FLUID MECHANICS I

SEMM 2313

DEFLECTOR

The application of the momentum equation to deflectors forms an integral part of the analysis of many turbomachines, such as turbines, pumps, and compressors. In this section we illustrate the steps in such an analysis. It will be separated into two parts: fluid jets deflected by stationary deflectors and fluid jets deflected by moving deflectors. For both problems we will assume the following:

- The pressure external to the fluid jets is everywhere constant so that the pressure in the fluid as it moves over a deflector remains constant.

- The frictional resistance due to the fluid-deflector interaction is negligible so that the relative speed between the deflector surface and the jet stream remains unchanged, a result of Bernoulli's equation.

- Lateral spreading of a plane jet is neglected.

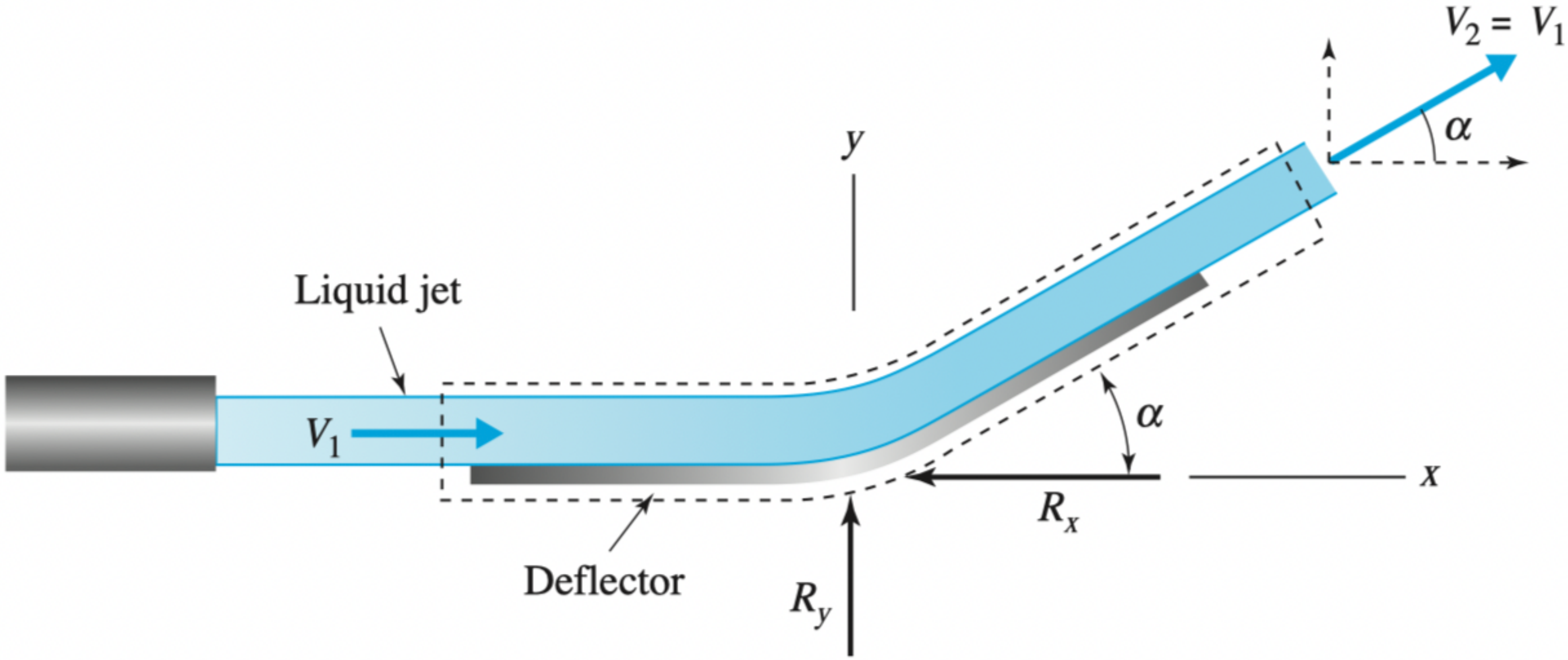
- The body force, the weight of the control volume, is small and will be neglected.

There are two types of deflector;

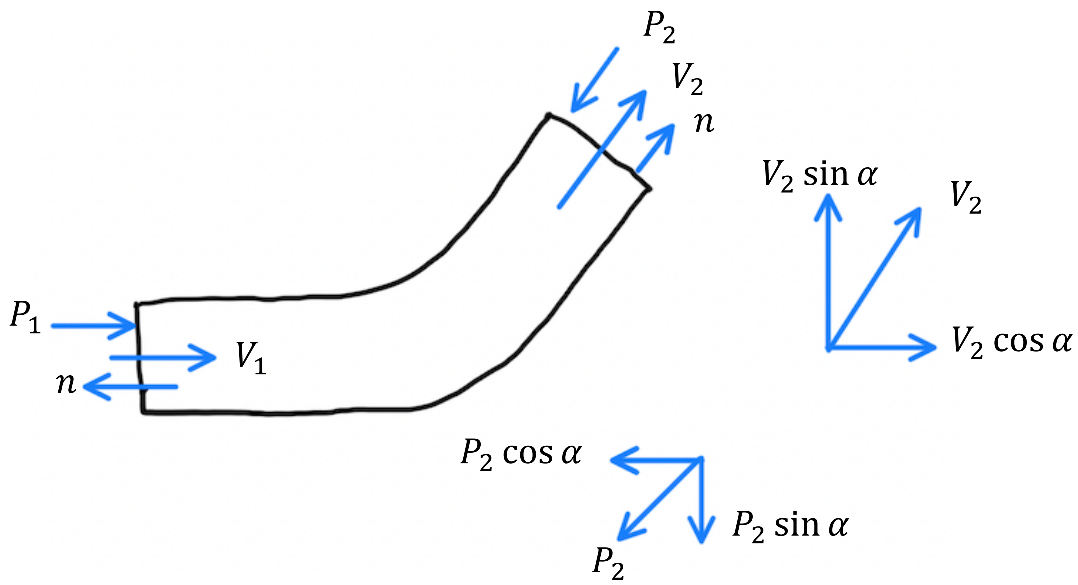
1. Stationary deflector: Deflector not moving (at any condition).

