Chapter 4

Compressible Flow

9.1 Introduction

- Not all gas flows are compressible flows, neither are all compressible flows gas flows.

- Examples: airflows around commercial and military aircraft, airflow through jet engines, and the flow of a gas in compressors and turbines.

9.1 Introduction

- Reminders from previous chapters:
- Continuity equation

$$\dot{m} = \rho_1 A_1 V_1 = \rho_2 A_2 V_2$$

- Momentum equation

$$\Sigma \mathbf{F} = \dot{m}(\mathbf{V}_2 - \mathbf{V}_1)$$

- Energy equation

$$\frac{\dot{Q} - \dot{W}_S}{\dot{m}} = \frac{V_2^2 - V_1^2}{2} + h_2 - h_1$$

9.1 Introduction

- Reminders from previous chapters:
- Thermodynamic relations

$$h_2 - h_1 = c_p(T_2 - T_1)$$

$$c_p = R + c_v$$

$$k = \frac{c_p}{c_v}$$

- Ideal gas law

$$p = \rho R T$$

- Isentropic flow

$$\frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{(k-1)/k} \qquad \frac{p_2}{p_1} = \left(\frac{\rho_2}{\rho_1}\right)^{k}$$

9.2 Speed of Sound and the Mach Number

- Sound wave:
  - Travels with a velocity *c* relative to a stationary observer.

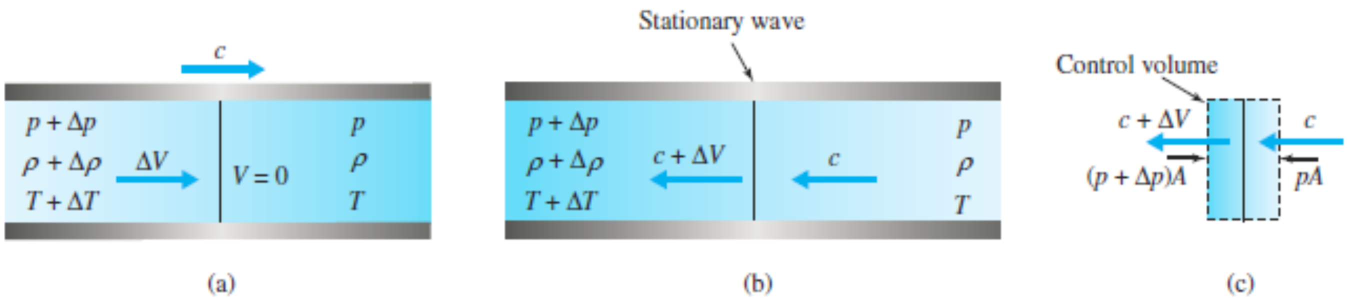


Figure 9.1 Sound wave: (a) stationary observer; (b) observer moving with the wave; (c) control volume enclosing the wave.

9.2 Speed of Sound and the Mach Number

- Speed of Sound

$$c = \sqrt{\frac{kp}{\rho}}$$

- Using the ideal gas law

$$c = \sqrt{kRT}$$

- Mach number (dimensionless velocity)

$$\mathrm{M} = \frac{V}{c}$$

9.2 Speed of Sound and the Mach Number

If M < 1, the flow is a *subsonic* flow, and if M > 1, it is a *supersonic* flow.

Angle α of the Mach cone is given by:

$$\alpha = \sin^{-1}\frac{c}{V} = \sin^{-1}\frac{1}{\mathrm{M}}$$

9.2 Speed of Sound and the Mach Number

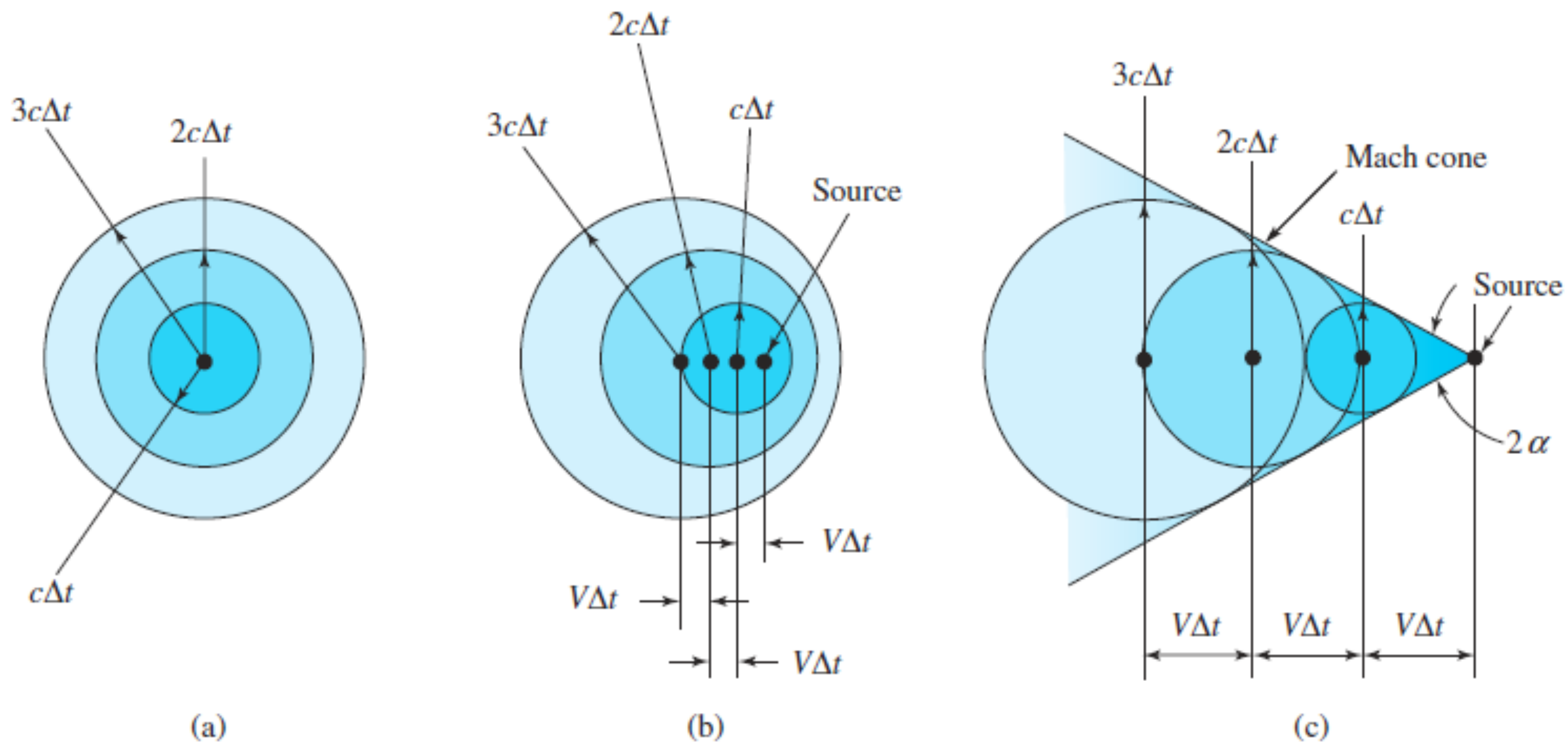


Figure 9.2 Sound waves propagating from a noise source: (a) stationary source; (b) moving source, $V < c$; (c) moving source, $V > c$.

9.2 Speed of Sound and the Mach Number

Example 9.1

A needle-nose projectile traveling at a speed with M = 3 passes 200 m above the observer of Figure E9.1. Calculate the projectile's velocity and determine how far beyond the observer the projectile will first be heard.

