.04)^2 (-400 \cos\theta)$$

$$= -804.352 - 402.176$$

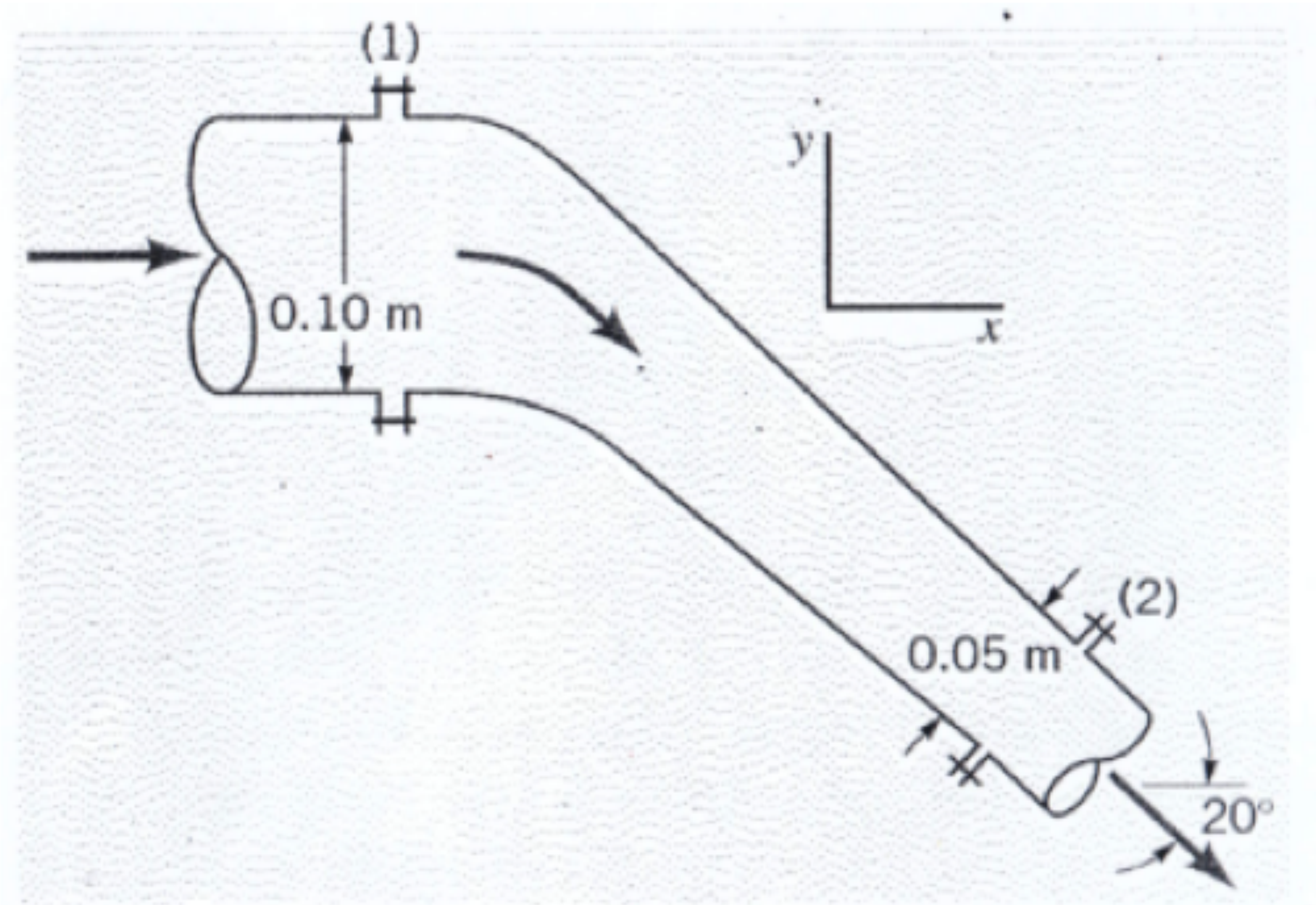
$$= -1206.528 \ (N)$$

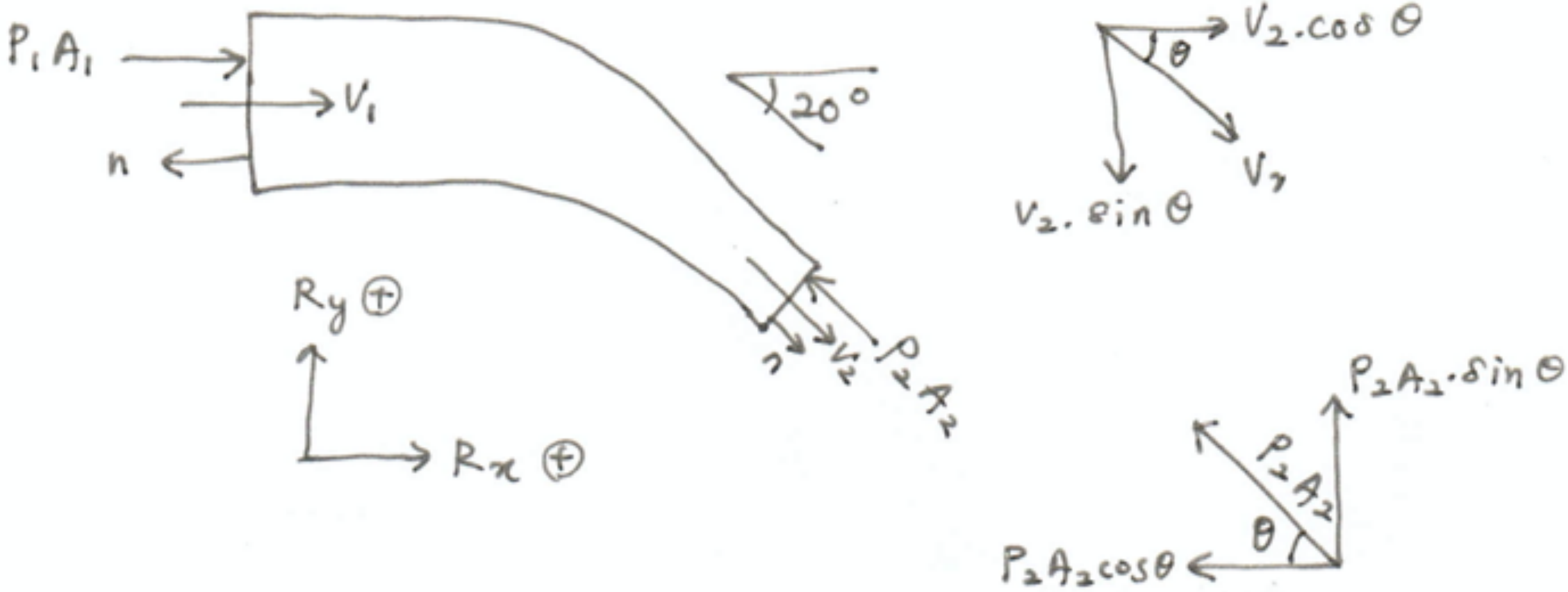
$$R_x (\leftarrow) = 1206.528 \ (N)$$

$$R_y (\downarrow) = 696 \ (N)$$

Tutorial 2

Water flows through the 20° reducing bend at rate 0.025 m^3/s. The flow is frictionless, gravitational effects are negligible and the pressure at section (1) is 150 kPa. Determine the x and y components of force required to hold the bend in place.





P1 A1
V1
n
20°
Ry ⊕
Rx ⊕
n
V2
P2 A2
V2.cos θ
θ
V2
V2. sin θ
P2A2.sin θ
P2A2
θ
P2A2cosθ

x - direction

$$F_x = \Sigma \rho A V_n \cdot V_x .$$

$$R_x + P_1 A_1 - P_2 A_2 \cos\theta = \rho A_1 (-V_1)(V_1) + \rho A_2 (V_2)(V_2 \cdot \cos\theta)$$

component P_1, P_2, V_1, V_2 perlu dicari

Pressure boleh dikira dengan menggunakan Bernoulli equation.

velocity boleh diselesaikan dengan menggunakan continuity equation.

$$Q = A_1 V_1 = A_2 V_2$$

$$0.025 = A_1 V_1 = \frac{\pi}{4}(0.1)^2 \cdot V_1$$

$$V_1 = 3.18 \text{ m/s.}$$

$$V_2 = 12.73 \text{ m/s.}$$

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} = \frac{P_2}{\rho g} + \frac{V_2^2}{2g}$$

$P_1 = 150$ kPa.

$\therefore$ $P_2 = 74$ kPa.

masukkan nilai yang diperolehi dalam persamaan linear momentum.

