Chapter 5

Turbomachinery

12.1 Introduction

Turbomachines are the commonly employed devices that either supply or extract energy from a flowing fluid by means of rotating propellers or vanes.

A turbopump/pump adds energy to a system, with the result that the pressure is increased.

A turbine extracts energy from a system and converts it to some other useful form, typically, to electric power.

A wind turbine converts the energy contained in the natural movement of atmospheric air into useful electrical
power.

12.2 Turbopumps

Turbopump: Consists of two major parts—an impeller and the pump housing.

Common types of pumps are radial flow, mixed flow, and axial flow.

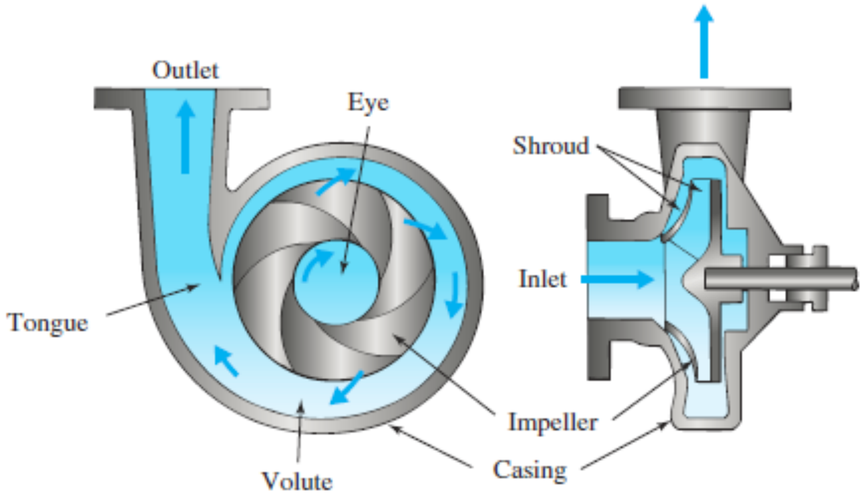

Figure 12.1 Single-suction pump.

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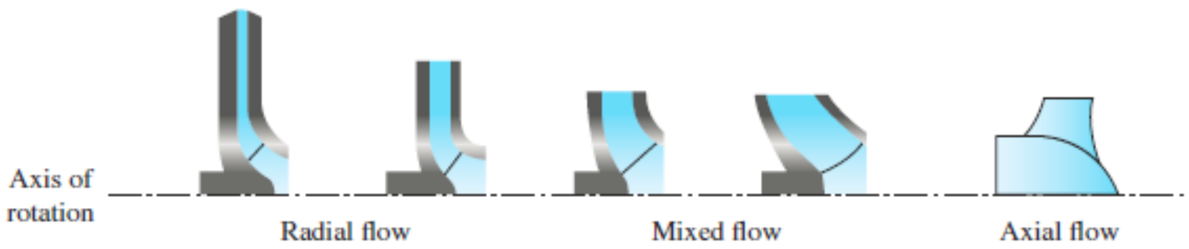

Figure 12.2 Various types of pump impellers.

In a radial-flow pump, the impeller vanes are commonly curved backward and the impeller is relatively narrow.

12.2 Turbopumps

Radial-Flow Pumps

A radial-flow, or centrifugal, pump is designed to deliver relatively low discharge at a high head.

To construct a simplified theory for the radial-flow pump, it is necessary to neglect viscosity and to assume idealized two-dimensional flow throughout the impeller region.

$$T = \rho Q(r_2 V_{t2} - r_1 V_{t1})$$

12.2 Turbopumps

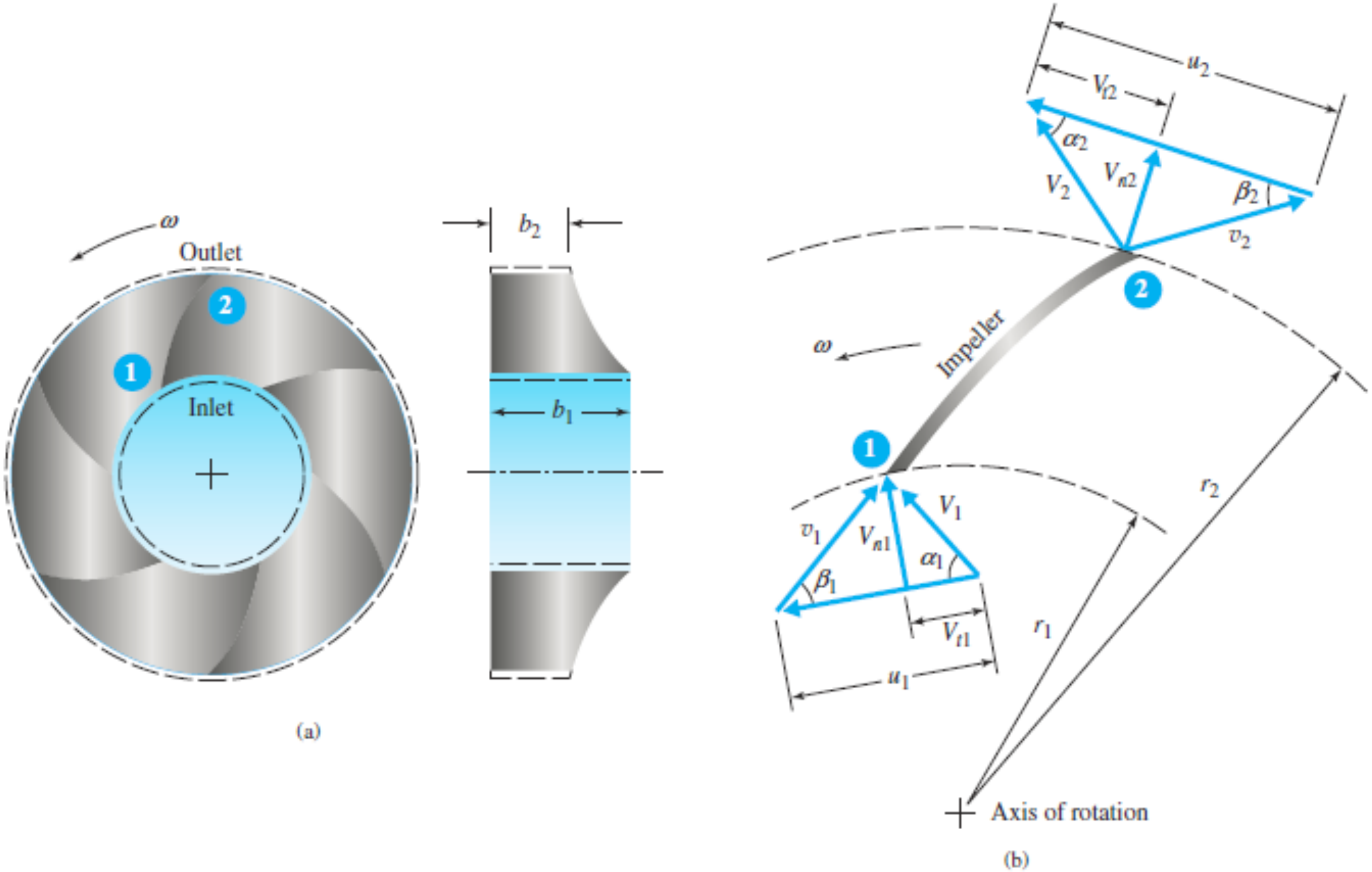

Figure 12.3 Idealized radial-flow impeller: (a) impeller control volume; (b) velocity diagrams at control surfaces.

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$$H_t = \frac{\omega T}{\gamma Q} = \frac{u_2 V_2 \cos\alpha_2 - u_1 V_1 \cos\alpha_1}{g}$$

$$H_t = \frac{\omega^2 r_2^2}{g} - \frac{\omega \cot\beta_2}{2\pi b_2 g} Q$$

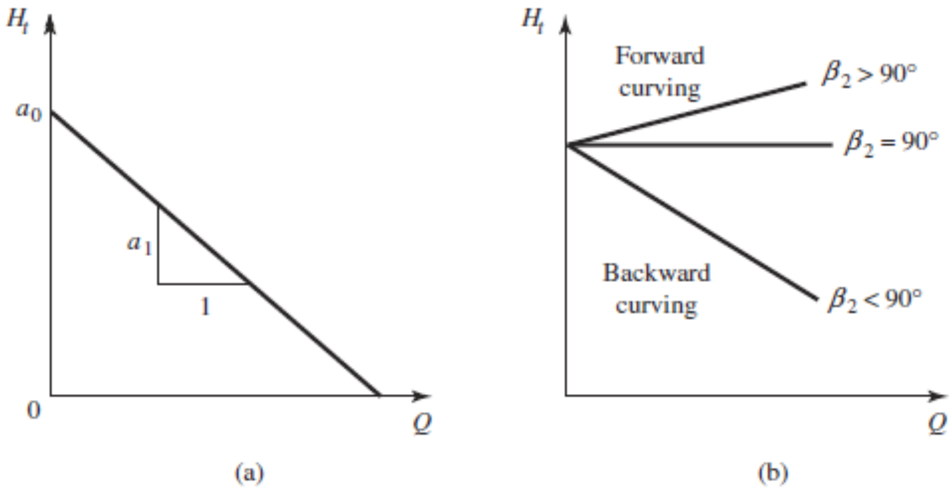

Figure 12.4 Ideal pump performance curves.