2. Moving deflector: Deflector moving at specific speed such as pump impeller.

STATIONARY DEFLECTOR



$$\sum F = \int_{cs} \rho(V) \cdot (V \cdot \hat{n}) \cdot dA = \sum \rho A (V \cdot \hat{n}) \cdot V$$

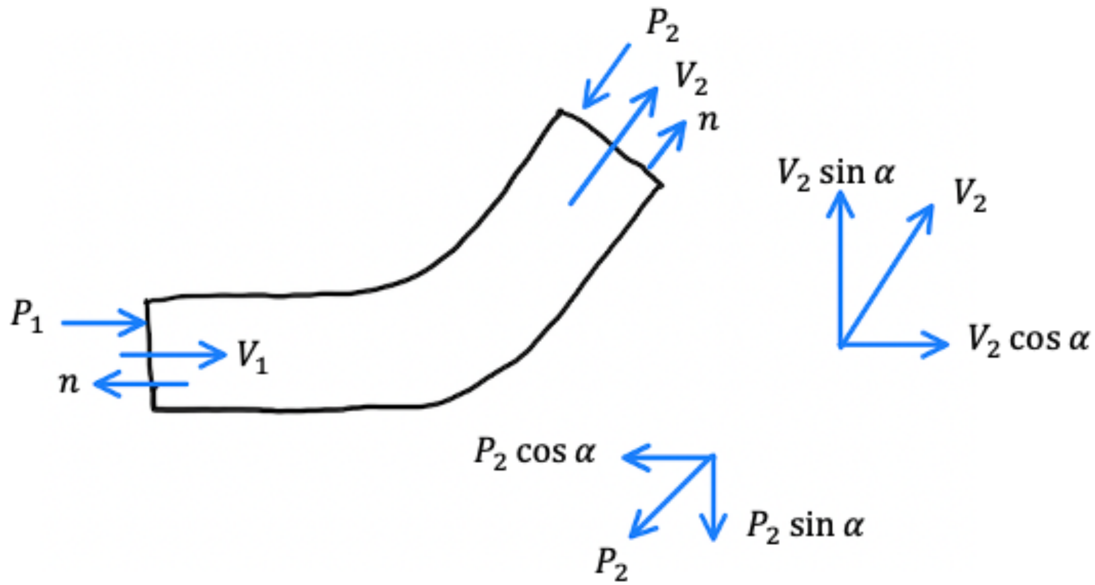


$$-R_x + P_1A_1 - P_2 \cos\alpha\, A_2 = \rho A_1(-V_1)V_1 + \rho A_2(V_2)V_2 \cos\alpha$$

$$P_1 = P_2 = P_{atm} = 0$$

In real situation, the velocity of V_2 is lower compare to the velocity V_1 due to the friction in fluid flow. It is good to understand that the $A(V \cdot \hat{n}) = Q$.

$$-R_x + P_1A_1 - P_2 \cos\alpha\, A_2 = \rho(-Q)V_1 + \rho(Q)V_2 \cos\alpha = (-\dot{m})V_1 + (\dot{m})V_2 \cos\alpha$$



$$-R_x = \rho A_1(-V_1)V_1 + \rho A_2(V_2)V_2 \cos\alpha$$

$$R_y = \rho A_2(V_2)V_2 \sin\alpha$$

$$R_y = \rho(Q)V_2 \sin\alpha = \dot{m}V_2 \sin\alpha$$

Example

A deflector turns a sheet of water through an angle of 30°as shown in Figure 1. What force components are necessary to hold the deflector in place if mass flow rate is 32 kg/s ?