$R_x = -822 \,(N)$

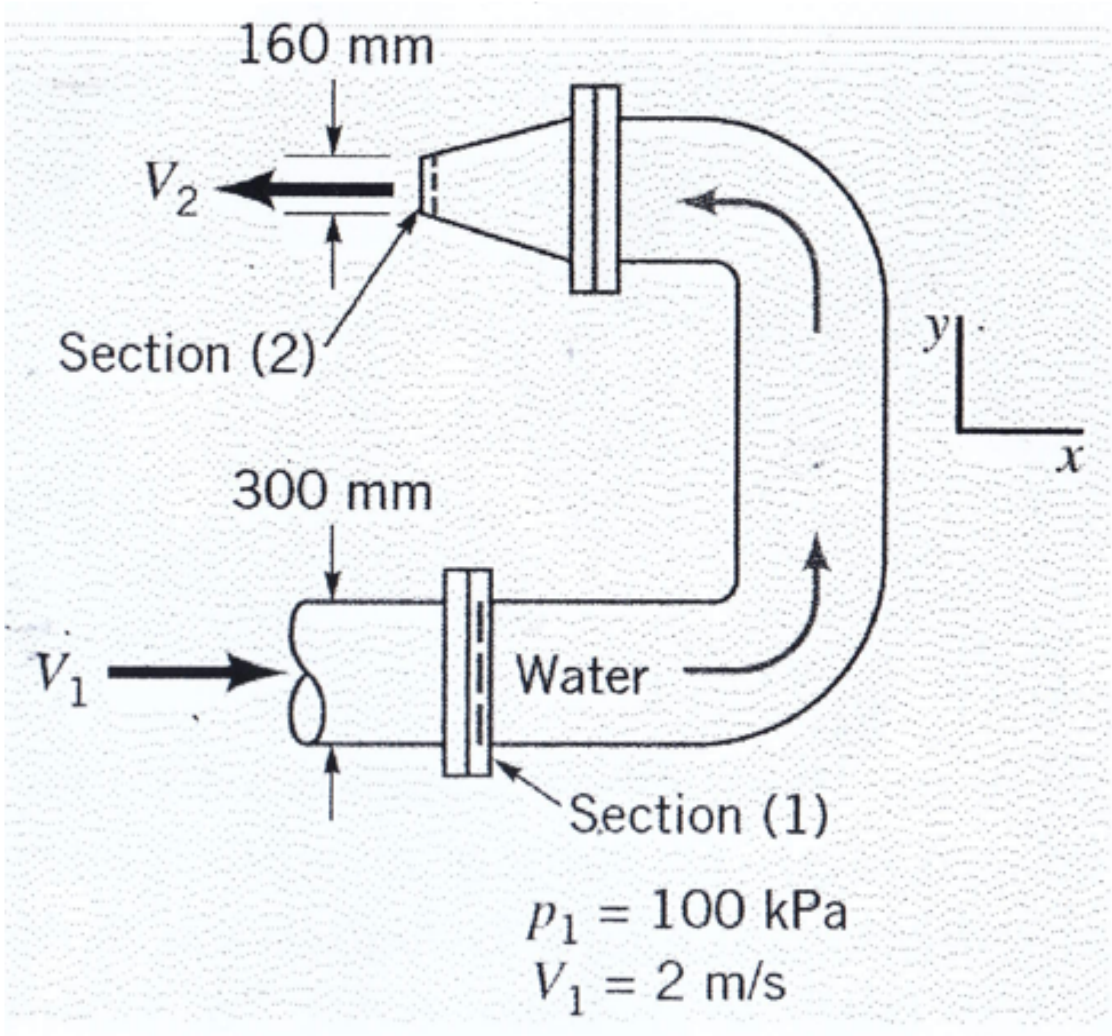
↑

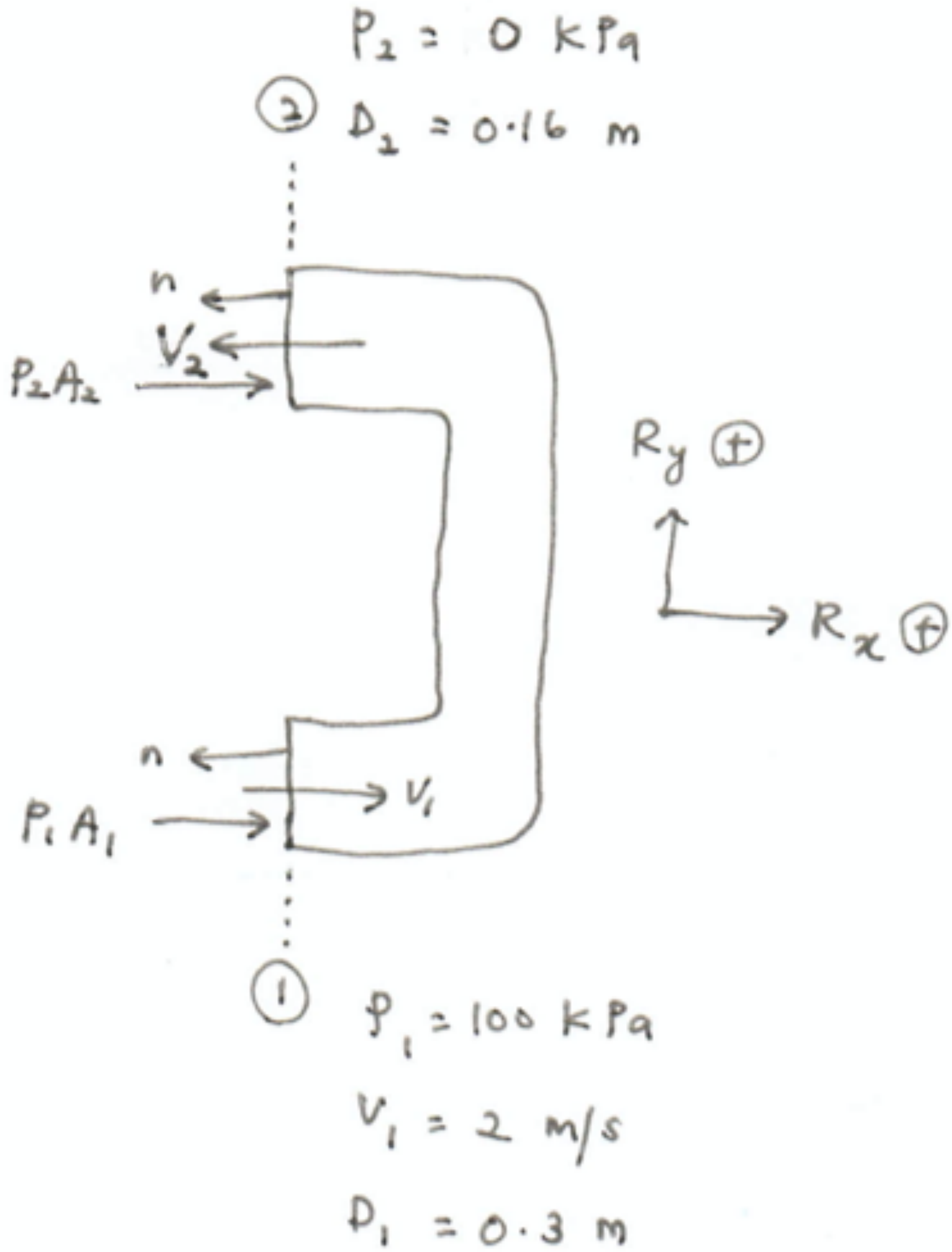
Tanda negatif menunjukkan R_x bertindak pada arah yang berlawanan.