12.2 Turbopumps

A radial-flow pump has the following dimensions:

$$\beta_1 = 44^\circ \qquad r_1 = 21 \text{ mm} \qquad b_1 = 11 \text{ mm}$$
$$\beta_2 = 30^\circ \qquad r_2 = 66 \text{ mm} \qquad b_2 = 5 \text{ mm}$$

For a rotational speed of 2500 rev/min, assuming ideal conditions (frictionless flow, negligible vane thickness, perfect guidance), with $\alpha_1 = 90^\circ$ (no prerotation), determine (a) the discharge, theoretical head, required power, and pressure rise across the impeller, and (b) the theoretical head-discharge curve. Use water as the fluid.

Solution

(a) Construct the velocity diagram at location 1, as shown in Figure E12.1a. The rotational speed is converted to the appropriate units as

$$\omega = 2500\frac{2\pi}{60} = 261.8 \text{ rad/s}$$

The impeller speed at r_1 is then

$$u_1 = \omega r_1 = 261.8 \times 0.021 = 5.50 \text{ m/s}$$

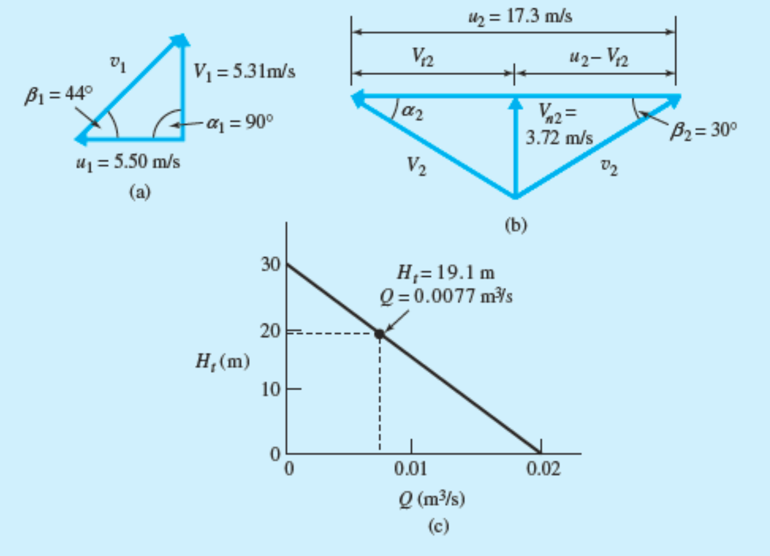

Figure E12.1

From the velocity diagram we see that

$$V_1 = u_1 \tan \beta_1 = 5.50 \tan 44^\circ = 5.31 \text{ m/s}$$

12.2 Turbopumps

and since $\alpha_1 = 90°$, $V_1 = V_{n1}$, or $V_{n1} = 5.31$ m/s. The discharge is computed to be

$$Q = 2\pi r_1 b_1 V_{n1}$$
$$= 2\pi \times 0.021 \times 0.011 \times 5.31 = 7.71 \times 10^{-3}\ \text{m}^3\text{/s} \quad \text{or} \quad \underline{7.71\ \text{L/s}}$$

The normal component of velocity at location 2 is

$$V_{n2} = \frac{Q}{2\pi r_2 b_2} = \frac{7.71 \times 10^{-3}}{2\pi \times 0.066 \times 0.005} = 3.72\ \text{m/s}$$

and the impeller speed at the outlet is

$$u_2 = \omega r_2 = 261.8 \times 0.066 = 17.28\ \text{m/s}$$

The velocity diagram at location 2 is now sketched as shown in Figure E12.1b. From the velocity diagram we see that

$$u_2 - V_{t2} = \frac{V_{n2}}{\tan\beta_2} = \frac{3.72}{\tan 30°} = 6.44\ \text{m/s}$$

Therefore,

$$V_{t2} = u_2 - 6.44 = 17.28 - 6.44 = 10.84\ \text{m/s}$$
$$\alpha_2 = \tan^{-1}\frac{V_{n2}}{V_{t2}} = \tan^{-1}\frac{3.72}{10.84} = 18.9°$$
$$V_2 = \frac{V_{t2}}{\cos\alpha_2} = \frac{10.84}{\cos 18.9°} = 11.46\ \text{m/s}$$

The theoretical head is computed with Eq. 12.2.10:

$$H_t = \frac{u_2 V_2 \cos\alpha_2}{g} = \frac{17.28 \times 11.46 \times \cos 18.9°}{9.81} = \underline{19.1\ \text{m}}$$

Hence the theoretical required power is

$$\dot{W}_p = \gamma Q H_t = 9810 \times (7.71 \times 10^{-3}) \times 19.1 = \underline{1440\ \text{W}}$$

The pressure rise is determined from the energy equation as follows:

$$p_2 - p_1 = \left(H_t + \frac{V_1^2 - V_2^2}{2g}\right)\gamma$$
$$= \left[19.1 + \frac{(5.31)^2 - (11.46)^2}{2 \times 9.81}\right] \times 9810 = 1.36 \times 10^5\ \text{Pa} \quad \text{or} \quad \underline{136\ \text{kPa}}$$

(b) The theoretical head-discharge curve is Eq. 12.2.13. For the present example we have

$$H_t = \frac{(\omega r_2)^2}{g} - \frac{\omega \cot\beta_2}{2\pi b_2 g}Q$$
$$= \frac{(261.8 \times 0.066)^2}{9.81} - \frac{261.8 \cot 30°}{2\pi \times 0.005 \times 9.81}Q$$
$$= 30.4 - 1471Q$$

The curve is shown in Figure E12.1c.

12.2 Turbopumps

Head-Discharge Relations: Performance Curves

Brake power: Power delivered to the impeller

$$H_P = \left(\frac{p}{\gamma} + \frac{V^2}{2g} + z\right)_2 - \left(\frac{p}{\gamma} + \frac{V^2}{2g} + z\right)_1$$
$$= H_t - h_L$$

$$\eta_P = \frac{\dot{W}_f}{\dot{W}_P} = \frac{\gamma Q H_P}{\omega T}$$

12.2 Turbopumps

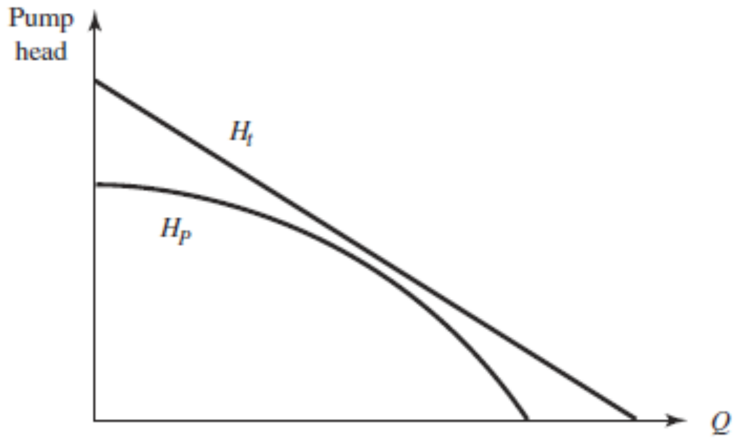

Figure 12.5 Comparison between theoretical and actual radial-flow pump performance curves.

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Axial- and Mixed-Flow Pumps

An axial-flow pump produces relatively large discharges at low heads.

$$H_t = \frac{u^2}{g} - \frac{uV_n}{g}(\cot \alpha_1 + \cot \beta_2)$$

If there is no prerotation

$$H_t = \frac{u^2}{g} - \frac{uV_n \cot \beta_2}{g}$$

12.2 Turbopumps

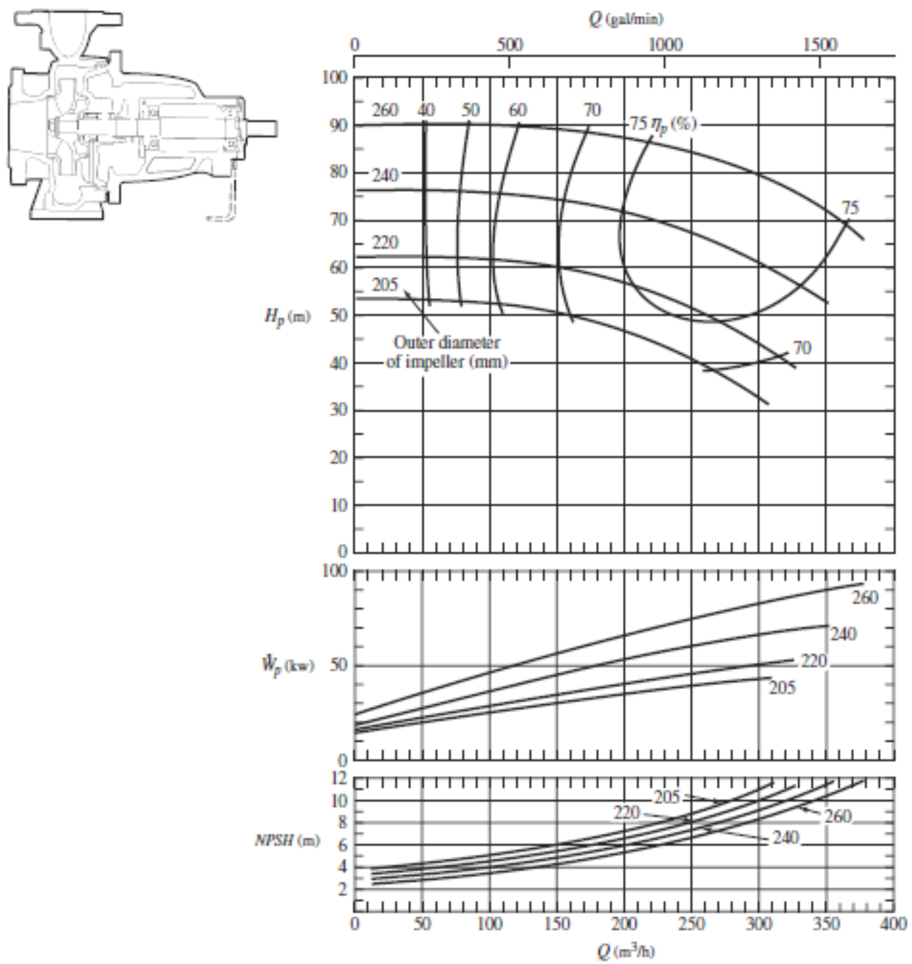


Figure 12.6 Radial-flow pump and performance curves for four different impellers with $N = 2900$ rpm ($\omega = 304$ rad/s). Water at 20°C is the pumped liquid. (Courtesy of Sulzer Pumps Ltd.)