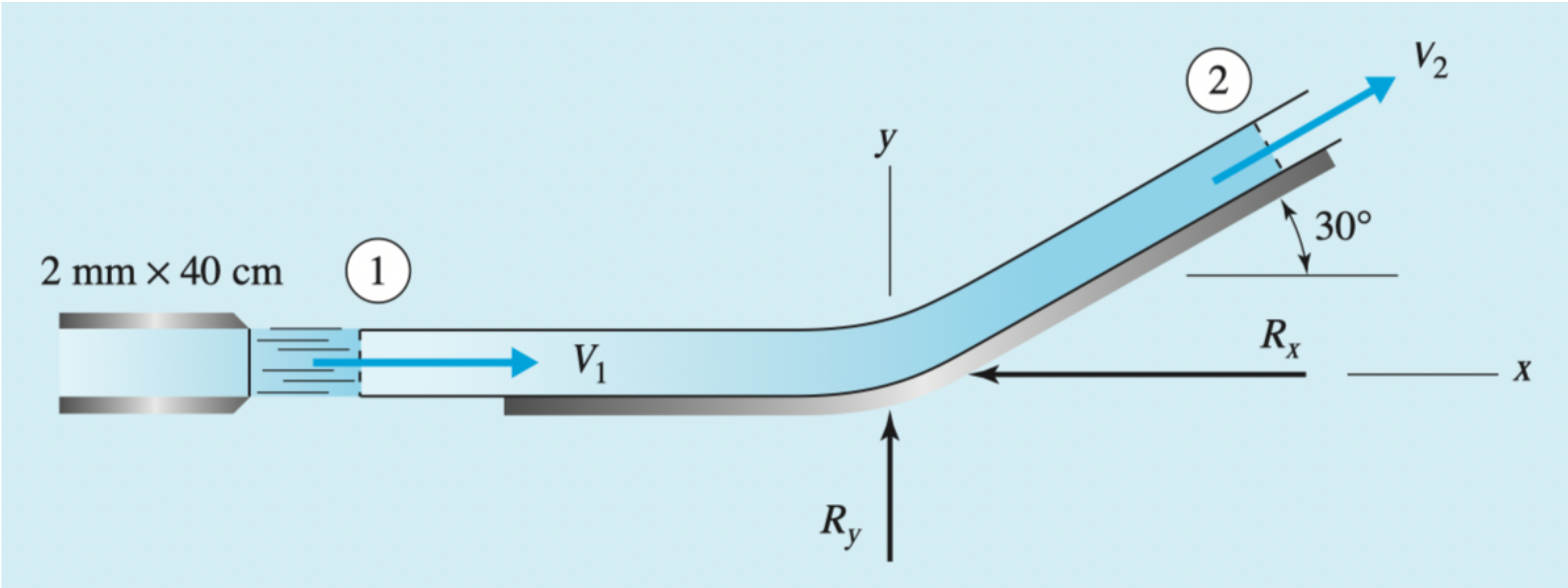
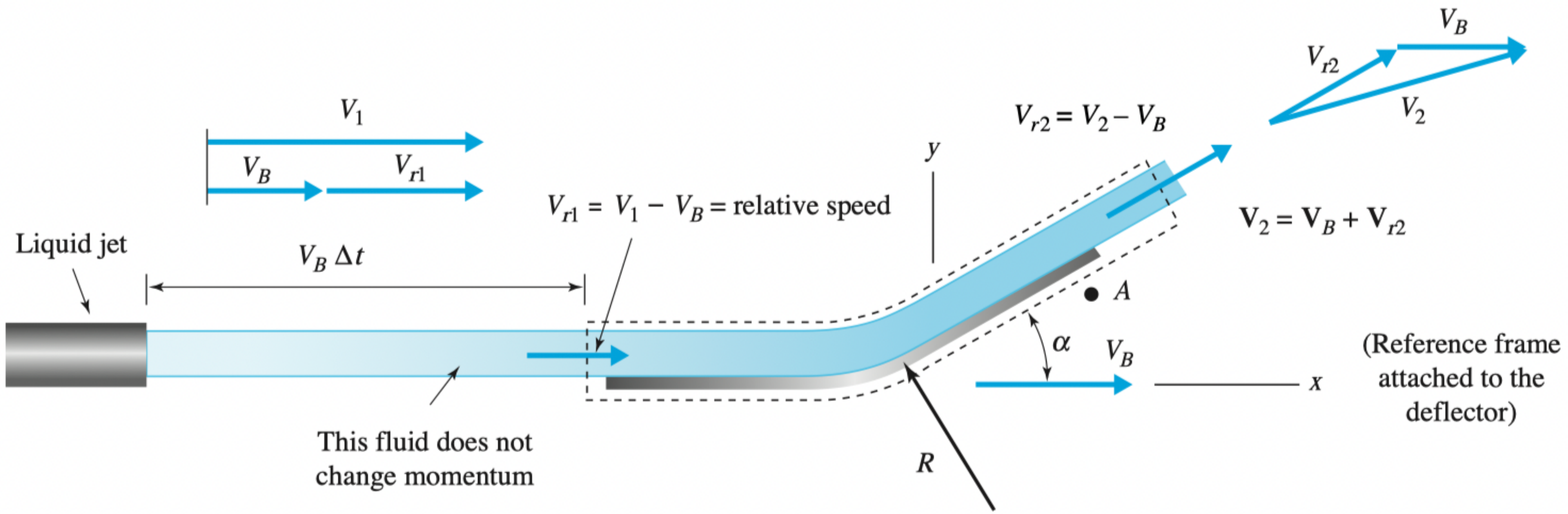