$R_y = -157.7 \,(N)$

Tutorial 3

Determine the magnitude and direction of the anchoring force needed to hold the horizontal elbow and nozzle combination shown below in place. The gage pressure at section (1) is 100 kPa. At section (2), the water exits to the atmosphere.





$P_2 = 0$ kPa
②
$D_2 = 0.16$ m
n
V_2
P_2A_2
R_y ⊕
R_x ⊕
n
V_1
P_1A_1
①
$P_1 = 100$ kPa
$V_1 = 2$ m/s
$D_1 = 0.3$ m

x - component

$$F_x = \sum \rho A V_n \cdot V_x.$$

$$R_x + P_1 A_1 + P_2 A_2 = \rho A_1(-V_1)(V_1) + \rho A_2 (V_2)(V_2)$$

↑ $P_{atm} = 0$

From $A_1 V_1 = A_2 V_2$

$V_2 = 7.03$ m/s

$$R_x = 1000 \left(\frac{\pi}{4}\right)(0.3)^2(-2)(2) + 1000\left(\frac{\pi}{4}\right)(0.16)^2(7.03)(7.03)$$

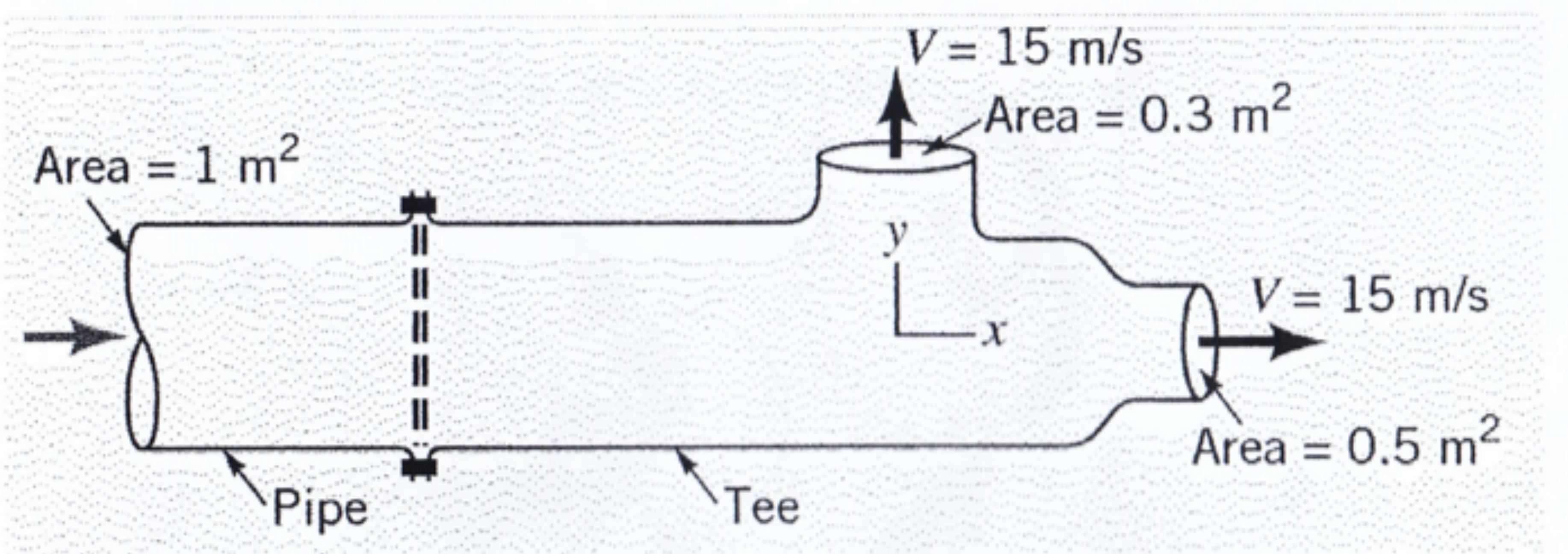
$$- 100 \times 10^3 \left(\frac{\pi}{4}\right)(0.3)^2$$