12.2 Turbopumps

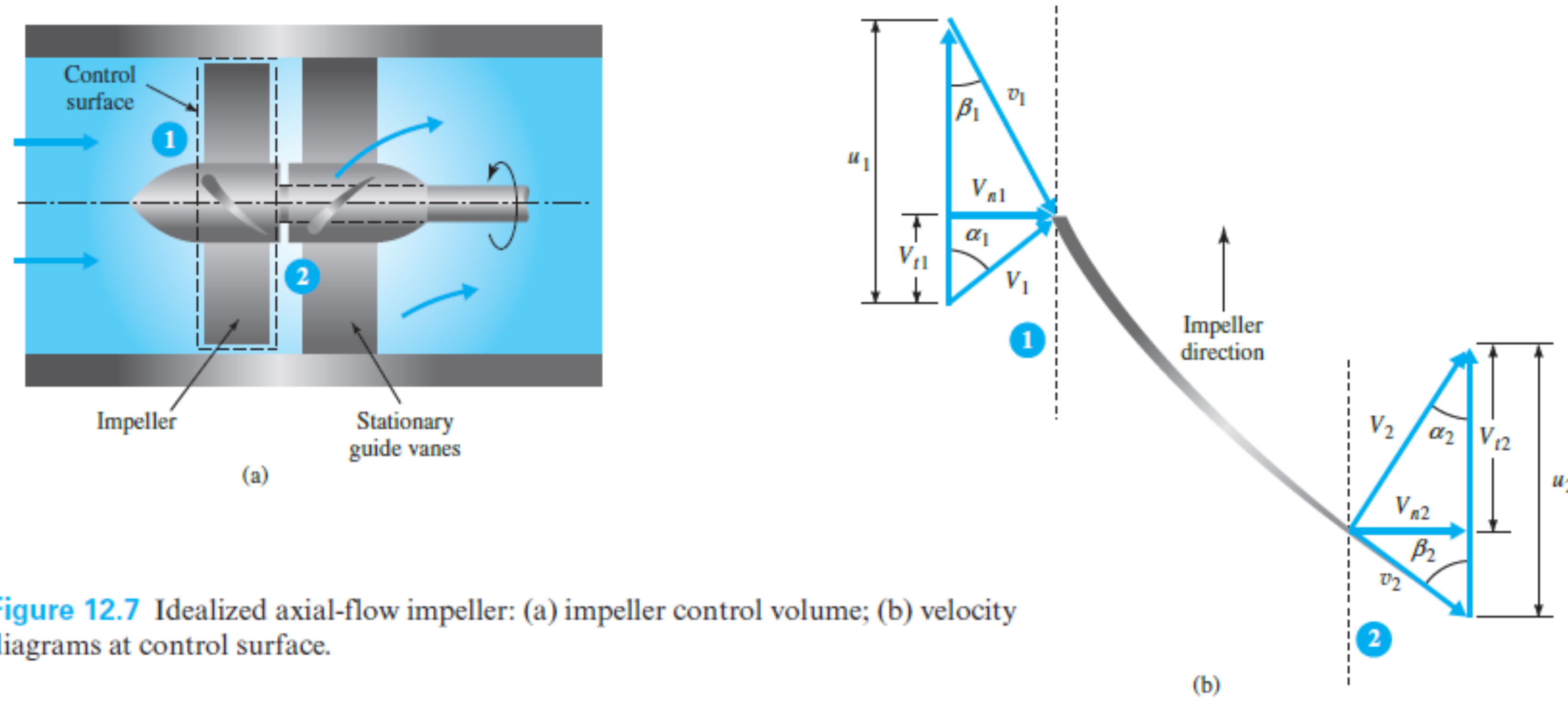


Figure 12.7 Idealized axial-flow impeller: (a) impeller control volume; (b) velocity diagrams at control surface.

12.2 Turbopumps

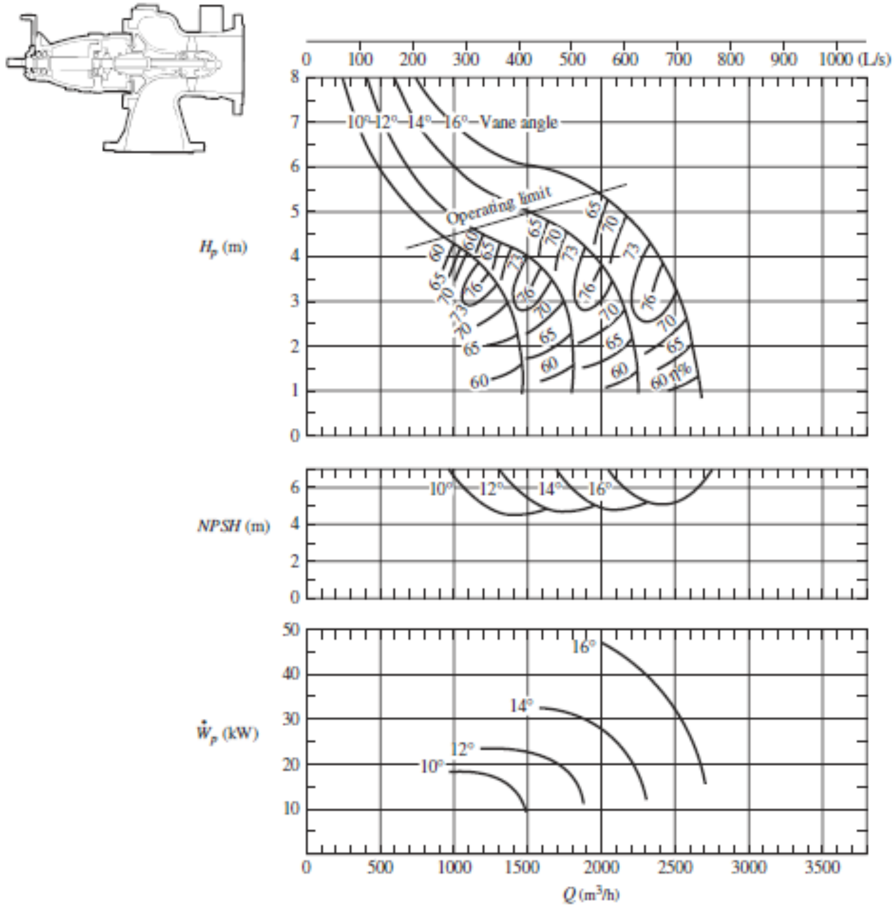


Figure 12.8 Axial-flow pump and performance curves for four different vane angles with $N = 880$ rpm ($\omega = 92.2$ rad/s). Propeller diameter is 500 mm. Water at 20°C is the pumped liquid. (Courtesy of Sulzer Pumps Ltd.)

12.2 Turbopumps

An axial-flow pump is designed with a fixed guide vane, or stator blade, located upstream of the impeller. The stator imparts an angle $\alpha_1 = 75°$ to the fluid as it enters the impeller region. The impeller has a rotational speed of 500 rpm with a blade exit angle of $\beta_2 = 70°$. The control volume has an outer diameter of $D_o = 300$ mm and an inner diameter of $D_i = 150$ mm. Determine the theoretical head rise and power required if 150 L/s of liquid ($S = 0.85$) is to be pumped.

Solution

First, the normal velocity component V_n is

$$V_n = \frac{Q}{A} = \frac{Q}{(\pi/4)(D_o^2 - D_i^2)} = \frac{0.15}{(\pi/4)(0.3^2 - 0.15^2)} = 2.83 \text{ m/s}$$

The peripheral speed u of the impeller is based on an average radius:

$$u \simeq \omega \frac{D_o + D_i}{4} = 500\left(\frac{2\pi}{60}\right)\frac{0.3 + 0.15}{4} = 5.89 \text{ m/s}$$

The theoretical head H_t is computed with Eq. 12.2.19 to be

$$H_t = \frac{u}{g}[u - V_n(\cot\alpha_1 + \cot\beta_2)] = \frac{5.89}{9.81}[5.89 - 2.83(\cot 75° + \cot 70°)] = \underline{2.46 \text{ m}}$$

Finally, for the assumed ideal conditions, the required power is

$$\dot{W}_P = \gamma Q H_t = (9810 \times 0.85) \times 0.15 \times 2.46 = 3080 \text{ W} \quad \text{or} \quad \underline{3.1 \text{ kW}}$$