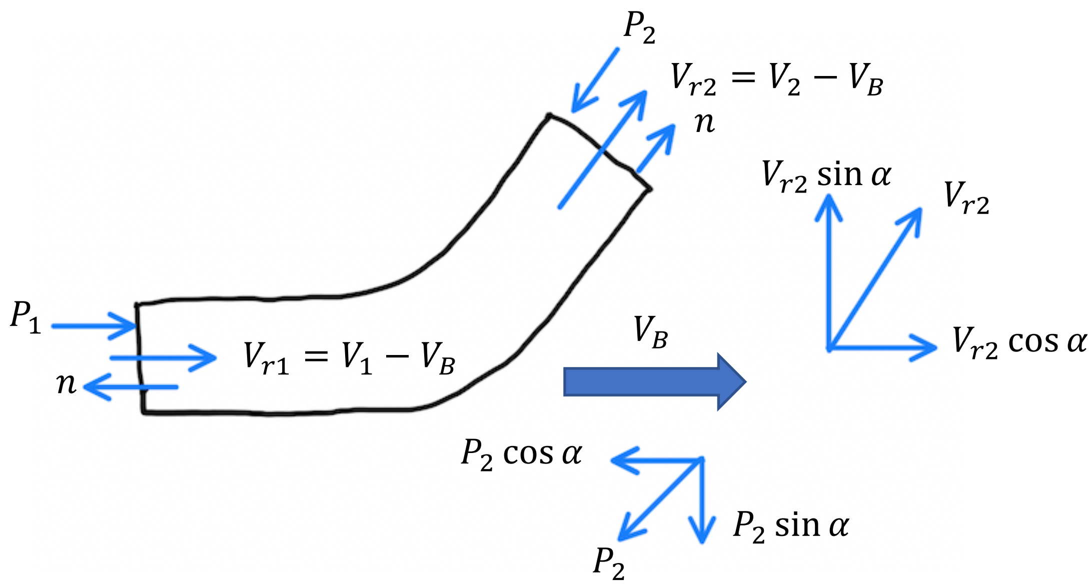
Figure 1

Answer

MOVING DEFLECTOR (FOR SINGLE DEFLECTOR)



$$P_1 = P_2 = P_{atm} = 0$$



$$-R_x + P_1 A_1 - P_2 \cos\alpha\, A_2 = \rho A_1(-V_{r1})V_{r1} + \rho A_2(V_{r2})V_{r2} \cos\alpha$$

$$-R_x = \rho(-Q)V_{r1} + \rho(Q)V_{r2} \cos\alpha$$

$$R_y - P_2 \sin\alpha\, A_2 = \rho A_2(V_{r2})V_{r2} \sin\alpha = \rho(Q)V_{r2} \sin\alpha$$

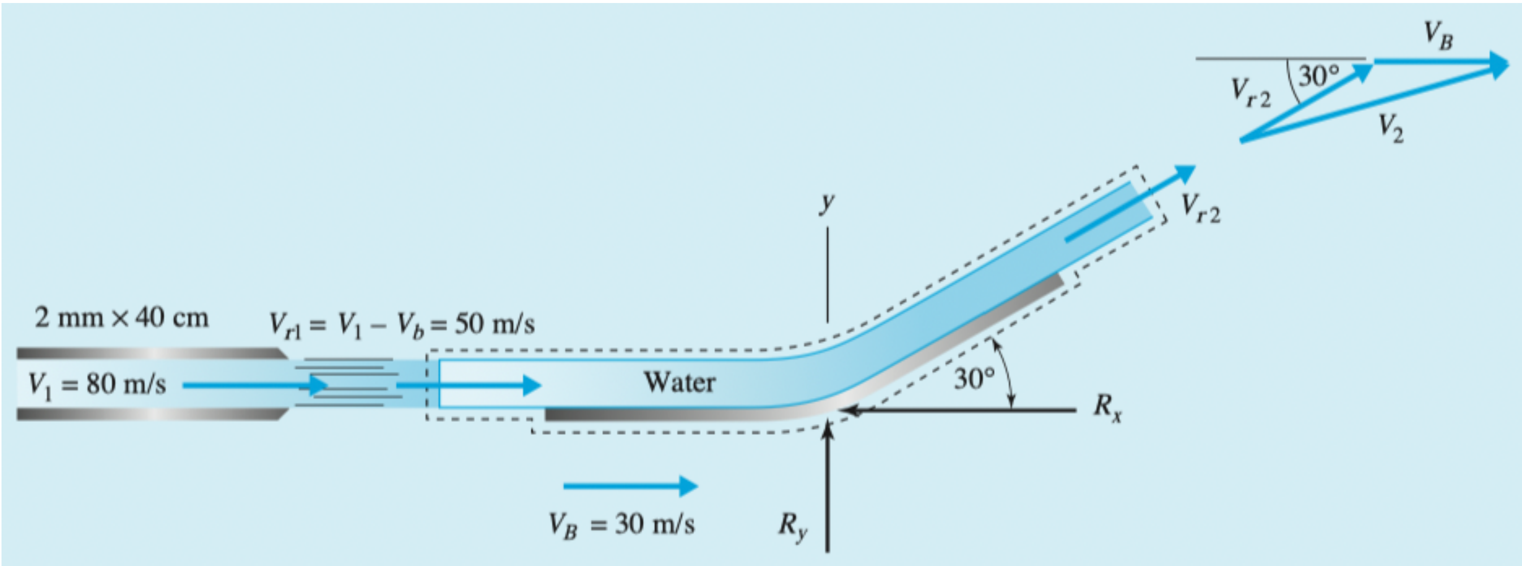
$$P_1 = P_2 = P_{atm} = 0$$
$$V_B = \text{Deflector velocity}$$

DISCUSSION

Here, the flowrate need to be calculated using the relative velocity because this only this relative velocity experiences a momentum change.

EXAMPLE

The deflector shown below moves to the right at 30 m/s while the nozzle remains stationary. Determine the force components needed to support the deflector and the power generated by the vane. The jet velocity is 80 m/s.