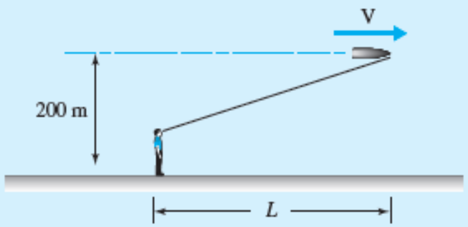


Figure E9.1

Solution

At a Mach number of 3 the velocity is

$$V = \mathrm{M}c = \mathrm{M}\sqrt{kRT}$$
$$= 3\sqrt{1.4 \times 287\ \mathrm{J/kg \cdot K} \times 288\ \mathrm{K}} = 1021\ \mathrm{m/s}$$

where a standard temperature of 15°C has been assumed since no temperature is given. (Remember, kg = N · s²/m and J = N · m.) Using h as the height and L as the distance beyond the observer (refer to Figure 9.2c), we have

$$\sin\alpha = \frac{h}{\sqrt{L^2 + h^2}} = \frac{1}{\mathrm{M}}$$

With the information given,

$$\frac{200}{\sqrt{L^2 + 200^2}} = \frac{1}{3}$$

giving

$$L = \underline{566\ \mathrm{m}}$$

Note: The units on kRT are $\frac{\mathrm{N \cdot m}}{\mathrm{kg \cdot K}} \times \mathrm{K} = \frac{\mathrm{N \cdot m}}{\mathrm{N \cdot s^2/m}} = \frac{\mathrm{m^2}}{\mathrm{s^2}}$. The quantity k is dimensionless.

9.3 Isentropic Nozzle Flow

Examples: diffuser near the front of a jet engine, exhaust gases passing through the blades of a turbine, the nozzles on a rocket engine, a broken natural gas line.

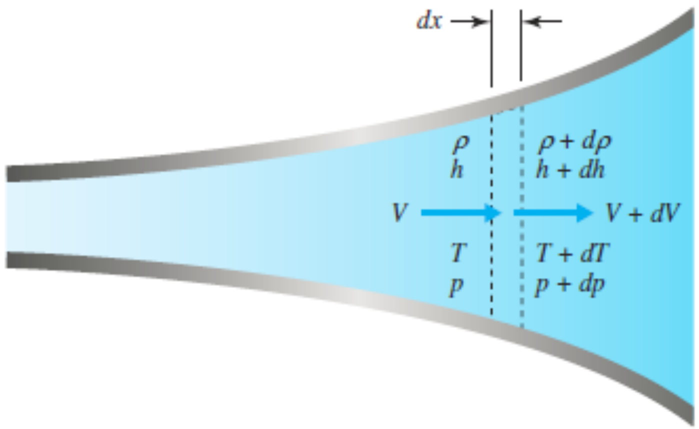


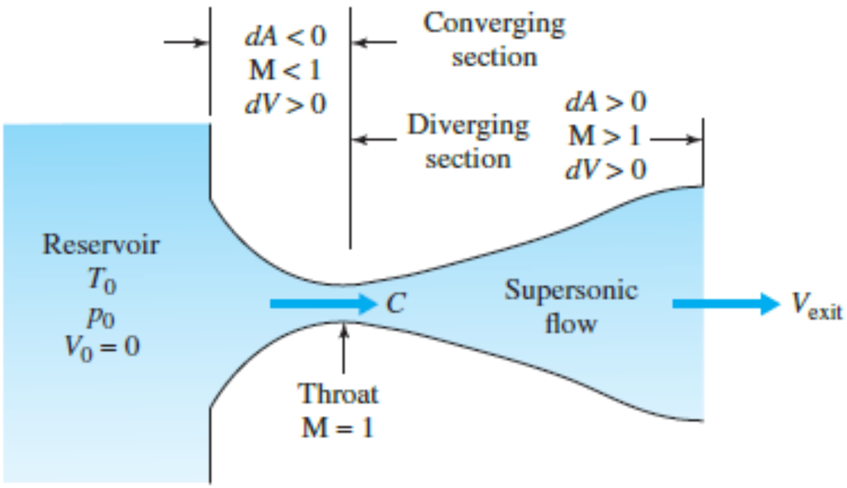
Figure 9.3 Uniform, isentropic flow.

9.3 Isentropic Nozzle Flow

$$\frac{dV}{V}\left(\mathrm{M}^2 - 1\right) = \frac{dA}{A}$$

1. If the area is increasing, $dA > 0$, and $\mathrm{M} < 1$, we see that dV must be negative, that is, $dV < 0$. The flow is decelerating for this subsonic flow.
2. If the area is increasing and $\mathrm{M} > 1$, we see that $dV > 0$; hence the flow is accelerating in the diverging section for this supersonic flow.
3. If the area is decreasing and $\mathrm{M} < 1$, then $dV > 0$, resulting in an accelerating flow.
4. If the area is decreasing and $\mathrm{M} > 1$, then $dV < 0$, indicating a decelerating flow.
5. At a throat where $dA = 0$, either $dV = 0$ or $\mathrm{M} = 1$, or possibly both.

9.3 Isentropic Nozzle Flow



Critical ratios:

$$\frac{T^*}{T_0} = \frac{2}{k+1}$$

$$\frac{p^*}{p_0} = \left(\frac{2}{k+1}\right)^{k/(k-1)}$$

$$\frac{\rho^*}{\rho_0} = \left(\frac{2}{k+1}\right)^{1/(k-1)}$$

9.3 Isentropic Nozzle Flow

For air with k = 1.4, the critical values are:

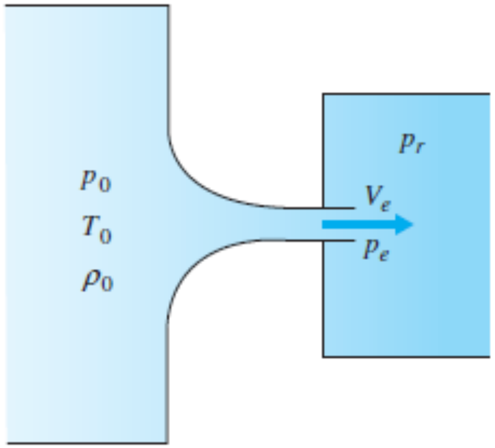
$$p^* = 0.5283\, p_0 \qquad T^* = 0.8333\, T_0 \qquad \rho^* = 0.6340\, \rho_0$$

$$\dot{m} = p_0 A^* \sqrt{\frac{k}{RT_0}} \left(\frac{k+1}{2}\right)^{(k+1)/2(1-k)}$$

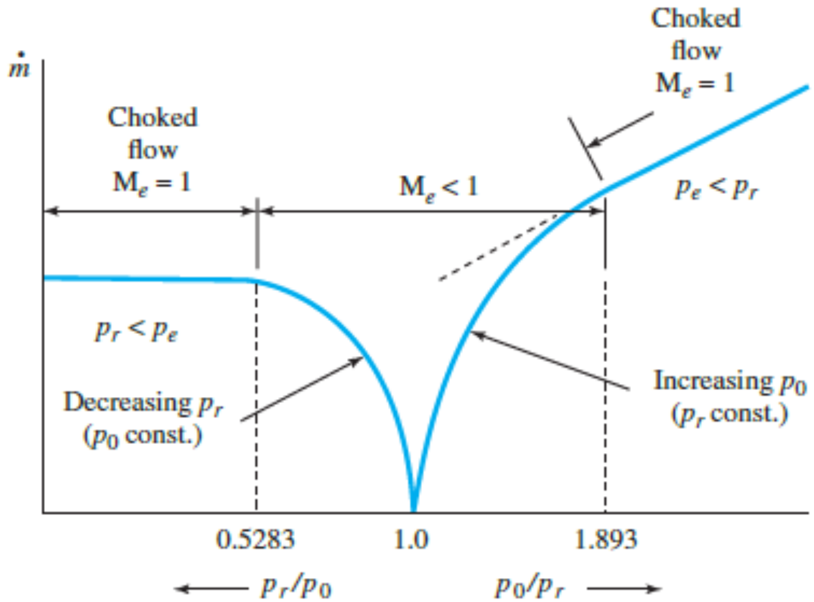
$$\frac{A}{A^*} = \frac{1}{\mathrm{M}} \left[\frac{2 + (k-1)\mathrm{M}^2}{k+1}\right]^{(k+1)/2(k-1)}$$

9.3 Isentropic Nozzle Flow

If $p_e > p_r$, the flow exiting the nozzle is able to turn sharply.