12.2 Turbopumps

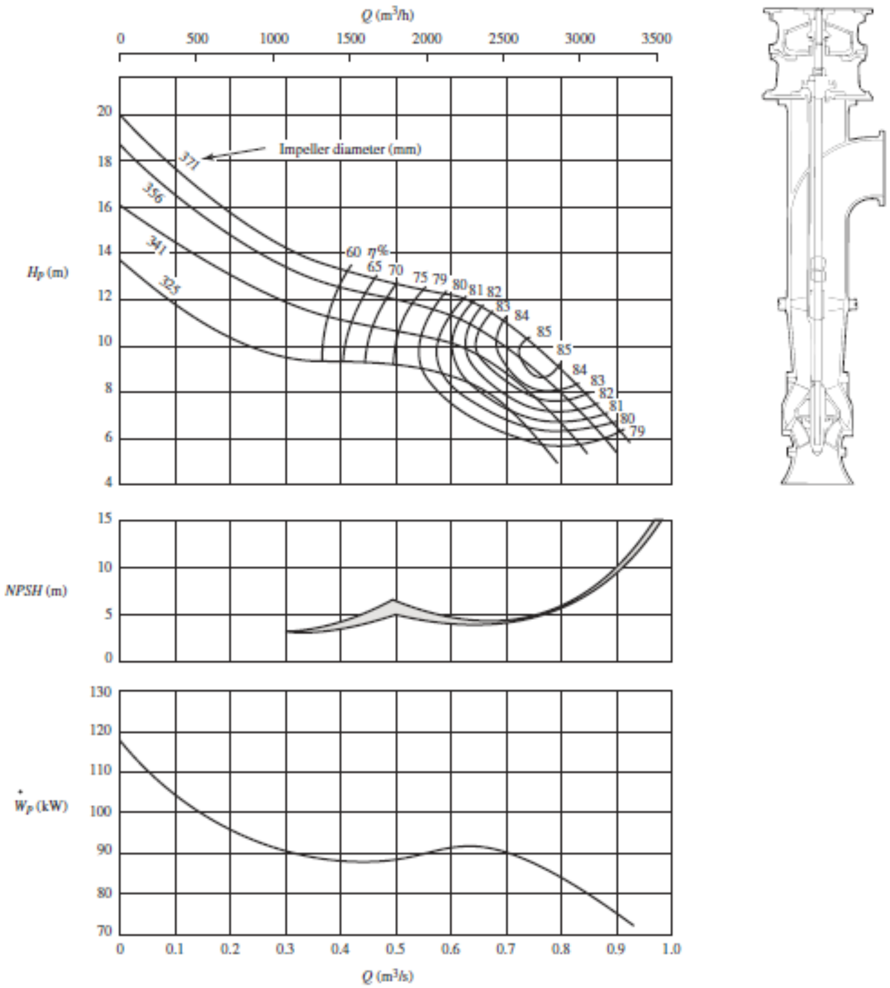


Figure 12.9 Mixed-flow pump and performance curves for four different impellers and vane angles with $N = 970$ rpm ($\omega = 102$ rad/s). Water at 20°C is the pumped liquid. (Courtesy of Sulzer Pumps Ltd.)

12.2 Turbopumps

Figure 12.10 Cavitation bubble distribution in an impeller region. (Courtesy of Sulzer Pumps Ltd.)

net positive suction head:

$$NPSH = \frac{p_{\text{atm}} - p_v}{\gamma} - \Delta z - h_L$$

12.2 Turbopumps

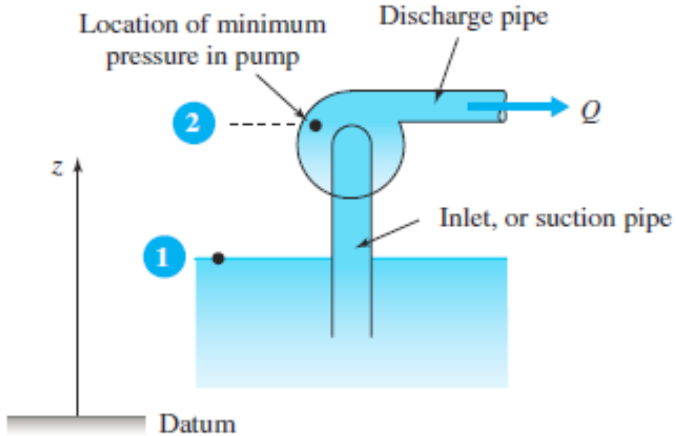


Figure 12.11 Cavitation setting for a pump.

Design requirement for a pump:

$$NPSH \leq \frac{p_{atm} - p_v}{\gamma} - \Delta z - h_L$$

12.2 Turbopumps

Determine the elevation at which the 240-mm-diameter pump of Figure 12.6 can be situated above the water surface of the suction reservoir without experiencing cavitation. Water at 15°C is being pumped at 250 m^3/h. Neglect losses in the system. Use $p_{atm} = 101$ kPa.

Solution

From Figure 12.6, at a discharge of 250 m^3/h, the *NPSH* for the 240-mm-diameter impeller is approximately 7.4 m. For a water temperature of 15°C, $p_v = 1666$ Pa absolute, and $\gamma = 9800$ N/m^3. Equation 12.2.22 with $h_L = 0$ is employed to compute Δz to be

$$\Delta z = \frac{p_{atm} - pv}{\gamma} - NPSH - h_L = \frac{101000 - 1666}{9800} - 7.4 - 0 = \underline{2.74 \text{ m}}$$

Thus the pump can be placed approximately 2.7 m above the suction reservoir water surface.

12.3 Dimensional Analysis and Similitude for Turbomachinery

Dimensionless Coefficients

$$f(\dot{W}, \omega, D, Q, \Delta p, \rho, \mu) = 0$$

Table 12.1 Turbomachinery Parameters

Parameter	*Symbol*	*Dimensions*
Power	$\dot{W}$	ML^2/T^3
Rotational speed	ω	T^{-1}
Outer diameter of impeller	D	L
Discharge	Q	L^3/T
Pressure change	Δp	M/LT^2
Fluid density	ρ	M/L^3
Fluid viscosity	μ	M/LT

12.3 Dimensional Analysis and Similitude for Turbomachinery

The discharge, head, and power coefficients are used to form the dimensionless performance curves.

$$C_{\dot{W}} = \frac{\dot{W}}{\rho\omega^3 D^5} \quad \text{power coefficient}$$

$$C_p = \frac{\Delta p}{\rho\omega^2 D^2} \quad \text{pressure coefficient}$$

$$C_Q = \frac{Q}{\omega D^3} \quad \text{flow rate coefficient}$$

$$\text{Re} = \frac{\omega D^2 \rho}{\mu} \quad \text{Reynolds number}$$

$$C_H = \frac{gH}{\omega^2 D^2} \quad \text{head coefficient}$$

$$\frac{C_Q C_H}{C_{\dot{W}}} = \frac{\gamma Q H}{\dot{W}} = \eta_P \qquad \text{(pump)}$$

$$\frac{C_{\dot{W}}}{C_Q C_H} = \frac{\dot{W}}{\gamma Q H} = \eta_T \qquad \text{(turbine)}$$

12.3 Dimensional Analysis and Similitude for Turbomachinery

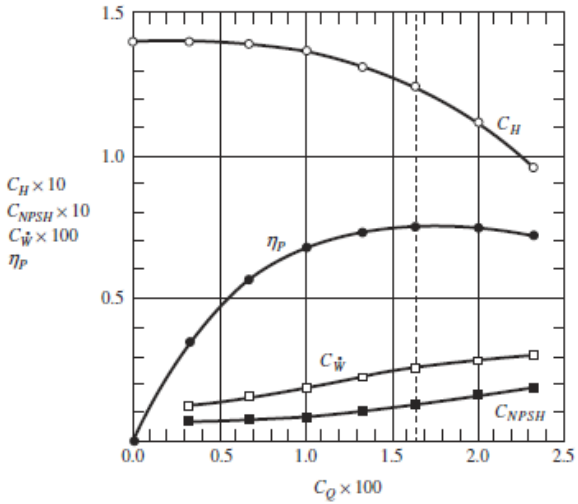


Figure 12.12 Dimensionless radial-flow pump performance curves for the pump presented in Figure 12.6; D = 240 mm; Ω_P = 0.61.

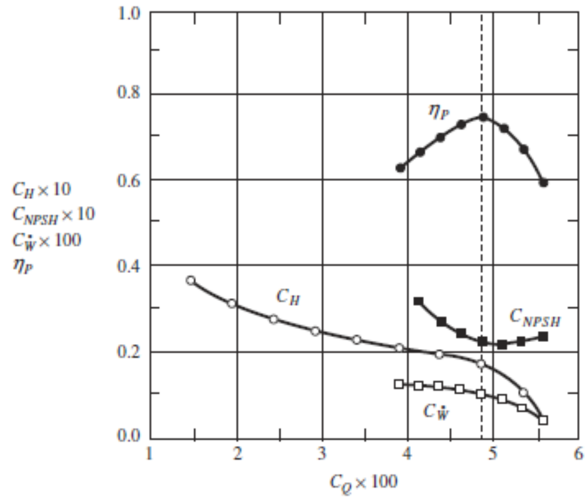


Figure 12.13 Dimensionless axial-flow pump performance curve for the pump presented in Figure 12.8; D = 500 mm; vane angle = 14°; Ω_P = 4.5.