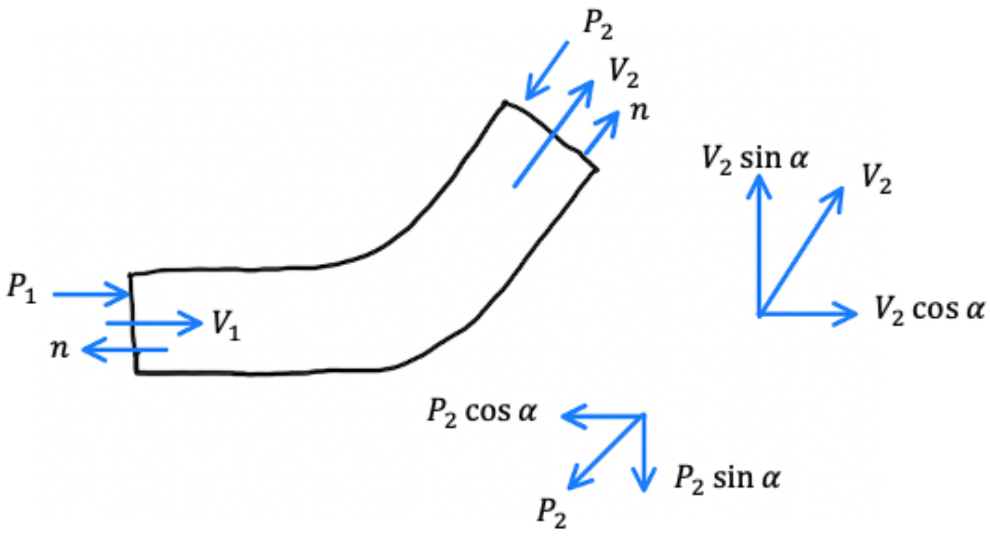


$$\text{relative velocity} = V_{r1} = V_1 - V_B = 80 - 30 = 50 \quad m/s$$

Assuming no friction on the reflector, the velocity of the water flowing at the outlet remains the same as the velocity of the water at the inlet. Therefore:

$$V_{r2} = V_2 - V_B = 80 - 30 = 50 \quad m/s$$

Then, we can make a free body diagram as usual, assuming the reflector does not move and the inlet velocity is only 50 m/s.



$$-R_x + P_1 A_1 - P_2 \cos\alpha\, A_2 = \rho A_1(-V_{r1})V_{r1} + \rho A_2(V_{r2})V_{r2}\cos\alpha$$

$$-R_x = (1000)(0.002 \times 0.4)(-50)50 + (1000)(0.002 \times 0.4)(50)50\cos 30$$

$$R_x = 268 \quad N$$

$$R_y - P_2 \sin\alpha\, A_2 = \rho A_2(V_{r2})V_{r2}\sin\alpha$$

$$R_y = (1000)(0.002 \times 0.4)(50)50\sin 30$$

$$R_y = 1000 \quad N$$

The power would be found by multiplying the x-component force by the blade speed for each jet.