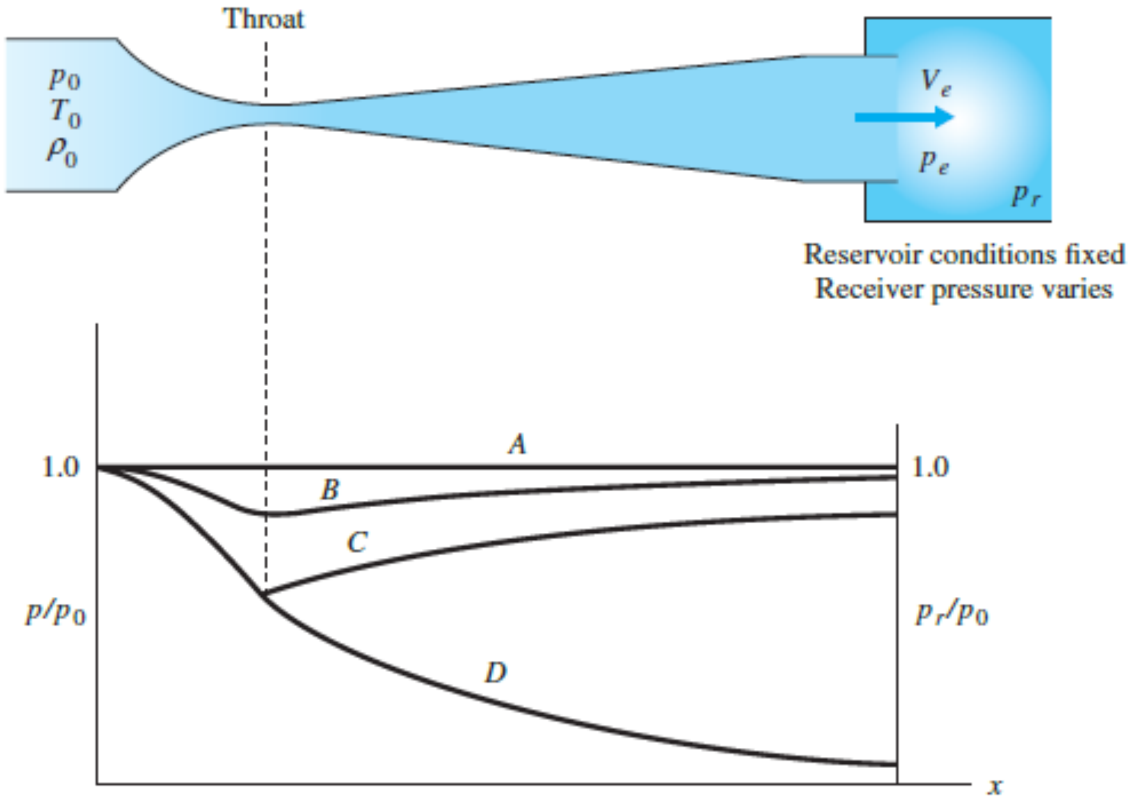


(a)



9.3 Isentropic Nozzle Flow

Converging–diverging nozzle:



9.3 Isentropic Nozzle Flow

The purpose of a nozzle is to convert stored energy into kinetic energy, while the purpose of a diffuser is to recover the pressure.

Example 9.2

Air exits from a reservoir maintained at 20°C and 500 kPa absolute into a receiver maintained at (a) 300 kPa absolute and (b) 200 kPa absolute. Estimate the mass flux if the exit area is 10 cm^2. Use the equations first and then the isentropic flow table, Table D.1. Refer to Figure 9.5.

Solution

To estimate the mass flux we will assume isentropic flow. For air the receiver pressure that would result in $M_e = 1$ is

$$p_r = 0.5283\, p_0 = 0.5283 \times 500 = 264.2 \text{ kPa}$$

For part (a) $M_e < 1$ since $p_r > 264.2$ kPa, and for part (b) choked flow occurs and $M_e = 1$ since $p_r < 264.2$ kPa.

(a) To find the exit Mach number, Eq. 9.3.13 gives

$$1 + \frac{k-1}{2} M^2 = \left(\frac{p_0}{p}\right)^{(k-1)/k}$$

$$M_e^2 = \frac{2}{k-1}\left(\frac{p_0}{p}\right)^{(k-1)/k} - \frac{2}{k-1}$$

$$= \frac{2}{0.4}\left(\frac{500}{300}\right)^{0.2857} - \frac{2}{0.4} = 0.7857 \quad \therefore M_e = 0.8864$$

The mass flux is given by Eq. 9.3.17 and is found to be

$$\dot{m} = p_0\sqrt{\frac{5}{RT_0}} MA \left(1 + \frac{k-1}{2} M^2\right)^{(k+1)/2(1-k)}$$

$$= 500\,000 \sqrt{\frac{1.4}{287 \times 93}} \times 0.8864 \times 0.001\left(1 + \frac{0.4}{2} \times 0.8864^2\right)^{-2.4/0.8}$$

$$= \underline{1.167 \text{ kg/s}}$$

9.3 Isentropic Nozzle Flow

(b) Choked flow occurs and thus $M_e = 1$; Eq. 9.3.18 yields

$$\dot{m} = p_0 A^* \sqrt{\frac{k}{RT_0}} \left(\frac{k+1}{2}\right)^{(k+1)/2(1-k)}$$

$$= 500\,000 \times 0.001 \sqrt{\frac{1.4}{287 \times 293}} \left(\frac{2.4}{2}\right)^{-2.4/0.8} = \underline{1.181\,\text{kg/s}}$$

Now, let us use the isentropic flow table (Table D.1) and solve parts (a) and (b).
(a) For a pressure ratio of $p/p_0 = 300/500 = 0.6$, we interpolate to find

$$M_e = \frac{0.6041 - 0.6}{0.6041 - 0.5913} \times 0.02 + 0.88 = 0.886$$

$$\frac{T_e}{T_0} = \frac{0.6041 - 0.6}{0.6041 - 0.5913}(0.8606 - 0.8659) + 0.8659 = 0.864$$

$$T_e = 0.864 \times 293 = 253\,\text{K}$$

The velocity and density are, respectively,

$$V = Mc = 0.886\sqrt{1.4 \times 287 \times 253} = 282\,\text{m/s}$$

$$\rho = \frac{p}{RT} = \frac{300}{0.287 \times 253} = 4.13\,\text{kg/m}^3$$

The mass flux is then

$$\dot{m} = \rho A V$$
$$= 4.13 \times 0.001 \times 282 = \underline{1.165\,\text{kg/s}}$$

(b) For choked flow we know that $M_e = 1$. The table gives

$$\frac{T_e}{T_0} = 0.8333 \quad \text{and} \quad \frac{p_e}{p_0} = 0.5283$$

Thus the temperature, velocity, and density are, respectively,

$$T = 0.8333 \times 293 = 244.2\,\text{K}$$

$$V = Mc = 1 \times \sqrt{1.4 \times 287 \times 244.2} = 313.2\,\text{m/s}$$

$$\rho = \frac{p}{RT} = \frac{0.5283 \times 500}{0.287 \times 244.2} = 3.769\,\text{kg/m}^3$$

The mass flux is calculated to be

$$\dot{m} = \rho A V$$
$$= 3.769 \times 0.001 \times 313.2 = \underline{1.180\,\text{kg/s}}$$

The results using the equations are essentially the same as those using the tables.