12.3 Dimensional Analysis and Similitude for Turbomachinery

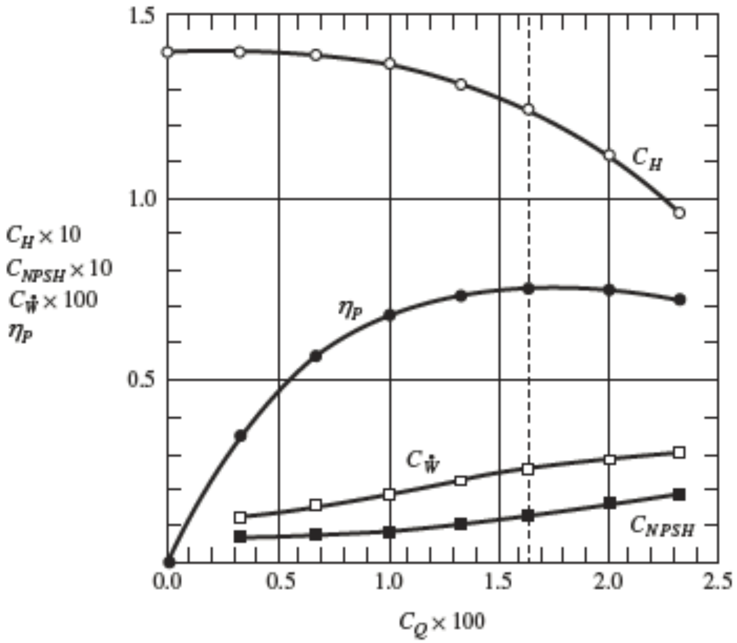


Figure 12.14 Dimensionless mixed-flow pump performance curve for the pump presented in Figure 12.9; $D = 371$ mm ; $\Omega_P = 3.1$.

12.3 Dimensional Analysis and Similitude for Turbomachinery

Determine the rotational speed, size, and required power for the axial-flow pump of Figure 12.13 to deliver 0.65 m^3/s of water at a head of 2.5 m.

Solution

The pump data are obtained from Figure 12.13. Reading from the figure we find that at the design, or maximum, efficiency of $\eta_P = 0.75$

$$C_Q = 0.048 \qquad C_H = 0.018 \qquad C_{\dot{W}} = 0.0012$$

Equations 12.3.4 and 12.3.6 are employed to determine the two unknowns ω and D. Rearrange Eq. 12.3.6 to solve for ωD:

$$\omega D = \sqrt{\frac{gH_P}{C_H}} = \sqrt{\frac{9.81 \times 2.5}{0.018}} = 36.9 \text{ m/s}$$

Equation 12.3.4 is rearranged to include ωD as a known quantity to provide D:

$$D = \sqrt{\frac{Q}{C_Q \omega D}} = \sqrt{\frac{0.65}{0.048 \times 36.9}} = \underline{0.61 \text{ m}}$$

Substituting $D = 0.61$ m into the relation $\omega D = 36.9$, one finds that

$$\omega = \frac{36.9}{D} = \frac{36.9}{0.61} = 60.5 \text{ rad/s} \qquad \text{or} \qquad \underline{578 \text{ rpm}}$$

The density for water is $\rho = 1000 \text{ kg/m}^3$. Then the pump power is determined by using Eq. 12.3.2:

$$\dot{W}_P = C_{\dot{W}} \rho \omega^3 D^5 = 0.0012 \times 1000 \times 60.5^3 \times 0.61^5 = 2.2 \times 10^4 \text{ W} \quad \text{or} \quad \underline{22 \text{ kW}}$$

Thus the speed, size, and required power are approximately 580 rpm, 610 mm, and 22 kW, respectively.

12.3 Dimensional Analysis and Similitude for Turbomachinery

Similarity Rules:

Similarity rules relate variables from a family of geometrically similar turbomachines, or are used to examine changes in variables for a given machine.

$$(C_{\dot{W}})_1 = (C_{\dot{W}})_2 \quad \text{or} \quad \frac{\dot{W}_2}{\dot{W}_1} = \frac{\rho_2}{\rho_1}\left(\frac{\omega_2}{\omega_1}\right)^3\left(\frac{D_2}{D_1}\right)^5$$

$$(C_H)_1 = (C_H)_2 \quad \text{or} \quad \frac{H_2}{H_1} = \frac{g_1}{g_2}\left(\frac{\omega_2}{\omega_1}\right)^2\left(\frac{D_2}{D_1}\right)^2$$

$$(C_Q)_1 = (C_Q)_2 \quad \text{or} \quad \frac{Q_2}{Q_1} = \frac{\omega_2}{\omega_1}\left(\frac{D_2}{D_1}\right)^3$$

$$\frac{1-(\eta_P)_2}{1-(\eta_P)_1} = \left(\frac{D_1}{D_2}\right)^{1/4}$$

12.3 Dimensional Analysis and Similitude for Turbomachinery

Determine the performance curve for the mixed-flow pump whose characteristic curves are shown in Figure 12.14. The required discharge is 2.5 m^3/s with the pump operating at a speed of 600 rpm. At design efficiency, what are the power and the *NPSH* requirements? Water is being pumped.

Solution

From Figure 12.14 the design efficiency is $\eta_P = 0.85$ and the corresponding design values for the coefficients are

$$C_Q = 0.15 \qquad C_H = 0.067 \qquad C_W = 0.0115 \qquad C_{NPSH} = 0.035$$

The rotational speed in radians per second is

$$\omega = 600 \times \frac{\pi}{30} = 62.8 \text{ rad/s}$$

The pump diameter is computed with Eq. 12.3.4:

$$D = \left(\frac{Q}{\omega C_Q}\right)^{1/3} = \left(\frac{2.5}{62.8 \times 0.15}\right)^{1/3} = 0.64 \text{ m}$$

With Eqs. 12.3.2 and 12.3.6, the required power and *NPSH* are computed:

$$\dot{W}_P = \rho\omega^3 D^5 C_{\dot{W}} = 1000 \times 62.8^3 \times 0.64^5 \times 0.0115 = 3.1 \times 10^5 \text{ W} \quad \text{or} \quad \underline{310 \text{ kW}}$$

$$NPSH = \frac{\omega^2 D^2}{g} C_{NPSH} = \frac{62.8^2 \times 0.64^2}{9.81} \times 0.035 = \underline{5.8 \text{ m}}$$

Hence, the required power is 310 kW and the required *NPSH* is 5.8 m.

The performance curve is constructed using Eqs. 12.3.10 and 12.3.11:

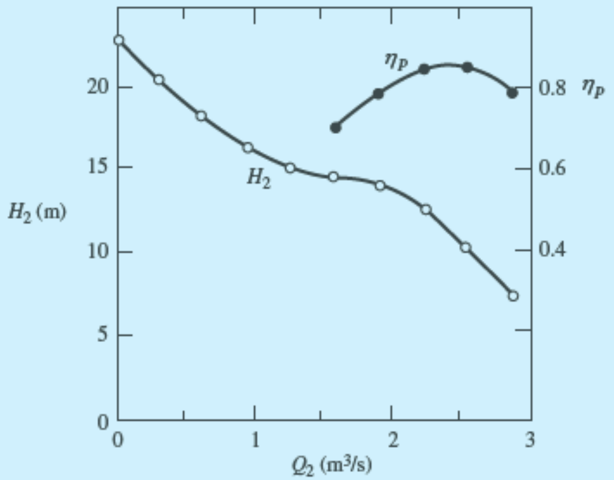
$$H_2 = \frac{\omega_2^2 D_2^2}{g}(C_H)_1 = \frac{62.8^2 \times 0.64^2}{9.81}(C_H)_1 = 165(C_H)_1$$

$$Q_2 = \omega_2 D_2^3 (C_Q)_1 = 62.8 \times 0.64^3 (C_Q)_1 = 16.5(C_Q)_1$$

12.3 Dimensional Analysis and Similitude for Turbomachinery

Values of $(C_H)_1$ and $(C_Q)_1$ are read from Figure 12.14, along with η_P and placed in the table shown below. Columns 4 and 5 are Q_2 and H_2 computed from the similarity equations. At best efficiency, $H_2 = 10.4$ m and $Q_2 = 2.54$ m³/s. The characteristic curve and efficiency curve are plotted in Figure E12.5.

$(C_Q)_1$	$(C_H)_1$	η_P	$Q_2(\text{m}^3/\text{s})$	H_2 (m)
0	0.138		0	22.8
0.019	0.123		0.31	20.3
0.039	0.110		0.64	18.2
0.058	0.099		0.96	16.3
0.077	0.091	0.59	1.27	15.0
0.096	0.088	0.70	1.58	14.5
0.116	0.085	0.78	1.91	14.0
0.135	0.076	0.84	2.23	12.5
0.154	0.063	0.85	2.54	10.4
0.174	0.046	0.79	2.87	7.6



12.3 Dimensional Analysis and Similitude for Turbomachinery

Specific speed: Dimensionless number that characterizes a turbomachine at maximum efficiency.

$$\Omega_P = \frac{C_Q^{1/2}}{C_H^{3/4}} = \frac{\omega Q^{1/2}}{(gH_P)^{3/4}}$$

$\Omega_P < 1$	radial-flow pump
$1 < \Omega_P < 4$	mixed-flow pump
$\Omega_P > 4$	axial-flow pump

$$\Omega_T = \frac{C_{\dot{W}}^{1/2}}{C_H^{5/4}} = \frac{\omega (\dot{W}_T/\rho)^{1/2}}{(gH_T)^{5/4}}$$