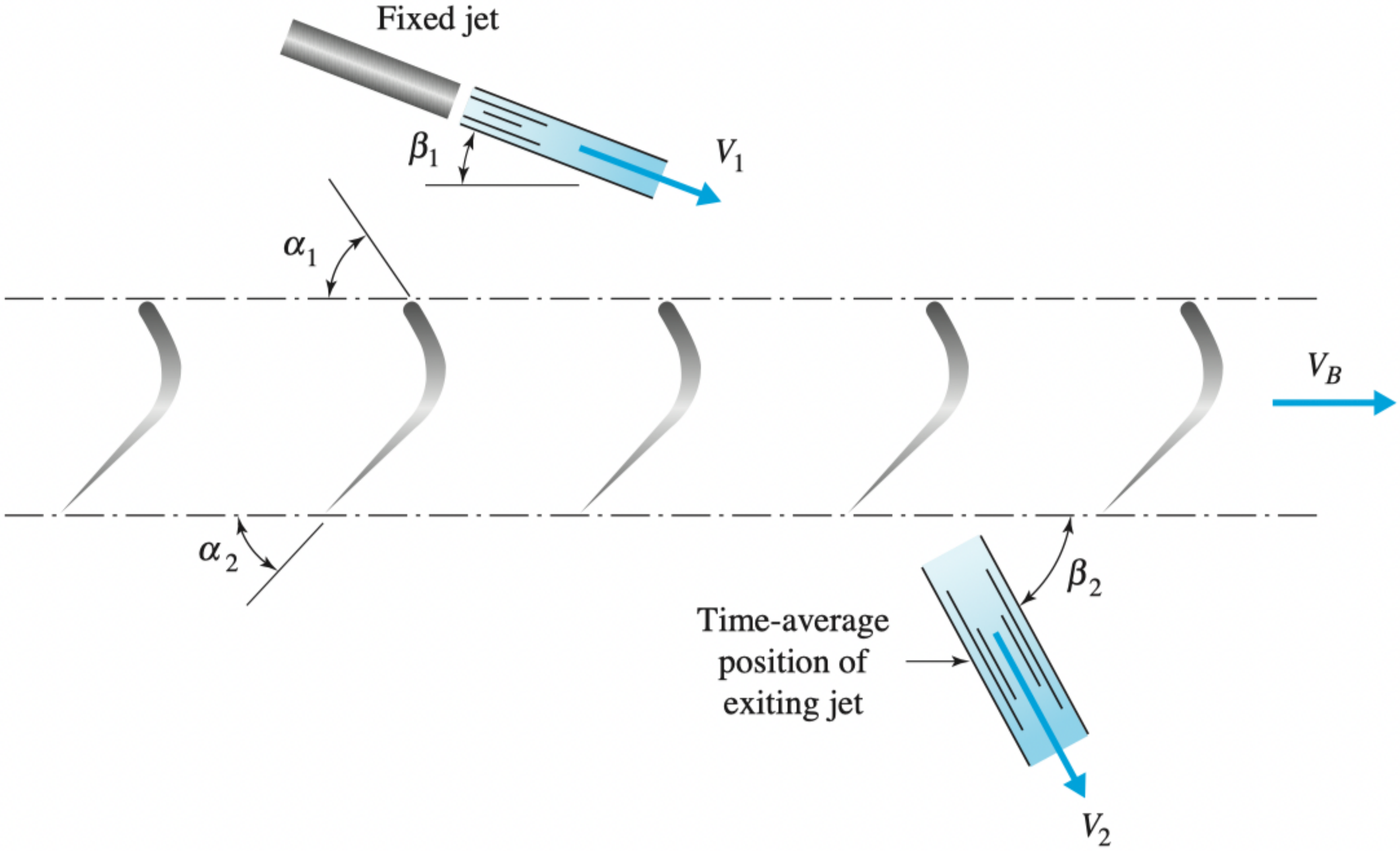
$$Power = N \cdot R_x \cdot V_B$$

N = Number of jets
R_x = Reaction force in x-direction
V_B = Deflector speed in x-direction

Here, number of jet is one, because there is only one deflector.

$$Power = (1)(268)(30) = 8040 \quad W$$

MOVING DEFLECTOR
(FOR MULTIPLE DEFLECTOR IN TURBINE BLADE APPLICATION)



As we know, a turbine is a device that absorbs energy from fluid flow. Since energy has been absorbed as the fluid flows over the reflector (turbine blade), the velocity V_2 will be slower than the velocity V_1. However, the flow rate at the inlet (point 1) and outlet (point 2) is still the same. This is because fluid flow must fulfill the law of conservation of mass (continuity equation). It means that the cross sectional area at the outlet (point 2) will increase because the velocity at the outlet (point 2) was decreased.

In blade application, the movement of turbine blade is not at the same direction with the water jet. It is difficult to suit with the idea of single moving deflector. In single moving deflector, usually, the deflector moves in the same direction of the water jet.

To solve this problem (movement of turbine blade), we need to understand the velocity triangle.

Velocity triangle in inlet and outlet is show below.

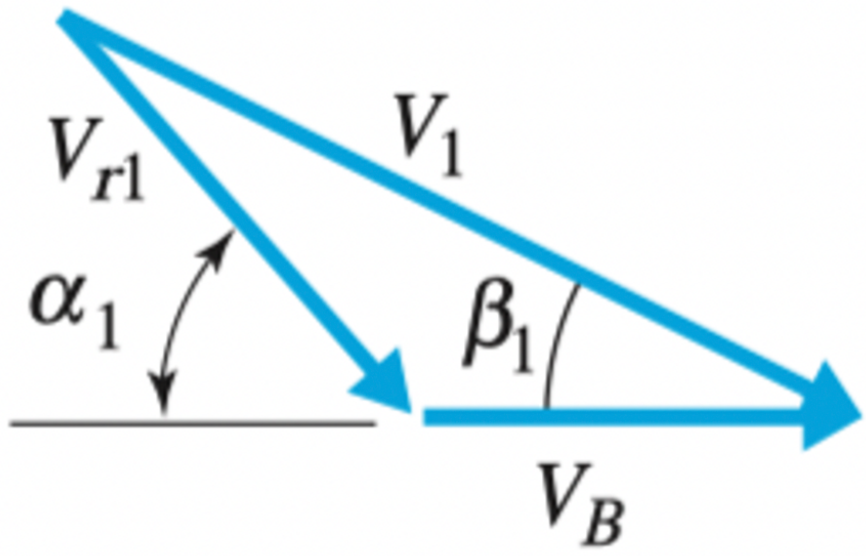
At the inlet:

Water jet enter as V_1 at the angle of β_1.
Point 1 means inlet and point 2 means outlet.

When it hit the blade, water jet will flows in the shape of blade. It means that, now water jet has new direction of flow (angle α_1). We could write the velocity relative to the blade angle as V_{r1} .

At the same time, blade moves (rotates) at certain velocity. We could write the velocity of blade as V_B .

Velocity is a vector. In this situation, V_1 is the resultant velocity. It is consist of V_{r1} and V_B .



$$\mathbf{V}_1 = \mathbf{V}_B + \mathbf{V}_{r1}$$

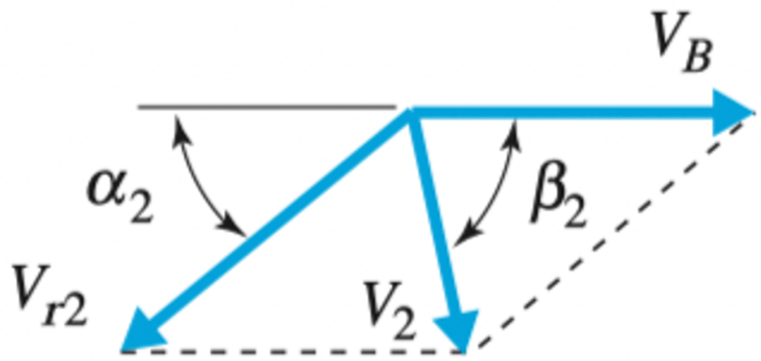
At the outlet:

Water jet flows out as V_2 at the angle of β_2.
Point 1 means inlet and point 2 means outlet.