9.3 Isentropic Nozzle Flow

A converging–diverging nozzle, with an exit area of 0.004 m^2 and a throat area of 0.001 m^2, is attached to a reservoir with $T = 20°C$ and $p = 500$ kPa absolute. Determine the two exit pressures that result in M = 1 at the throat for an isentropic flow. Also, determine the associated exit temperatures and velocities. See Figure 9.7.

Solution

The exit pressures we seek are associated with curves C and D of Figure 9.7. The area ratio is

$$\frac{A}{A^*} = \frac{40}{10} = 4$$

We could solve Eq. 9.3.19 for M using a trial-and-error technique; however, let us use the isentropic flow table, Table D.1. There are two entries for $A/A^* = 4$. Interpolation gives

$$\left(\frac{p}{p_0}\right)_C = \frac{4.182 - 4.0}{4.182 - 3.673}(0.9823 - 0.9864) + 0.9864 = 0.9849$$

$$\left(\frac{p}{p_0}\right)_D = \frac{4.0 - 3.999}{4.076 - 3.999}(0.02891 - 0.02980) + 0.02980 = 0.02979$$

Hence the two exit pressures that will result in isentropic flow are

$$p_C = \underline{492.4\text{ kPa}} \quad \text{and} \quad p_D = \underline{14.9\text{ kPa}}$$

Note the very small pressure difference (7.6 kPa) between receiver and reservoir necessary to create the flow condition of curve C of Figure 9.7.

The exit temperature ratios and Mach numbers are interpolated to be

$$\left(\frac{T}{T_0}\right)_C = 0.3576(0.9949 - 0.9961) + 0.9961 = 0.9957$$

$$\left(\frac{T}{T_0}\right)_D = 0.01299(0.3633 - 0.3665) + 0.3665 = 0.3665$$

$$M_C = 0.3576 \times 0.02 + 0.14 = 0.147$$

$$M_D = 0.01299 \times 0.02 + 2.94 = 2.94$$

The exit temperatures associated with curves C and D are thus

$$T_C = 0.9957 \times 293 = \underline{291.7\text{ K}}$$

$$T_D = 0.3665 \times 293 = \underline{107.4\text{ K}}$$

The exit velocities are found from $V = Mc$ to be

$$V_C = 0.147\sqrt{1.4 \times 287 \times 291.7} = \underline{50.3\text{ m/s}}$$

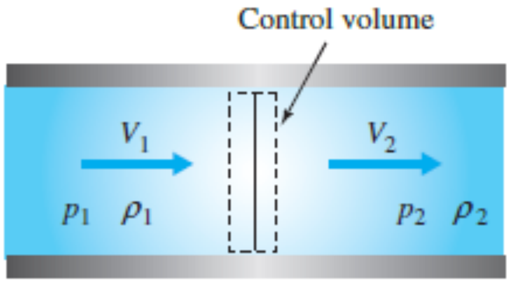
$$V_D = 2.94\sqrt{1.4 \times 287 \times 107.4} = \underline{611\text{ m/s}}$$

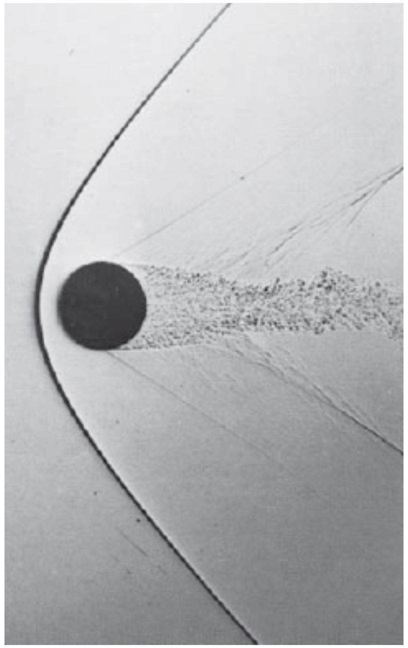
9.4 Normal Shock Wave

Shock wave: A large disturbance that propagates through a gas.

The changes that occur across a shock wave take place over an extremely short distance.

Stationary shock wave in a tube:





9.4 Normal Shock Wave

The three equations below allow us to determine three unknowns:

Continuity equation

$$\rho_1 V_1 = \rho_2 V_2$$

Energy equation

$$\frac{V_2^2 - V_1^2}{2} + \frac{k}{k-1}\left(\frac{p_2}{\rho_2} - \frac{p_1}{\rho_1}\right) = 0$$

Momentum equation

$$p_1 - p_2 = \rho_1 V_1 (V_2 - V_1)$$

9.4 Normal Shock Wave

A finite wave that converts a subsonic flow into a supersonic flow is an impossibility.

$$M_2^2 = \frac{M_1^2 + 5}{7M_1^2 - 1}$$

$$\frac{p_2}{p_1} = \frac{7M_1^2 - 1}{6}$$

$$\frac{T_2}{T_1} = \frac{(M_1^2 + 5)(7M_1^2 - 1)}{36M_1^2}$$

$$\begin{aligned} s_2 - s_1 &= c_p \ln\frac{T_2}{T_1} - R\ln\frac{p_2}{p_1} \\ &= c_p \ln\frac{2 + (k-1)M_1^2}{2 + (k-1)M_2^2} - R\ln\frac{1 + kM_1^2}{1 + kM_2^2} \end{aligned}$$

9.4 Normal Shock Wave

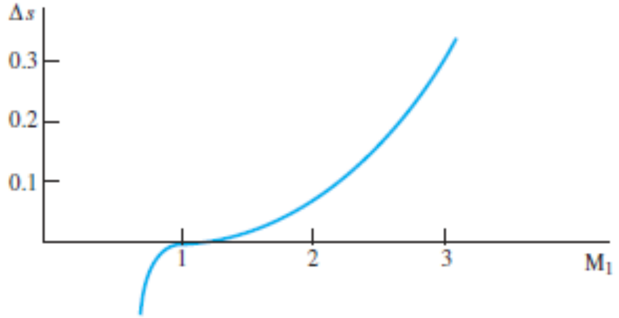


Figure 9.10 Entropy change for a normal shock in air.

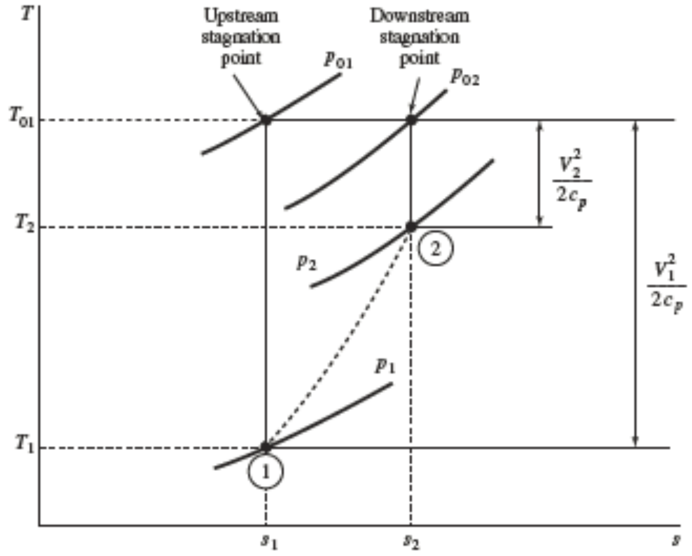


Figure 9.11 T–s diagram for a normal shock wave.

9.4 Normal Shock Wave

A normal shock wave passes through stagnant air at 16°C and atmospheric pressure of 82 kPa with a speed of 453 m/s. Calculate the pressure and temperature downstream of the shock wave. Use (a) the equations and (b) the gas tables.

