FLUID MECHANICS I

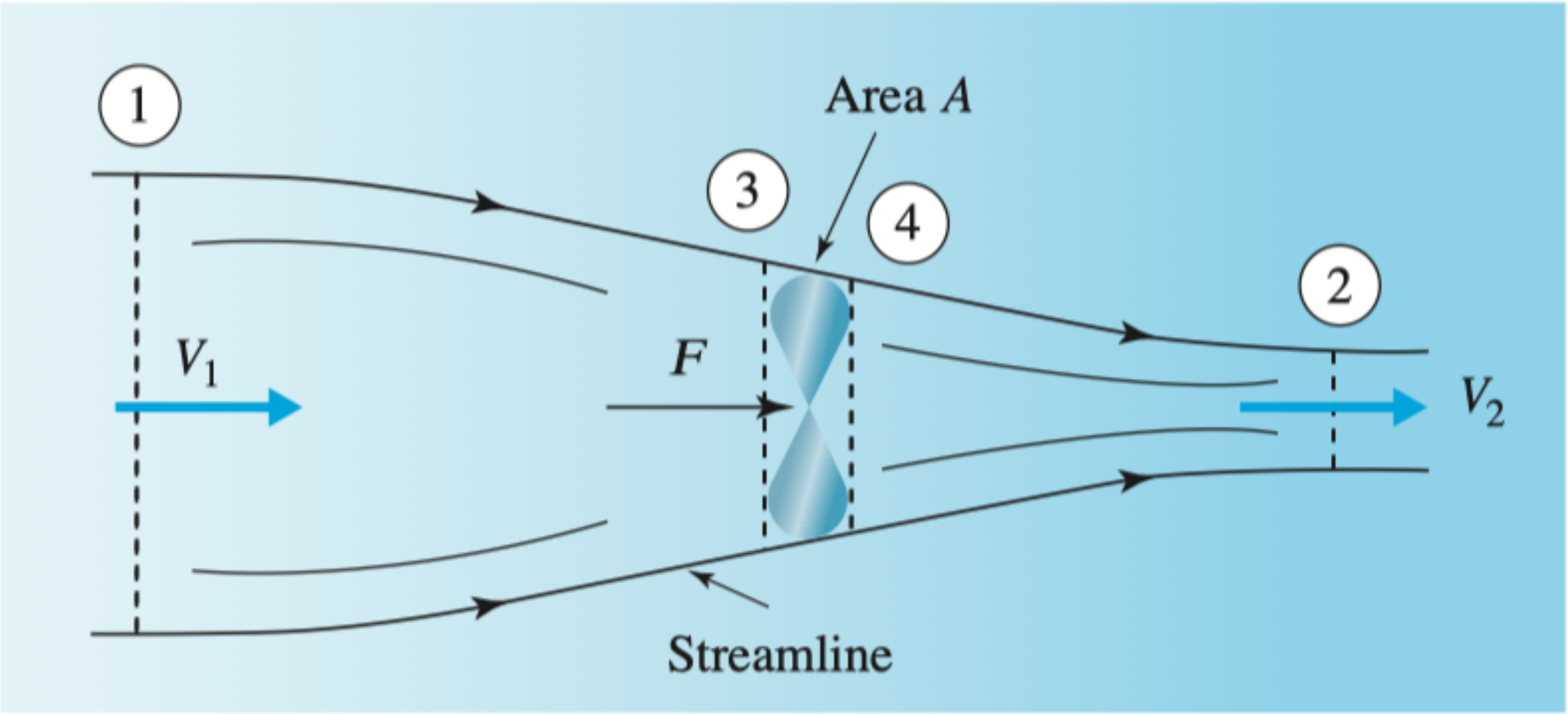
SEMM 2313

MOMENTUM EQUATION APPLIED TO PROPELLERS

Consider the propeller with the streamlines shown forming the surface of a control volume in which the fluid enters with a uniform velocity V_1 and exits with a uniform velocity V_2 .

The outer streamlines just touch the tips of the propeller. This flow situation can be seen to be identical to that of a propeller moving with velocity V_1 in a stagnant fluid by adding V_1 to the left (as shown in the figure). The momentum equation, applied to the large control volume shown, gives:

$$F = \dot{m}(V_2 - V_1)$$


1
Area A
3
4
2
V_1
F
V_2
Streamline

This control volume is not sufficient, however, since the areas A_1 and A_2 are unknown. We know the flow area A of the propeller.

So a control volume is drawn close to the propeller such that $V_3 \cong V_4$ and
$V_3 \cong V_4 = A$.