12.3 Dimensional Analysis and Similitude for Turbomachinery

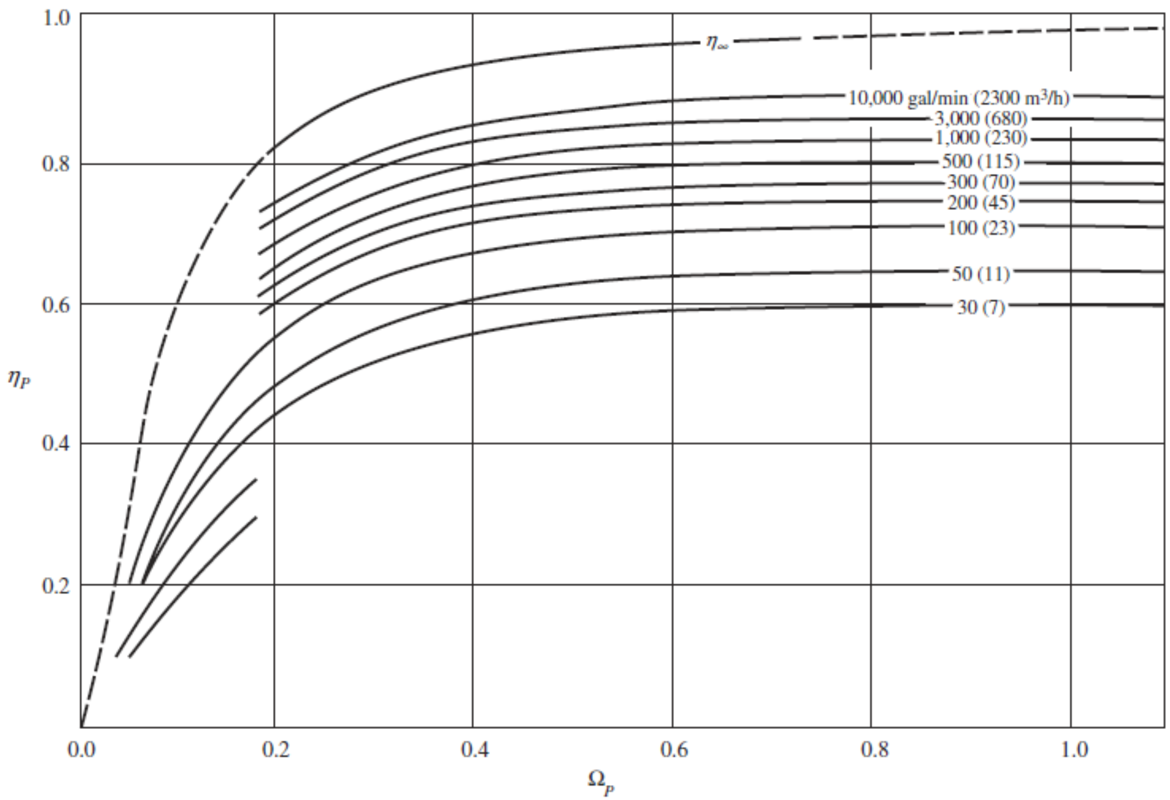


Figure 12.15 Maximum efficiency as a function of specific speed and discharge for radial-flow pumps. (Based on Karassik, *Pump Handbook*, 2nd edition, 1986, McGraw-Hill Companies, Inc.)

12.3 Dimensional Analysis and Similitude for Turbomachinery

Select a pump to deliver 30 L/s of water with a pressure rise of 450 kPa. Assume a rotational speed not to exceed 3600 rpm.

Solution

To estimate the specific speed we need the following:

$$\omega = 3600 \times \pi/30 = 377 \text{ rad/s}$$

$$H_P = \frac{\Delta p}{\rho g} = \frac{450 \times 10^3}{1000 \times 9.81} = 45.9 \text{ m}$$

$$Q = 30 \times 10^{-3} \text{ m}^3\text{/s} = 0.03$$

Use Eq. 12.3.13 to find the specific speed to be

$$\Omega_P = \frac{\omega\sqrt{Q}}{(gH_P)^{3/4}} = \frac{377\sqrt{0.03}}{(9.81 \times 45.9)^{3/4}} = 0.67$$

From Eq. 12.3.14, this would indicate a radial-flow pump. The pump of Figure 12.12 could be used, even though the specific speed (0.61) would result in a lower efficiency. With $\Omega_P \simeq 0.61$ (Figure 12.12), the speed is estimated with Eq. 12.3.13 as

$$\omega = \frac{\Omega_P(gH_P)^{3/4}}{\sqrt{Q}} = \frac{0.61 \times (9.81 \times 45.9)^{3/4}}{\sqrt{0.03}} = 344 \text{ rad/s}$$

Hence the required speed is $344 \times 30/\pi$ = 3300 rpm, which does not exceed 3600 rpm. The required diameter is determined with use of Figure 12.12, where at maximum efficiency, $C_Q = 0.0165$. This is substituted into Eq. 12.3.4 to determine the diameter:

$$D = \left(\frac{Q}{C_Q \omega}\right)^{1/3} = \left(\frac{0.03}{0.0165 \times 344}\right)^{1/3} = \underline{17.4 \text{ cm}}$$

12.4 Use of Turbopumps in Piping Systems

Matching Pumps to System Demand

System demand curve:

$$H_P = (z_2 - z_1) + \left(f\frac{L}{D} + \Sigma K\right)\frac{Q^2}{2gA^2}$$

The intersection of the system demand curve with the pump head-discharge curve specifies the operating head and discharge.

12.4 Use of Turbopumps in Piping Systems

Pumps in Parallel and in Series

Overall efficiency
of pumps in parallel is

$$\eta_P = \frac{\gamma H_D \Sigma Q}{\Sigma \dot{W}_P}$$

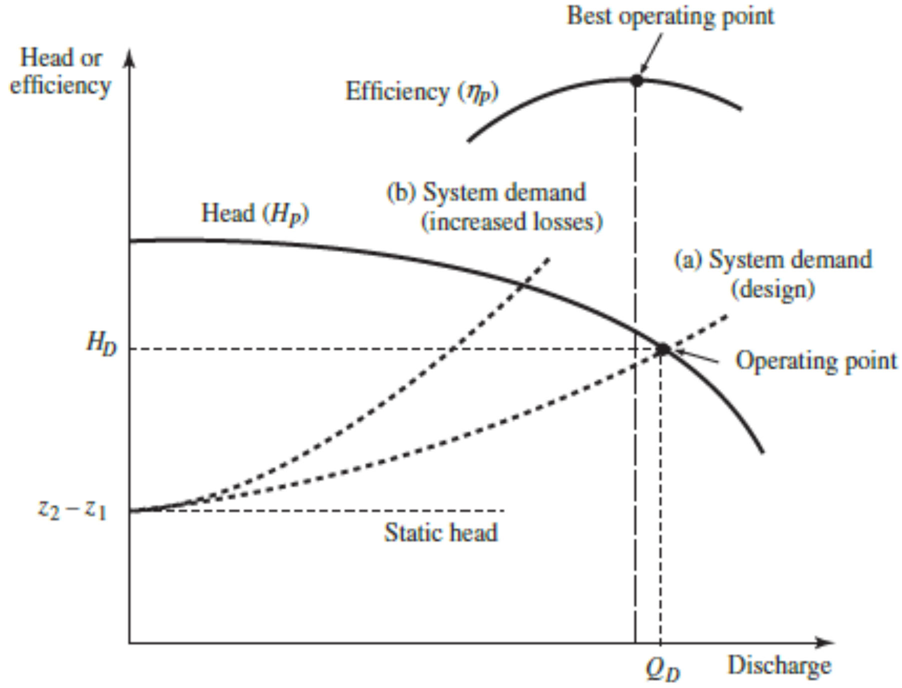


Figure 12.16 Pump characteristic curve and system demand curve.

12.4 Use of Turbopumps in Piping Systems

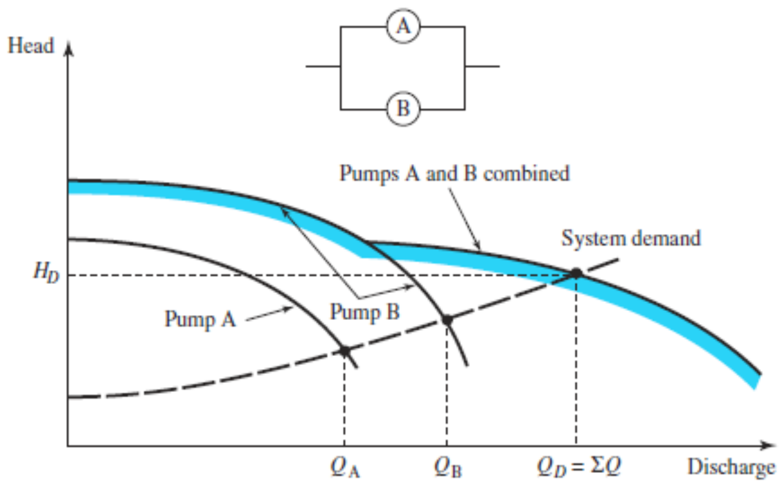


Figure 12.17 Characteristic curves for pumps operating in parallel.

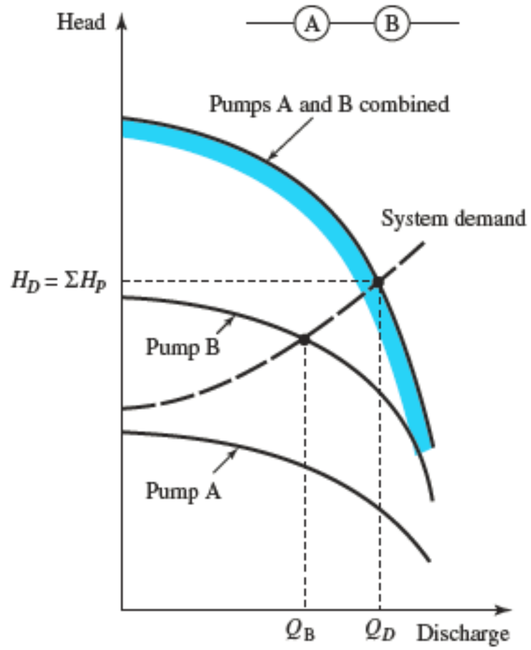


Figure 12.18 Characteristic curves for pumps operating in series.