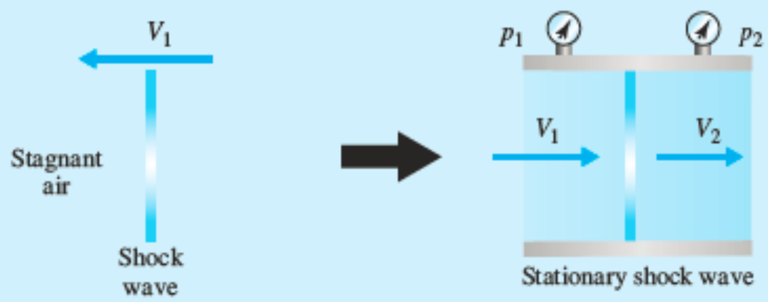


Figure E9.5

Solution

We consider the shock wave to be stationary with $V_1 = 453$ m/s and $p_1 = 82$ kPa. (a) To use the simplified equations (9.4.14) we must know the upstream Mach number. It is

$$M_1 = \frac{V_1}{c_1} = \frac{V_1}{\sqrt{kRT_1}}$$

$$= \frac{453 \text{ m/s}}{\sqrt{1.4 \times 287 \text{ J/kg} \cdot \text{K} \times 289 \text{ K}}} = 1.33$$

9.4 Normal Shock Wave

The pressure and temperature are then found to be

$$p_2 = \frac{p_1(7M_1^2 - 1)}{6}$$
$$= \frac{82(7 \times 1.3^2 - 1)}{6} = \underline{155.5 \text{ kPa}}$$
$$T_2 = \frac{T_1(M_1^2 + 5)(7M_1^2 - 1)}{36M_1^2}$$
$$= \frac{289(1.33^2 + 5)(7 \times 1.33^2 - 1)}{36 \times 1.33^2} = \underline{349.6 \text{ K}}$$

(b) From part (a), we use $M_1 = 1.33$. Interpolation in Table D.2 yields

$$\frac{p_2}{p_1} = \frac{1.33 - 1.32}{1.34 - 1.32}(1.928 - 1.8) + 1.866 = 1.897$$
$$\frac{T_2}{T_1} = \frac{1.33 - 1.32}{1.34 - 1.32}(1.216 - 1.204) + 1.204 = 1.21$$

Using the information given, we have

$$p_2 = 82 \text{ kPa} \times 1.897 = \underline{155.5 \text{ kPa}}$$
$$T_2 = 289 \times 1.21 = \underline{349.7 \text{ K}}$$

9.5 Shock Waves in Converging-Diverging Nozzles

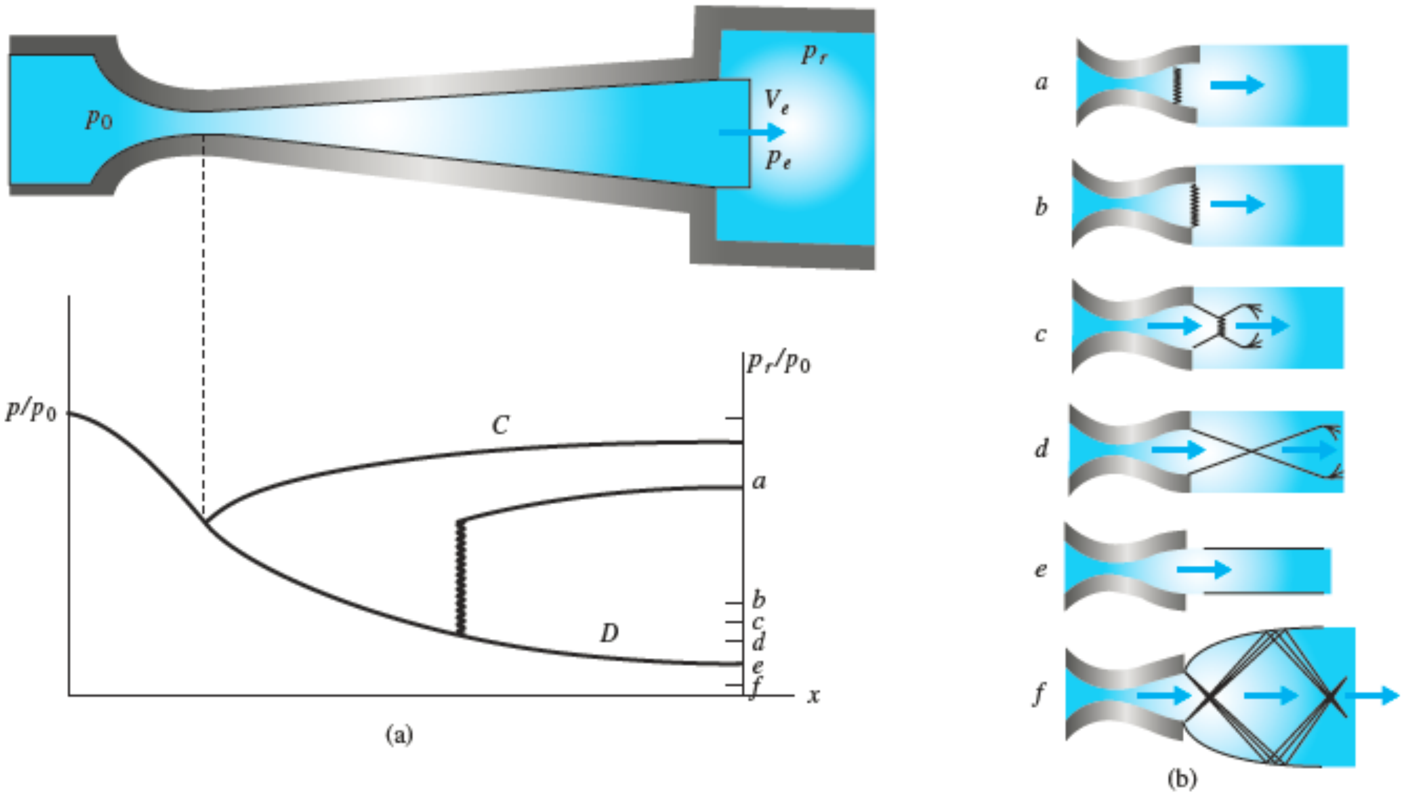


Figure 9.12 Converging-diverging nozzle.

9.5 Shock Waves in Converging-Diverging Nozzles

A converging–diverging nozzle has a throat diameter of 50 mm and an exit diameter of 100 mm. The reservoir is the laboratory, maintained at atmospheric conditions of 20°C and 90 kPa absolute. Air is constantly pumped from a receiver so that a normal shock wave stands across the exit plane of the nozzle. Determine the receiver pressure and the mass flux.

Solution

Isentropic flow occurs from the reservoir, to the throat, to the exit plane in front of the normal shock wave at state 1. Supersonic flow occurs downstream of the throat making the throat the critical area. Hence

$$\frac{A_1}{A^*} = \frac{10^2}{5^2} = 4$$

Interpolation in the isentropic flow table (Table D.1) gives

$$\mathrm{M}_1 = 2.94 \qquad \frac{p_1}{p_0} = 0.0298$$

9.5 Shock Waves in Converging-Diverging Nozzles

Hence the pressure in front of the normal shock is

$$p_1 = p_0 \times 0.0298$$
$$= 90 \times 0.0298 = 2.68\,\text{kPa}$$

From the normal shock table (Table D.2), using $M_1 = 2.94$, we find that

$$\frac{p_2}{p_1} = 9.918$$
$$\therefore p_2 = 9.918 \times 2.68 = \underline{26.6\ \text{kPa}}$$

This is the receiver pressure needed to orient the shock across the exit plane as shown for $p_r/p_0 = b$ in Figure 9.12.