The momentum equation in the x-direction gives:

$$F + P_3 A - P_4 A = 0$$

$$F = (P_4 - P_3) A$$

Now, since viscous effects would be quite small in this flow situation the energy equation up to the propeller and then downstream from the propeller is used to obtain

$$\frac{V_1^2 - V_3^2}{2} + \frac{P_1 - P_3}{\rho} = 0$$

$$\frac{V_4^2 - V_2^2}{2} + \frac{P_4 - P_2}{\rho} = 0$$

$$P_1 = P_2 = P_{atm}$$

So, we have,

$$(V_2^2 - V_1^2)\frac{\rho}{2} = P_4 - P_3$$

From all the equations, we could determine:

$$V_3 = \frac{1}{2}(V_2 + V_1)$$

where we have used $\dot{m} = \rho A V_3$ since the propeller area is the only area known.

This result shows that the velocity of the fluid moving through the propeller is the average of the upstream and downstream velocities.

The input power needed to produce this effect is found by applying the energy equation between sections 1 and 2, where the pressures are atmospheric pressure and neglecting losses.

$$\text{Power input} = \frac{V_2^2 - V_1^2}{2}(\dot{m})$$

The moving propeller requires power given by:

$$\text{Power propeller} = F \times V_1 = \dot{m}V_1(V_2 - V_1)$$

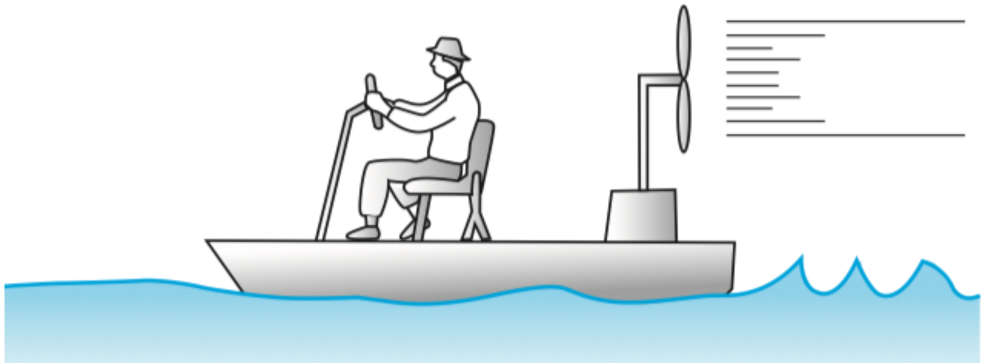
The theoretical propeller efficiency is then:

$$\text{Efficiency}, \eta_p = \frac{\text{power propeller}}{\text{power input}} = \frac{V_1}{V_3}$$

In contrast to the propeller, a wind machine extracts energy from the airflow; the downstream velocity is reduced and the diameter is increased.

Question 1

The swamp boat shown in Fig. P4.150 is propelled at 50 km/h by a 2-m-diameter propeller that requires a 20-kW engine. Calculate the thrust on the boat, the flow rate of air through the propeller, and the propeller efficiency.



Answer 1

For this steady-state flow, we fix the boat and move the upstream air. This provides us with the steady-state flow of Fig. 4.17. This is the same as observing the flow while standing on the boat.

$$\dot{W} = FV_1. \quad 20\,000 = F \times \frac{50 \times 1000}{3600}. \quad \therefore F = \underline{1440\text{ N}}. \quad (V_1 = 13.89\text{ m/s})$$

$$F = \dot{m}(V_2 - V_1). \quad 1440 = 1.23\pi \times 1^2 \times \frac{V_2 + 13.89}{2}(V_2 - 13.89). \quad \therefore V_2 = 30.6\text{ m/s}.$$

$$\therefore Q = A_3 V_3 = \pi \times 1^2 \times \frac{30.6 + 13.89}{2} = \underline{69.9\text{ m}^3/\text{s}}$$

$$\eta_p = \frac{V_1}{V_3} = \frac{13.89}{22.24} = 0.625 \quad \text{or} \quad \underline{62.5\%}$$

Question 2

An aircraft is propelled by a 2.2-m-diameter propeller at a speed of 200 km/h. The air velocity downstream of the propeller is 320 km/h relative to the aircraft. Determine the pressure difference across the propeller blades and the required power. Use $\rho = 1.2\ \text{kg/m}^3$.

Answer 2

Fix the reference frame to the aircraft so that $V_1 = \dfrac{200 \times 1000}{3600} = 55.56$ m/s.

$$V_2 = \frac{320 \times 1000}{3600} = 88.89 \text{ m/s.} \qquad \therefore \dot{m} = 1.2 \times \pi \times 1.1^2 \times \frac{55.56 + 88.89}{2} = 329.5 \text{ kg/s.}$$

$$F = 329.5(88.89 - 55.56) = 10\ 980 \text{ N} = \Delta p \pi \times 1.1^2. \qquad \therefore \Delta p = \underline{2890 \text{ Pa}}.$$

$$\dot{W} = F \times V_1 = 10\ 980 \times 55.56 = 610\ 000 \text{ W} \quad \text{or} \quad \underline{818 \text{ hp}}.$$