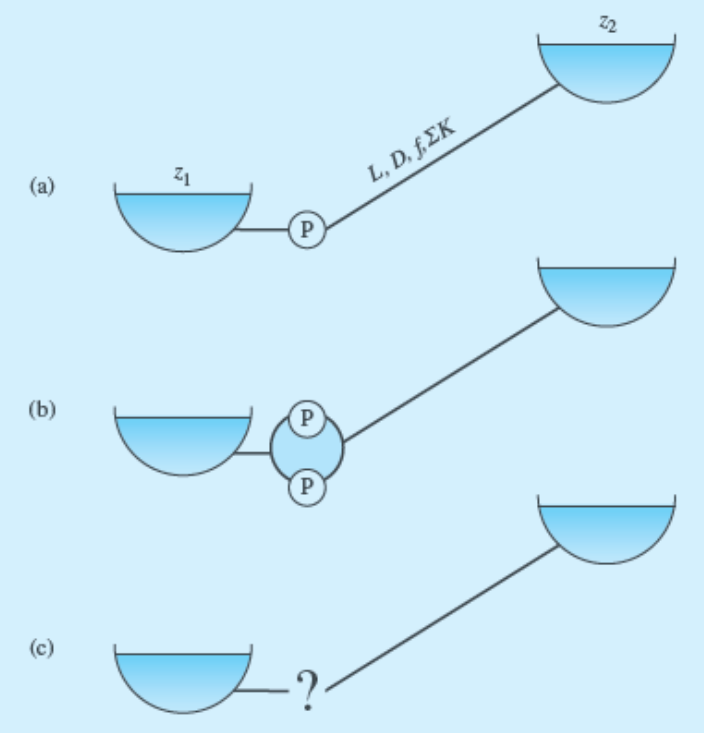
12.4 Use of Turbopumps in Piping Systems

Water is pumped between two reservoirs in a pipeline with the following characteristics: $D = 300$ mm, $L = 70$ m, $f = 0.025$, $\Sigma K = 2.5$. The radial-flow pump characteristic curve is approximated by the formula

$$H_P = 22.9 + 10.7Q - 111Q^2$$

where H_P is in meters and Q is in m³/s.

Determine the discharge Q_D and pump head H_D for the following situations: (a) $z_2 - z_1 = 15$ m, one pump placed in operation; (b) $z_2 - z_1 = 15$ m, with two identical pumps operating in parallel; and (c) the pump layout, discharge, and head for $z_2 - z_1 = 25$ m.



12.4 Use of Turbopumps in Piping Systems

Solution

(a) The system demand curve (Eq. 12.4.1) is developed first:

$$H_P = (z_2 - z_1) + \left(f\frac{L}{D} + \Sigma K\right)\frac{Q^2}{2gA^2}$$
$$= 15 + \left(\frac{0.025 \times 70}{0.3} + 2.5\right)\frac{Q^2}{2 \times 9.81(\pi/4 \times 0.3^2)^2}$$
$$= 15 + 85Q^2$$

To find the operating point, equate the pump characteristic curve to the system demand curve,

$$15 + 85Q_D^2 = 22.9 + 10.7Q_D - 111Q_D^2$$

Reduce and solve for Q_D:

$$195Q_D^2 - 10.7Q_D - 7.9 = 0$$

$$\therefore Q_D = \frac{1}{2 \times 195}\left[10.7 + \sqrt{10.7^2 + 4 \times 195 \times 7.9}\right] = \underline{0.23 \text{ m}^3/\text{s}}$$

Using the system demand curve, H_D is computed as

$$H_D = 15 + 85 \times 0.23^2 = \underline{19.5 \text{ m}}$$

(b) For two pumps in parallel, the characteristic curve is

$$H_P = 22.9 + 10.7\left(\frac{Q}{2}\right) - 111\left(\frac{Q}{2}\right)^2$$
$$= 22.9 + 5.35Q - 27.75Q^2$$

Equate this to the system demand curve and solve for Q_D:

$$15 + 85Q_D^2 = 22.9 + 5.35Q_D - 27.75Q_D^2$$

$$112.8Q_D^2 - 5.35Q_D - 7.9 = 0$$

$$\therefore Q_D = \frac{1}{2 \times 112.8}\left[5.35 + \sqrt{5.35^2 + 4 \times 112.8 \times 7.9}\right] = \underline{0.29 \text{ m}^3/\text{s}}$$

The design head is calculated to be

$$H_D = 15 + 85 \times 0.29^2 = \underline{22.2 \text{ m}}$$

12.4 Use of Turbopumps in Piping Systems

(c) Since $z_2 = z_1$ is greater than the single pump shutoff head (i.e., 25 > 22.9 m), it is necessary to operate with two pumps in series. The combined pump curve is

$$H_D = 2(22.9 + 10.7Q - 111Q^2)$$
$$= 45.8 + 21.4Q - 222Q^2$$

The system demand curve is changed since $z_2 - z_1 = 25$ m. It becomes

$$H_D = 25 + 85Q^2$$

Equating the two relations above and solving for Q_D and H_P results in

$$25 + 85Q_D^2 = 45.8 + 21.4Q_D - 222Q_D^2$$

or

$$307Q_D^2 - 21.4Q_D = 20.8 = 0$$

$$\therefore Q_D = \frac{1}{2 \times 307}\left[21.4 + \sqrt{21.4^2 + 4 \times 307 \times 20.8}\right] = \underline{0.30 \text{ m}^3/\text{s}}$$

and

$$H_D = 25 + 85 \times 0.30^2 = \underline{32.7 \text{ m}}$$

12.5 Turbines

In contrast to pumps, turbines extract useful energy from the water flowing in a piping system.

Runner: Moving component of a turbine.

Reaction turbine: Turbine using both flow energy and kinetic energy.

Impulse turbine: The flow energy is converted into kinetic energy through a nozzle before the liquid impacts the runner.

12.5 Turbines

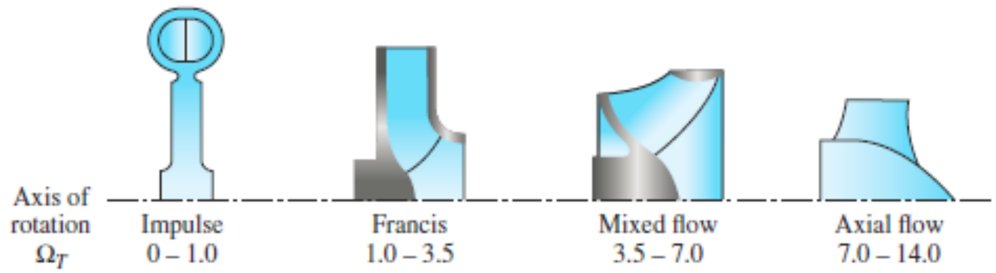
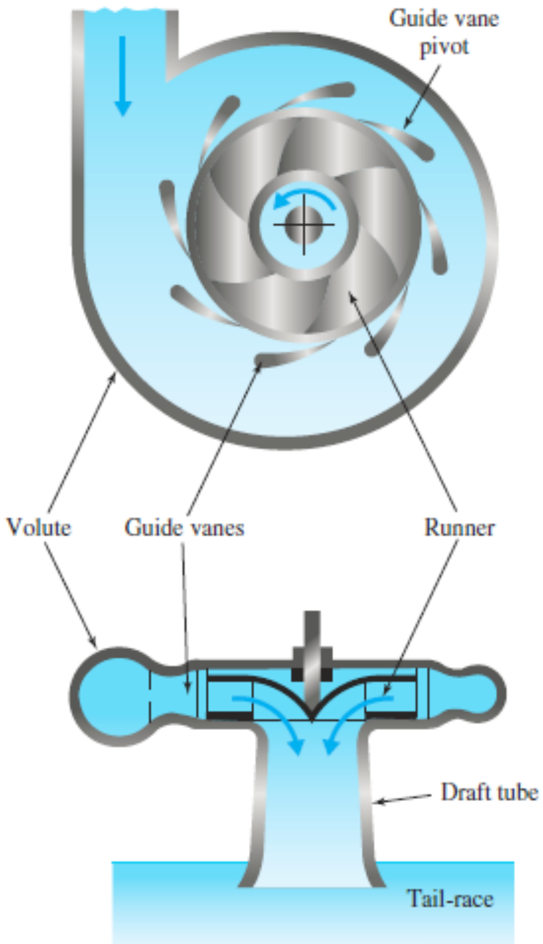


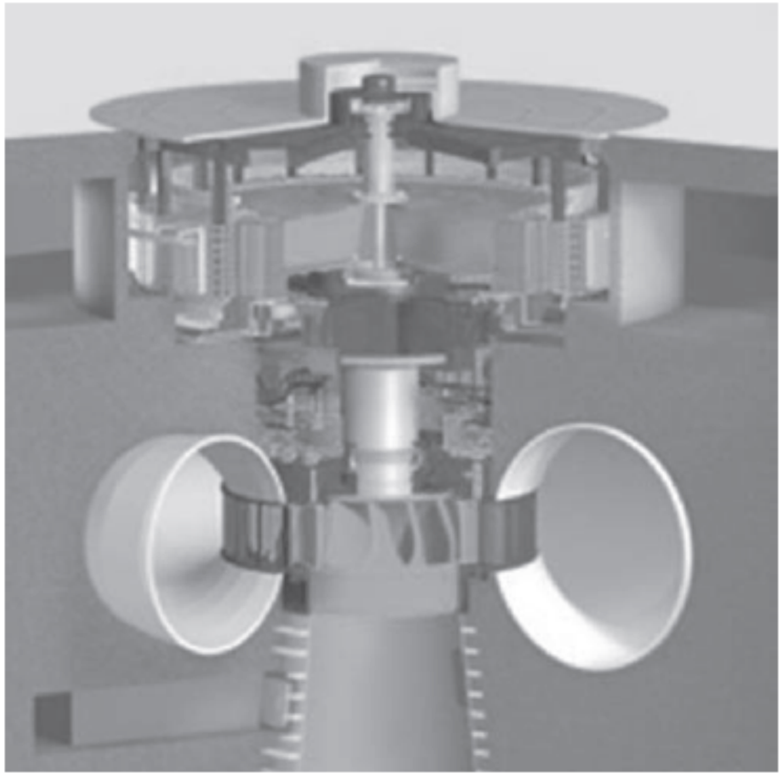
Figure 12.20 Various types of turbine runners.