To find the mass flux through the nozzle, we need only consider the throat. Recognizing that $M_t = 1$, so that $V_t = c_t$, we can write

$$\dot{m} = \rho_t A_t V_t = \frac{p_t}{RT_t} A_t \sqrt{kRT_t} = p_t A_t \sqrt{\frac{k}{RT_t}}$$

The isentropic flow table yields

$$\frac{p_t}{p_0} = 0.5283 \qquad \frac{T_t}{T_0} = 0.8333$$

Thus the mass flux becomes

$$\dot{m} = (0.5283 \times 90\,000) \times \frac{\pi \times 0.05^2}{4} \sqrt{\frac{1.4}{287 \times (0.8333 \times 293)}}$$
$$= \underline{0.417\,\text{kg/s}}$$

Remember, the pressure must be measured in pascals in the equation above. Check the units to make sure that the units on $\dot{m}$ are kg/s.

9.6 Vapor Flow Through a Nozzle

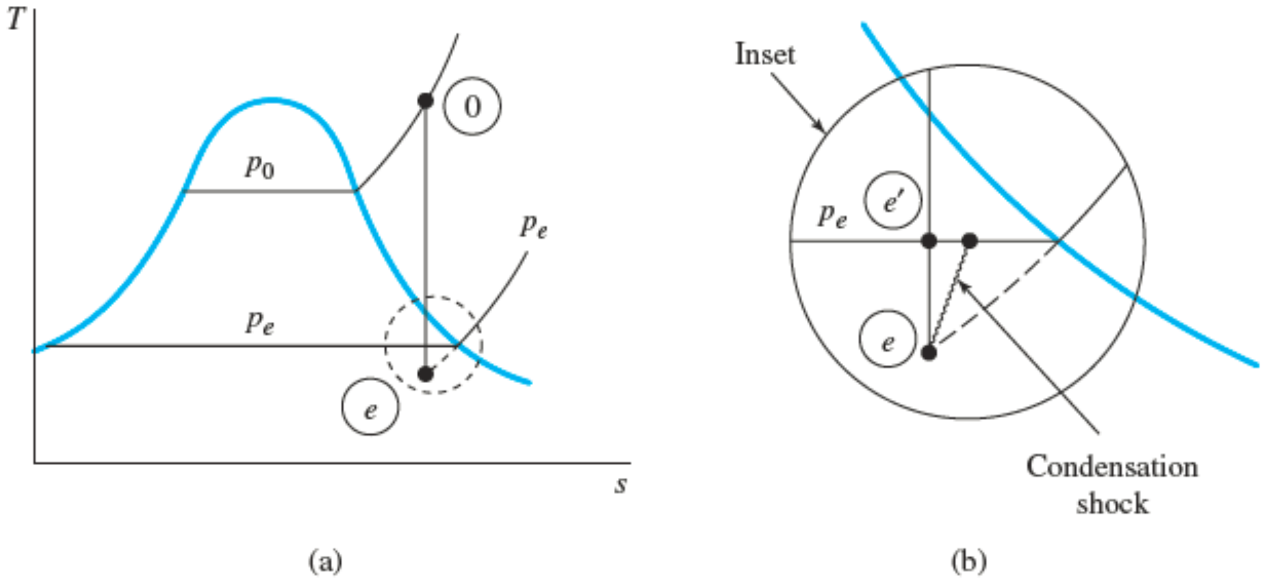


Figure 9.13 Isentropic expansion of a vapor.

The critical pressure ratio for steam

$$\frac{p^*}{p_0} = \left(\frac{2}{k+1}\right)^{k/(k-1)} = 0.546$$

9.6 Vapor Flow Through a Nozzle

Steam is to be expanded isentropically from reservoir conditions of 300°C and 800 kPa absolute to an exit condition of 100 kPa absolute. If supersonic flow is desired, calculate the necessary throat and exit diameters if a mass flux of 2 kg/s is demanded.

Solution

From the steam tables (found in any thermodynamics textbook) we find that

$$s_0 = s_e = 7.2336 \text{ kJ/kg} \cdot \text{K}$$
$$h_0 = 3056.4 \text{ kJ/kg}$$

To estimate the temperature of the metastable exit state, we use

$$T_e = T_0 \left(\frac{p_e}{p_0} \right)^{(k-1)/k} = 593 \left(\frac{100}{800} \right)^{0.3/1.3} = 367 \text{ K} \quad \text{or} \quad 94°\text{C}$$

Using the steam tables at this temperature, we interpolate, using the exit quality x_e, to find that

$$7.2336 = 1.239 + 6.90 x_e$$
$$\therefore x_e = 0.968$$

Thus we have the enthalpy and specific volume at the exit:

$$h_e = 394 + 0.968 \times 2273 = 2594 \text{ kJ/kg}$$
$$v_e = 0.001 + 0.968 \times (2.06 - 0.001) = 1.99 \text{ m}^3\text{/kg}$$

Using the energy equation, the exit velocity is estimated as follows:

$$\cancel{\frac{V_0^2}{2}}^{\,0} + h_0 = \frac{V_e^2}{2} + h_e$$
$$\therefore V_e = \sqrt{2(h_0 - h_e)} = \sqrt{2(3056 - 2594) \times 1000} = 961 \text{ m/s}$$

9.6 Vapor Flow Through a Nozzle

where the 1000 converts kJ to J. From the definition of mass flux we have

$$m_e = \rho_e A_e V_e$$

$$2 = \frac{1}{1.99} \times \frac{\pi d_e^2}{4} \times 961$$

$$\therefore d_e = 0.0726 \text{ m} \qquad \text{or} \qquad \underline{72.6 \text{ mm}}$$

To determine the diameter of the throat, we recognize the throat to be the critical area; thus Eq. 9.6.1 gives

$$p^* = 0.546\, p_0 = 437 \text{ kPa}$$

Using this pressure and $s^* = s_0 = 7.2336 \text{ kJ/kg} \cdot \text{K}$ we could use the steam tables to find h^* and v^*; the energy equation would then allow us to find V^* and thus d_t. However, a simpler, approximate technique, assuming constant specific heats, is to use Eq. 9.3.18 with $k = 1.3$ and obtain the following:

$$\dot{m} = p_0 A^* \sqrt{\frac{1.3}{RT_0}} \left(\frac{2.3}{2}\right)^{2.3/-0.6}$$

$$2 = 800\,000 \frac{\pi d_t^2}{4} \sqrt{\frac{13}{462 \times 573}} \times 0.585$$

$$\therefore d_t = 0.049 \text{ m} \quad \text{or} \quad \underline{49 \text{ mm}}$$

This is reasonable since we have already assumed constant specific heats in predicting T_e. Obviously, the above is approximate; using $k = 1.3$ does, however, give reasonable predictions.