When water leave the blade, it still flows on the shape of blade. It will flows as V_{r2} with angle of α_2 .

At the same time, blade moves (rotates) at certain velocity. We could write the velocity of blade as V_B .

At outlet also, outlet velocity V_2 is the resultant velocity.

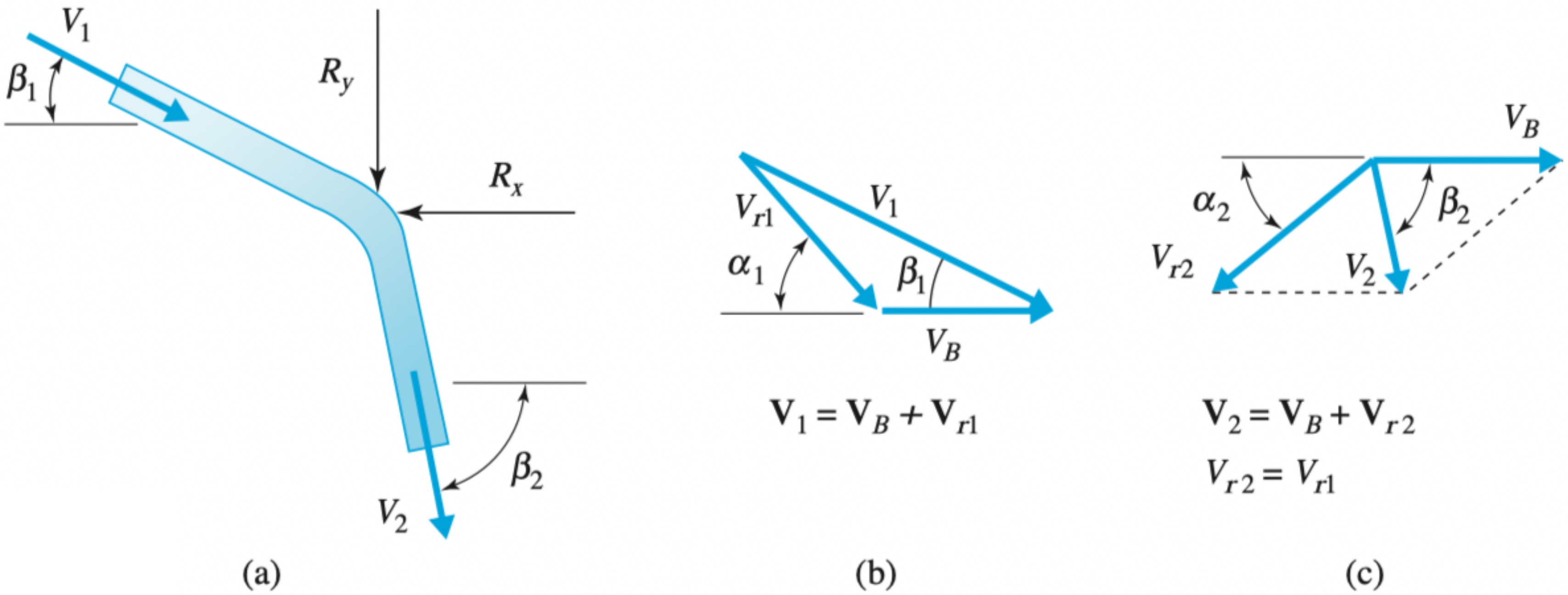


$$\mathbf{V}_2 = \mathbf{V}_B + \mathbf{V}_{r2}$$

$$V_{r2} = V_{r1}$$

In this situation V_2 and V_2 are the actual velocity. It can be measured. The V_r and the V_B are the component velocity only. They are just imaginary velocity to construct the velocity triangle.

VELOCITY COMPONENT FOR MULTIPLE MOVING DEFLECTOR



Detail of the flow situation involving a series of vanes: (a) average position of jet; (b) entrance velocity polygon; (c) exit velocity polygon.