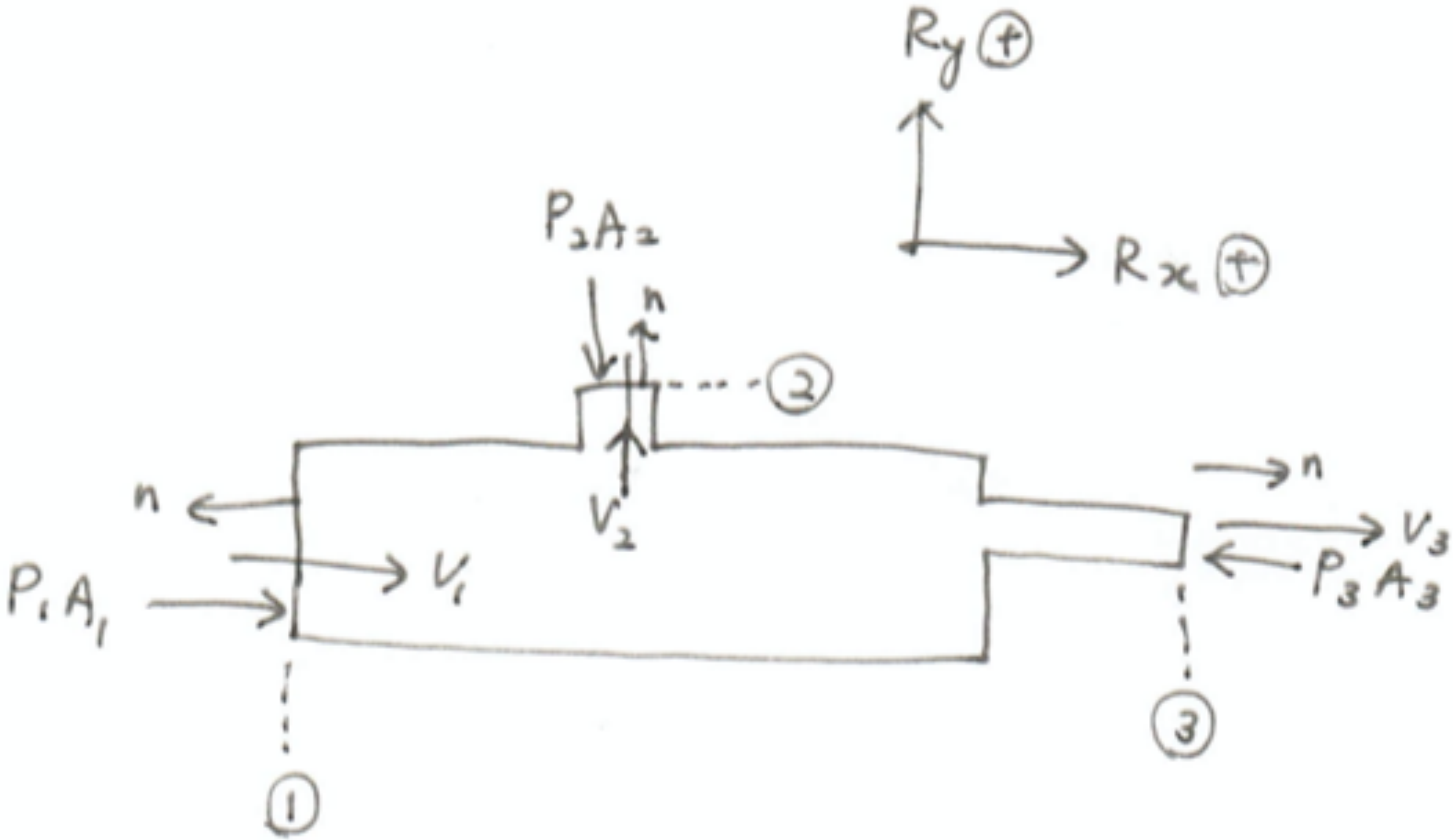
$R_x = -6358.5$ (N)