9.7 Oblique Shock Wave

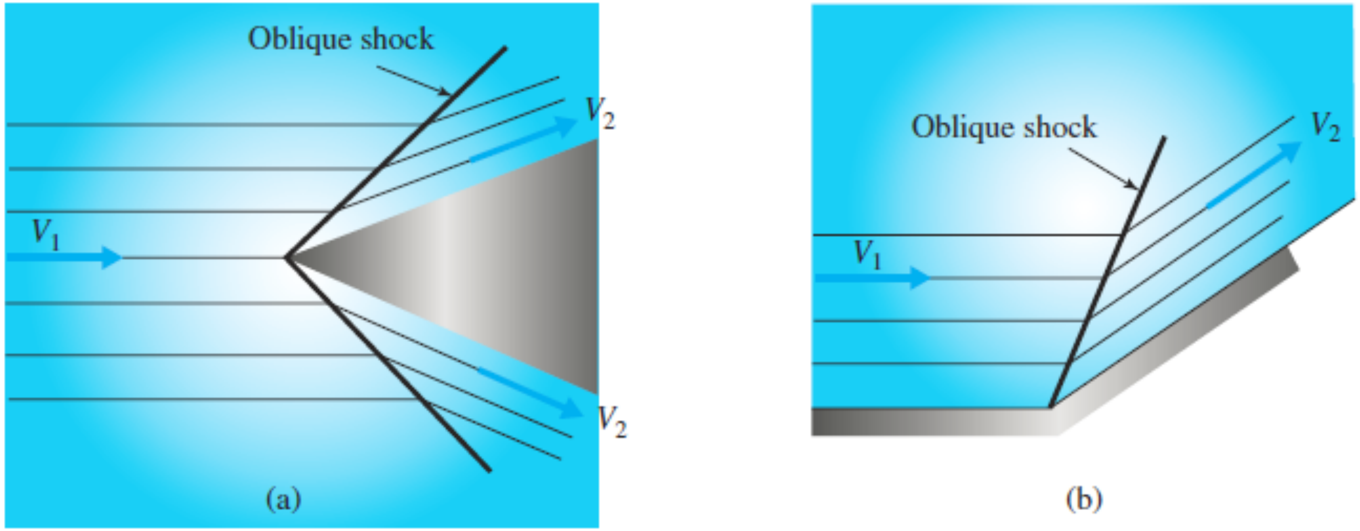


Figure 9.14 Oblique shock waves in a supersonic flow: (a) flow over a symmetrical wedge; (b) flow in a corner.

The oblique shock wave turns the flow so that the velocity vector is parallel to the plane wall.

9.7 Oblique Shock Wave

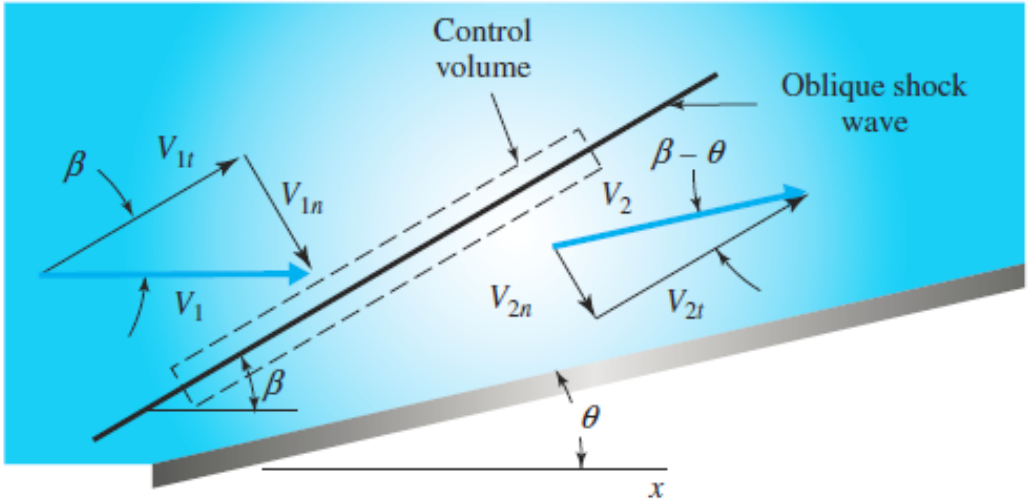


Figure 9.15 Control volume enclosing a small portion of an oblique shock wave.

The tangential components of the two velocity vectors do not enter the equations.

$$\frac{V_{1n}^2}{2} + \frac{k}{k-1}\frac{p_1}{\rho_1} = \frac{V_{2n}^2}{2} + \frac{k}{k-1}\frac{p_2}{\rho_2}$$

$$\tan(\beta - \theta) = \frac{\tan\beta}{k+1}\left[k - 1 + \frac{2}{M_1^2 \sin^2\beta}\right]$$

9.7 Oblique Shock Wave

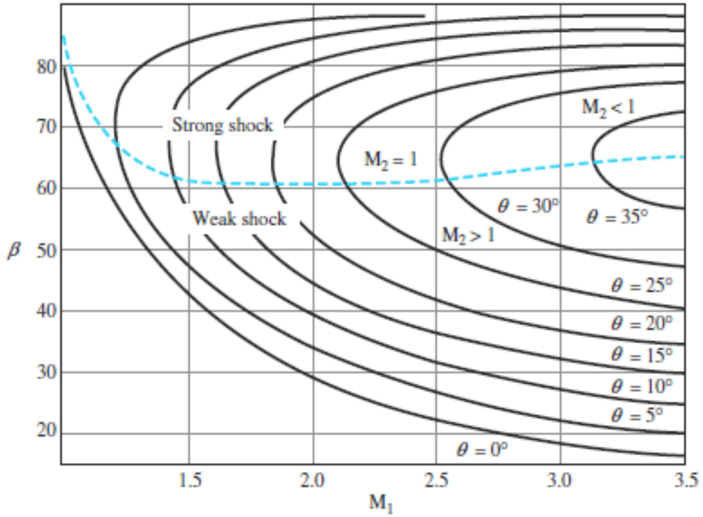


Figure 9.16 Oblique shock wave relationships for $k = 1.4$.

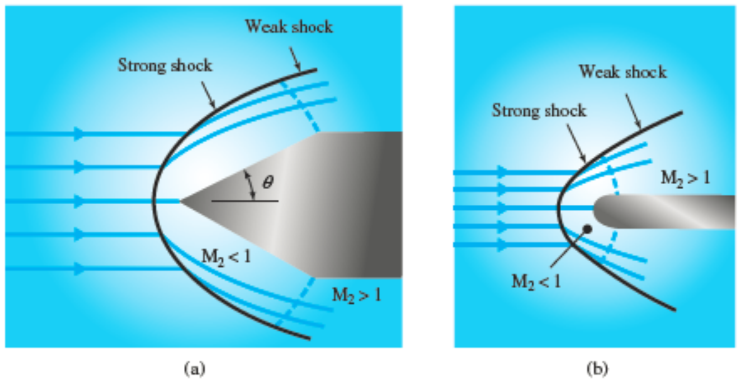


Figure 9.17 Detached shock waves: (a) flow around a wedge; (b) flow around a blunt object.

9.7 Oblique Shock Wave

Air flows over a wedge with $M_1 = 3$, as shown in Figure E9.11. A weak shock reflects from the wall. Determine the values of M_3 and β_3 for the reflected wave.

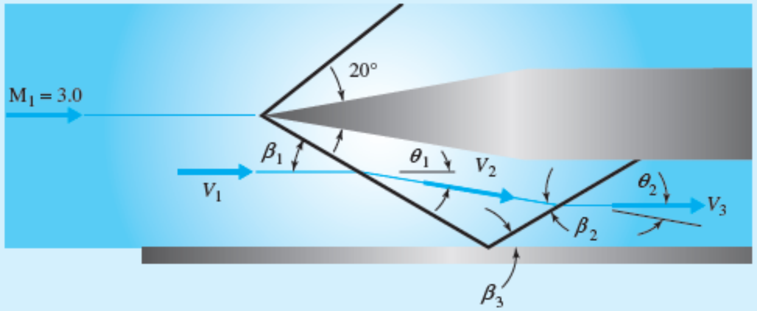


Figure E9.11

Solution

From Figure 9.16 with $\theta_1 = 10°$ and $M_1 = 3.0$, we find for the weak shock that $\beta_1 = 27.5°$. This yields

$$M_{1n} = 3 \sin 27.5° = 1.39$$

From the shock table we interpolate to find

$$M_{2n} = 0.744 = M_2 \sin(27.5° - 10°)$$
$$\therefore M_2 = 2.48$$

The reflected shock must again turn the flow through an angle of 10°, that is, $\theta_2 = 10°$. For this wedge angle and $M_2 = 2.48$ from Figure 9.16 for a weak shock, we see that $\beta_2 = 33°$. This results in

$$M_{2n} = 2.48 \sin 33° = 1.35$$

From the shock table

$$M_{3n} = 0.762 = M_3 \sin 23°$$
$$\therefore M_3 = \underline{1.95}$$

The desired angle is calculated to be

$$\beta_3 = \beta_2 - 10° = \underline{23°}$$

Note that Figure 9.16 does not allow precise calculations. Equation 9.7.8 could be used, by trial and error, to improve the accuracy of the β's and hence the quantities that follow.