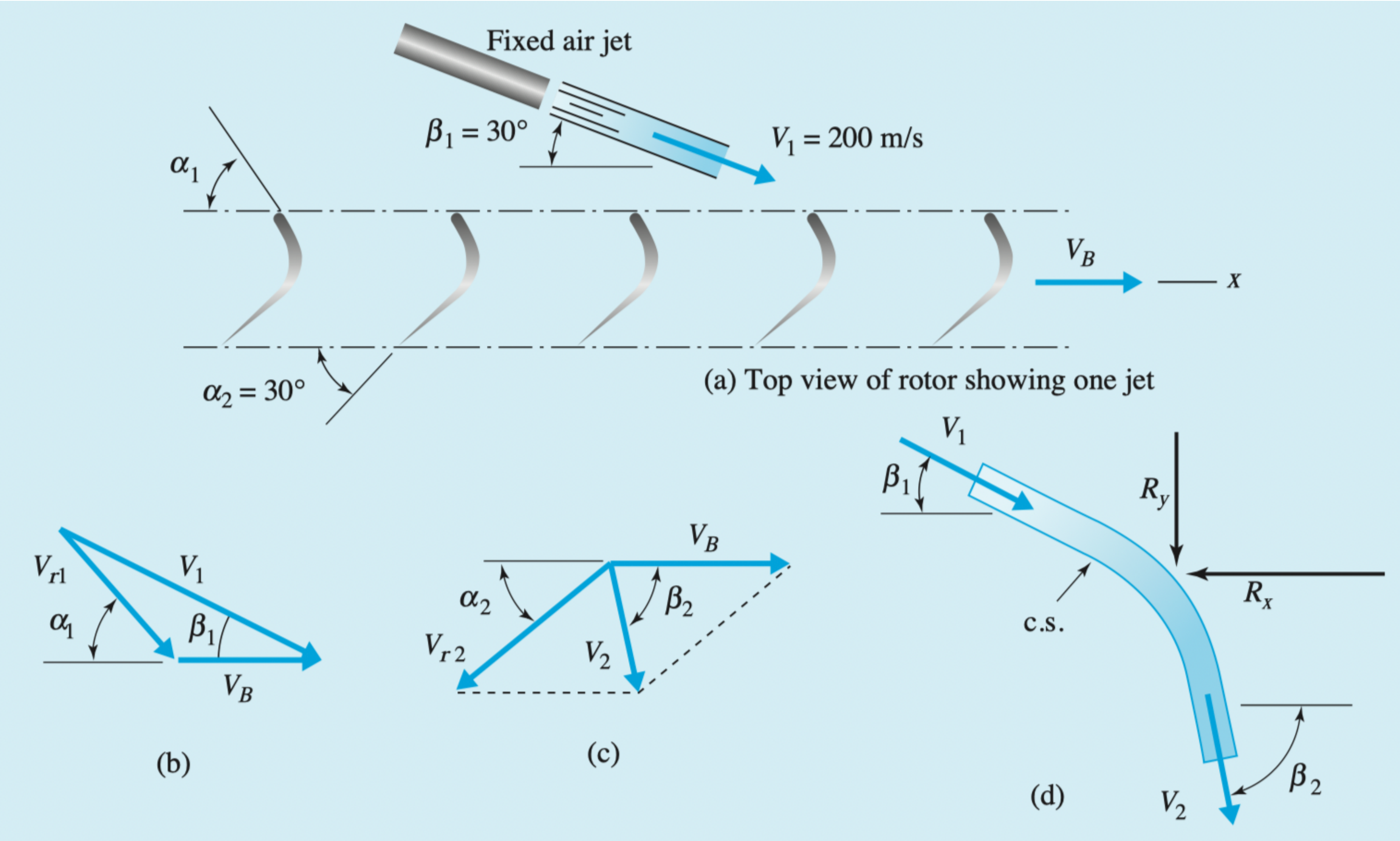
The power would be found by multiplying the x-component force by the blade speed for each jet.

$$Power = N \cdot R_x \cdot V_B$$

$$N = \text{Number of jets}$$
$$R_x = \text{Reaction force in x-direction}$$
$$V_B = \text{Deflector speed in x-direction}$$

Example 3

High-speed air jets strike the blades of a turbine rotor tangentially while the 1.5-mdiameter rotor rotates at 140 rad/s. There are 10 such 4-cm-diameter jets. Calculate the maximum power output. The air density is 2.4 kg/m³ .



Answer 3

From the polygon at the entrance we have

$$V_1 \sin \beta_1 = V_{r1} \sin \alpha_1$$

$$V_1 \cos \beta_1 = V_{r1} \cos \alpha_1 + V_B$$

$$\therefore 200 \sin 30^\circ = V_{r1} \sin \alpha_1$$

$$200 \cos 30^\circ = V_{r1} \cos \alpha_1 + 0.75 \times 140$$

where V_B is the radius multiplied by the angular velocity. A simultaneous solution yields

$$V_{r1} = 121 \text{ m/s} \qquad \alpha_1 = 55.7^\circ$$

The friction between the air and the blade is quite small and can be neglected when calculating the maximum output. This allows us to assume $V_{r2} = V_{r1}$. From the exiting velocity polygon we can write

$$V_B - V_{r2} \cos \alpha_2 = V_2 \cos \beta_2$$

$$V_{r2} \sin \alpha_2 = V_2 \sin \beta_2$$

$$\therefore 0.75 \times 140 - 121 \cos 30° = V_2 \cos \beta_2$$

$$121 \sin 30° = V_2 \sin \beta_2$$

A simultaneous solution results in

$$V_2 = 60.5 \text{ m/s} \qquad \beta_2 = 89.8°$$

The momentum equation applied to the control volume, shown in Fig. E4.17d, gives

$$-R_x = \dot{m}(V_{2x} - V_{1x})$$
$$= 2.4\ \text{kg/m}^3 \times \pi \times 0.02^2\ \text{m}^2 \times 200\ \text{m/s}(60.5 \cos 89.8° - 200 \cos 30°)\ \text{m/s}$$
$$\therefore R_x = 104.3\ \text{N}$$

There are 10 jets, each producing the force above. The maximum power output is then

$$\text{power} = 10 \times R_x \times V_B$$
$$= 10 \times 104.3\ \text{N} \times (0.75 \times 140)\ \text{m/s} = 109\,600\ \text{W} \quad \text{or} \quad 109.6\ \text{kW}$$