$R_y =$ zero (tiada)

Tutorial 4

Water flows as two free jets from the tee attached to the pipe. The exit speed is 15 m/s. If viscous effects and gravity are negligible, determine the x and y components of force that the pipe exerts on the tee





Ry ⊕
Rx ⊕
P2A2
n
2
n
V2
n
V1
P1A1
V3
P3A3
3
1

$$P_2 = P_3 = P_{atm} = 0$$

$$Q_{in} = Q_{out}$$

$$Q_1 = Q_2 + Q_3$$

$$A_1 V_1 = A_2 V_2 + A_3 V_3$$

$$\therefore \quad V_1 = 12 \text{ m/s.}$$

Energy ① = Energy ②

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} = \frac{P_2}{\rho g} + \frac{V_2^2}{2g}.$$

$$P_1 = 40500 \quad (N)$$

x - direction :

$$F_x = \sum \rho A V_n \cdot V_x.$$

$$R_x + P_1 A_1 - P_3 A_3 = \rho A_1 (-V_1)(V_1) + \rho A_2 (V_2)(0) + \rho A_3 (V_3)(V_3)$$

↑ Tiada velocity arah -x pada point 2.

$$R_x = -72 \text{ kN}.$$

y - direction

$$F_y = \sum \rho A V_n \cdot V_y$$

$$R_y + (-P_2 A_2) = \rho A_2 (V_2)(V_2)$$

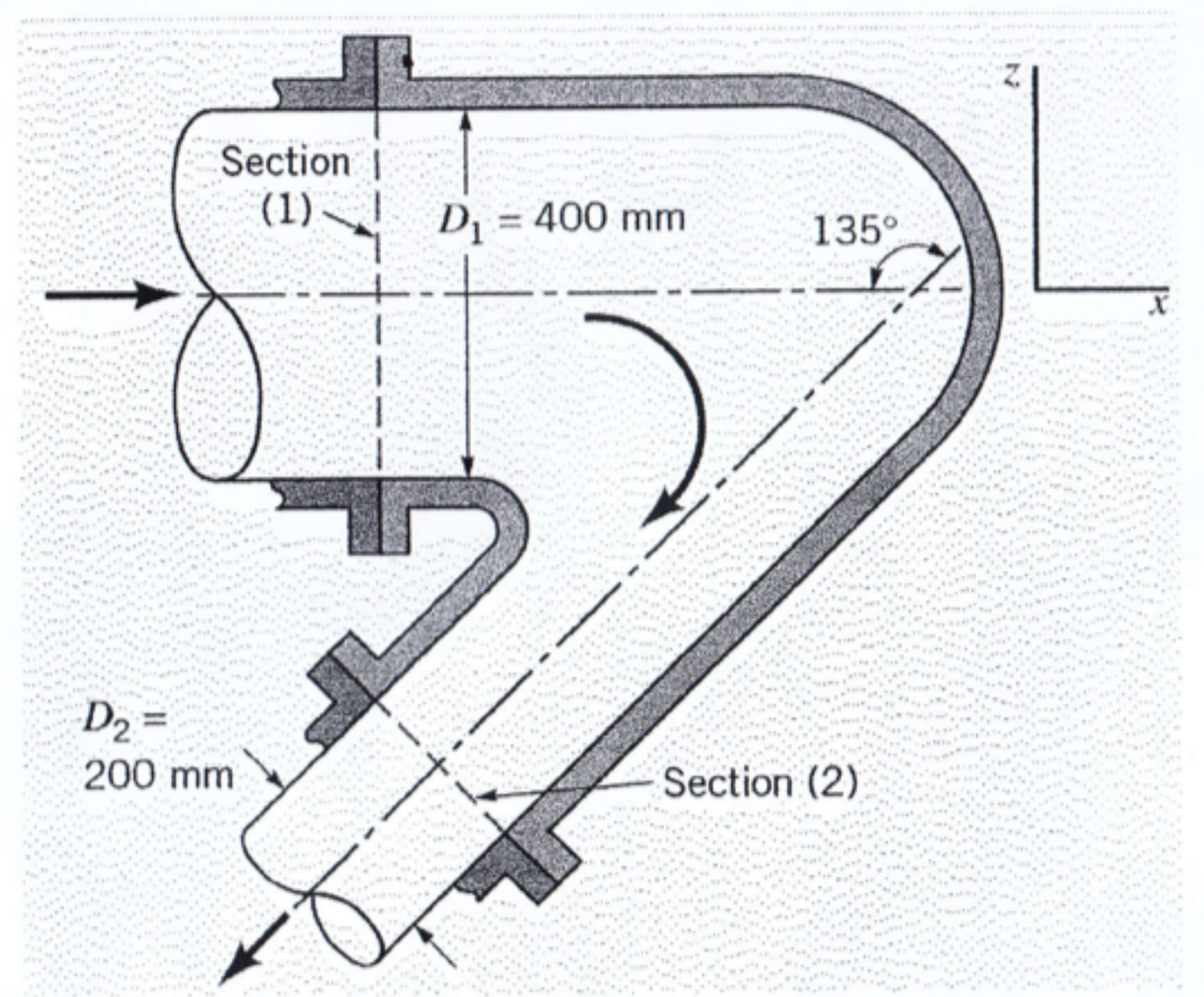
↑

$P_{atm} = 0$

$$R_y = 67500 \text{ (N)}$$

Tutorial 5

A converging elbow turns water through an angle of 135° in a vertical plane. The flow cros section diameter is 400 mm at the elbow inlet (section 1), and 200 mm at the elbow outlet (section 2). The elbow flow passage volume is 0.2 m^3 between section 1 and 2. The water volume flow rate is 0.4 m3/s and the elbow inlet and outlet pressures are 150 kPa and 90 kPa. The elbow mass is 12 kg. Calculate the horizontal (x-direction) and vertical (z-direction) anchoring forces required to hold the elbow in place.

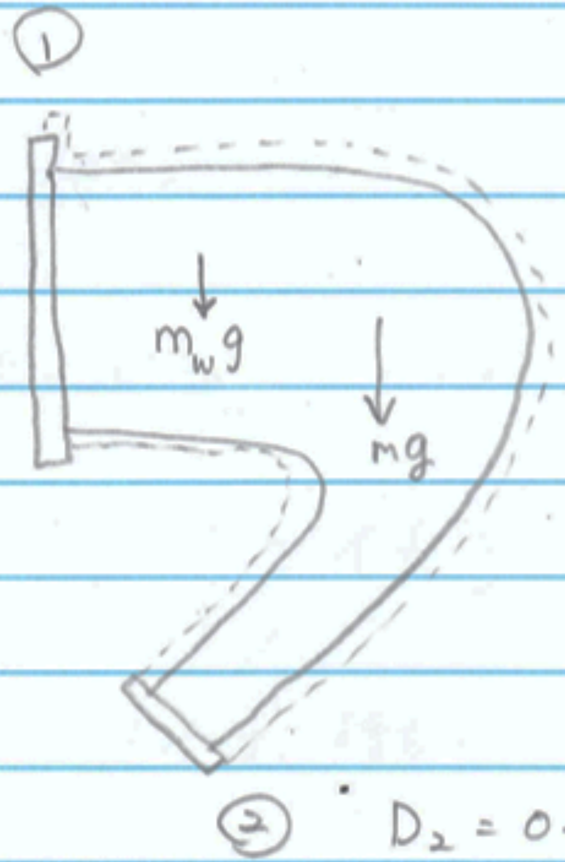


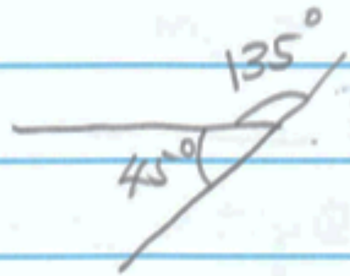
Section
(1)
D_1 = 400 mm
135°
z
x
D_2 =
200 mm
Section (2)