9.8 Isentropic Expansion Waves

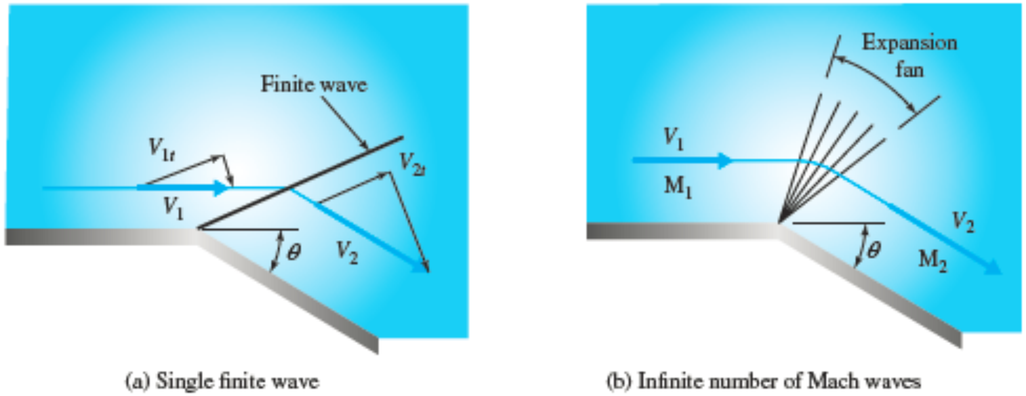


Figure 9.18 Supersonic flow around a convex corner.

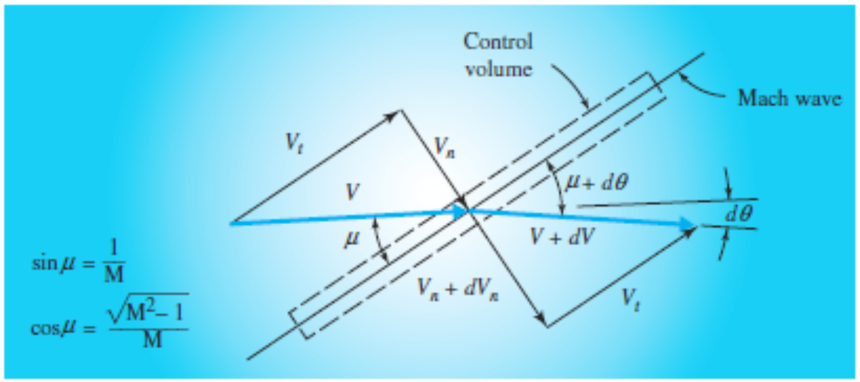


Figure 9.19 A single Mach wave.

9.8 Isentropic Expansion Waves

Prandtl-Meyer function: The angle θ through which the supersonic flow turns.

$$\theta = \left(\frac{k+1}{k-1}\right)^{1/2} \tan^{-1}\left[\frac{k-1}{k+1}(\mathrm{M}^2 - 1)\right]^{1/2} - \tan^{-1}(\mathrm{M}^2 - 1)^{1/2}$$

The collection of Mach waves that turn the flow are referred to as an *expansion fan*.

The supersonic flow remains attached to the wall as it turns the corner, even for large abrupt angles.

Turning angles greater than 90˚ are possible in supersonic flows.

9.8 Isentropic Expansion Waves

Air at a Mach number of 2.0 and a temperature and pressure of 500°C and 200 kPa absolute, respectively, flows around a corner with a convex angle of 20° (Figure E9.12). Find M_2, p_2, T_2, and V_2.

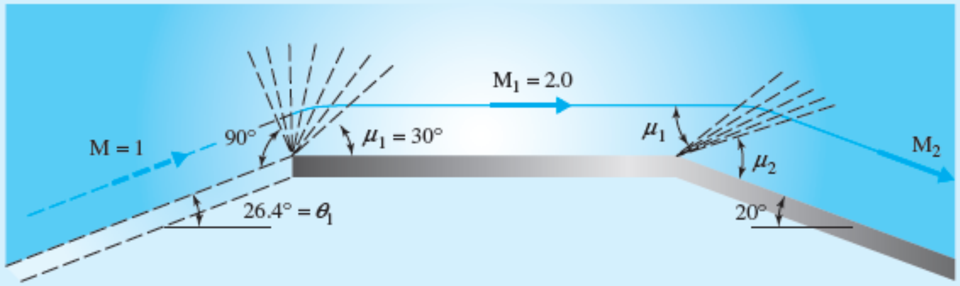


Figure E9.12

Solution

Table D.3 uses M = 1 as a reference condition; thus we visualize the flow as originating from a flow with M = 1 and turning through the angle θ_1 to $M_1 = 2$, as shown in the sketch. From the table, adding an additional 20° to the deflection angle, we find that $\theta_2 = 46.4°$. This would be equivalent to the flow at M = 1 turning a convex corner with $\theta = 46.4°$. Since the flow is isentropic, we can simply superimpose in this manner. Now, for an angle of $\theta = 46.4°$ from the table, we find that

$$M_2 = \underline{2.83}$$

From the isentropic flow table (Table D.1) we find that

$$\begin{aligned} p_2 &= p_1 \frac{p_0}{p_1} \frac{p_2}{p_0} \\ &= 200 \times \frac{1}{0.1278} \times 0.0352 = \underline{55.1 \text{ kPa}} \\ T_2 &= T_1 \frac{T_0}{T_1} \frac{T_2}{T_0} \\ &= 773 \times \frac{1}{0.5556} \times 0.3844 = 534.8 \text{ K} \quad \text{or} \quad \underline{261.8°\text{C}} \end{aligned}$$

The velocity V_2 is found to be

$$\begin{aligned} V_2 &= M_2 \sqrt{kRT_2} \\ &= 2.83\sqrt{1.4 \times 287 \times 534.8} = \underline{1312 \text{ m/s}} \end{aligned}$$

9.9 Summary

$$\sin\alpha = \frac{1}{\mathrm{M}}$$

$$\mathrm{M} = V/c \text{ and } c = \sqrt{kRT}$$

$$\frac{dV}{V}(\mathrm{M}^2 - 1) = \frac{dA}{A}$$

$$\dot{m} = p_0 A^* \sqrt{\frac{k}{RT_0}}\left(\frac{k+1}{2}\right)^{(k+1)/2(1-k)}$$

$$\rho_1 V_1 = \rho_2 V_2, \qquad p_1 - p_2 = \rho_1 V_1 (V_2 - V_1),$$

$$\frac{V_2^2 - V_1^2}{2} + \frac{k}{k-1}\left(\frac{p_2}{\rho_2} - \frac{p_1}{\rho_1}\right) = 0$$

$$\tan(\beta - \theta) = \frac{\tan\beta}{k+1}\left[k - 1 + \frac{2}{\mathrm{M}_1^2 \sin^2\beta}\right]$$

$$\theta = \left(\frac{k+1}{k-1}\right)^{1/2} \tan^{-1}\left(\frac{k-1}{k+1}(\mathrm{M}^2 - 1)\right)^{1/2} - \tan^{-1}(\mathrm{M}^2 - 1)^{1/2}$$