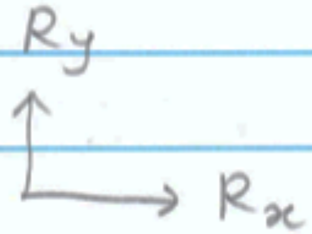
$D_1 = 0.4$ m

$Q = 0.4$ m^3/s

$P_1 = 150$ kPa







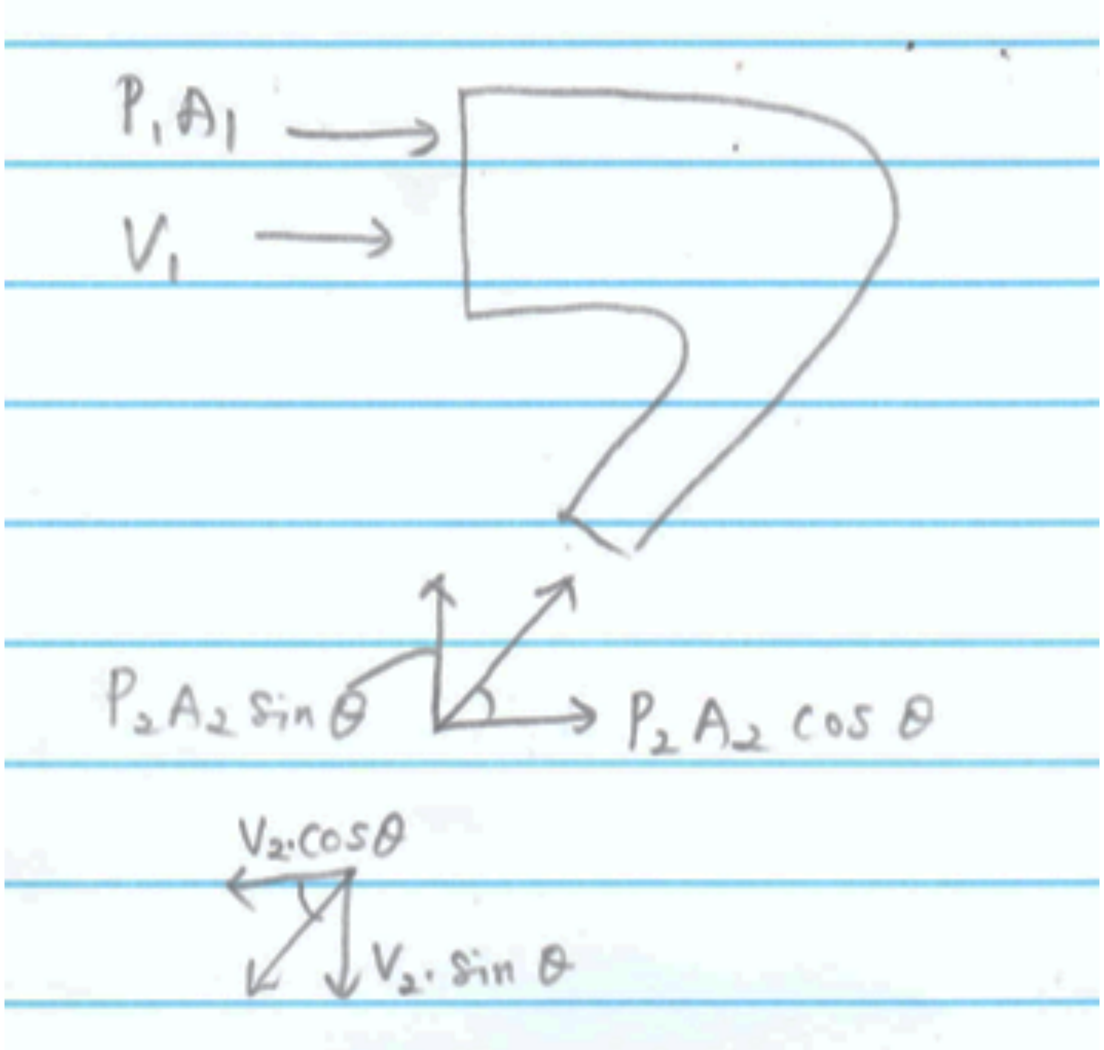
elbow mass = 12 kg.

air dalam elbow = 0.2 m^3

$D_2 = 0.2$ m

$P_2 = 90$ kPa

$Q = A_1 V_1$



P1 A1
V1
P2 A2 sin θ
P2 A2 cos θ
V2·cos θ
V2· sin θ

$$Q = A_1 V_1$$

$$\therefore V_1 = \frac{Q}{A_1} = \frac{0.4}{\frac{\pi}{4}(0.4)^2} = 3.18 \text{ (m/s)}$$

$$V_2 = \frac{Q}{A_2} = \frac{0.4}{\frac{\pi}{4}(0.2)^2} = 12.73 \text{ (m/s)}$$

$$R_x + P_1A_1 + P_2A_2\cos\theta = V_1\rho(-V_1A_1) + (-V_2\cos\theta)\rho(A_2V_2)$$

$$R_x + P_1A_1 + P_2A_2\cos\theta = -V_1\cdot\rho Q - V_2\cos\theta\cdot\rho Q$$

$$R_x = -V_1\rho Q - V_2\cos\theta\cdot\rho Q - P_1A_1 - P_2A_2\cos\theta$$

$$= -(3.18)(1000)(0.4) - (12.73)(0.707)(1000)(0.4) - (150\times10^3)\left(\frac{\pi}{4}(0.4)^2\right) - (90\times10^3)\left(\frac{\pi}{4}(0.2)^2\right)$$

$$= -1272 - 3600 - 18849.6 - 2827.4$$

$$= -26.55 \text{ kN}$$

negative menunjukkan arah berlawanan

$\leftarrow R_x$

$$R_y + P_2 A_2 \sin\theta - m_{pipe}\cdot g - m_w \cdot g = -V_2 \cdot \sin\theta \cdot \rho A_2 V_2$$

$$R_y = m_{pipe}\cdot g + m_{water}\cdot g - V_2 \sin\theta \rho Q - P_2 A_2 \sin\theta$$

$$= 12(9.81) + \rho \bar{V} \cdot g - (12.73)(0.707)(1000)(0.4) - (90\times10^3)\left(\frac{\pi}{4}(0.2)^2\right)(0.707)$$

$$= 117.72 + 1962 - 3600 - 1999$$

$$= -3519.28$$

$\downarrow R_y$