In reaction turbines, the flow is contained in a volute that channels the liquid into the runner. Adjustable guide vanes (also called wicket gates) are situated upstream of the runner.

12.5 Turbines



(a)



12.5 Turbines

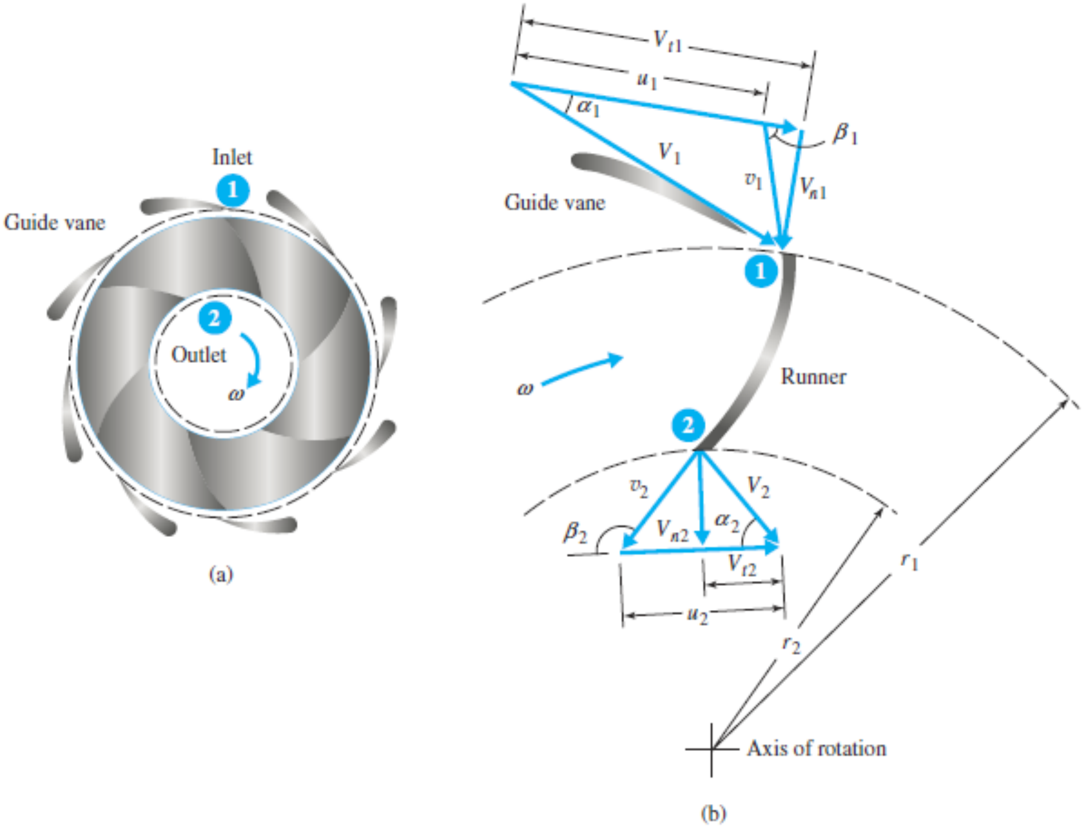


Figure 12.22 Idealized Francis turbine runner: (a) runner control volume; (b) velocity diagrams at control surfaces.

12.5 Turbines

$$T = \rho Q(r_1 V_{t1} - r_2 V_{t2})$$

$$\eta_T = \frac{\dot{W}_T}{\dot{W}_f} = \frac{\omega T}{\gamma Q H_T}$$

$$\cot \alpha_1 = \frac{2\pi r_1^2 b_1 \omega}{Q} + \cot \beta_1$$

12.5 Turbines

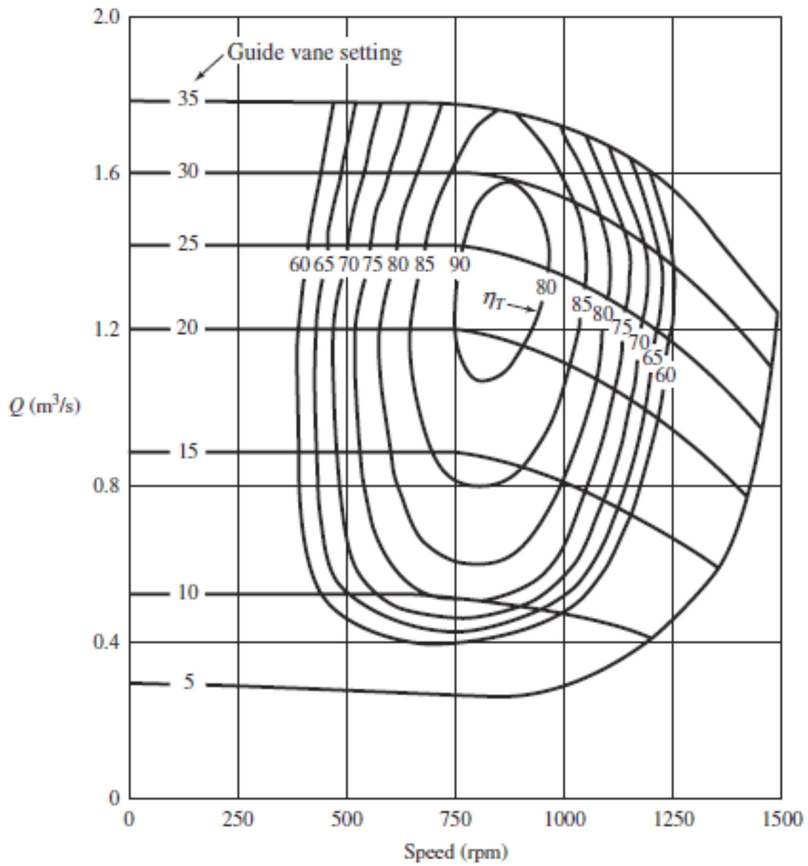


Figure 12.23 Isoefficiency curve for a Francis turbine: $D = 500$ mm, $H_T = 50$ m. (Courtesy of Gilbert Gilkes and Gordon, Ltd.)

12.5 Turbines

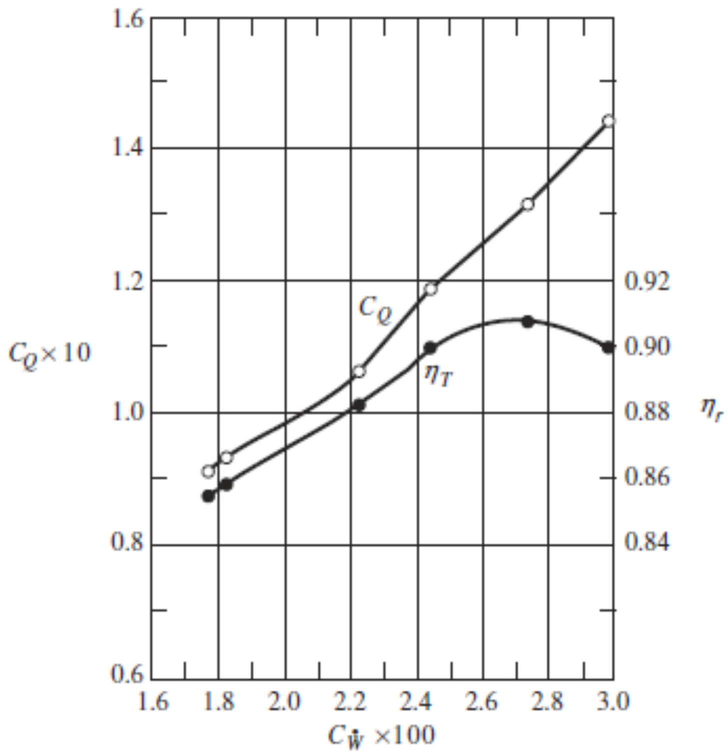
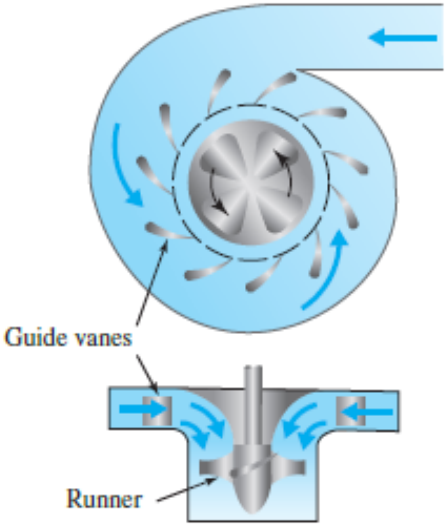
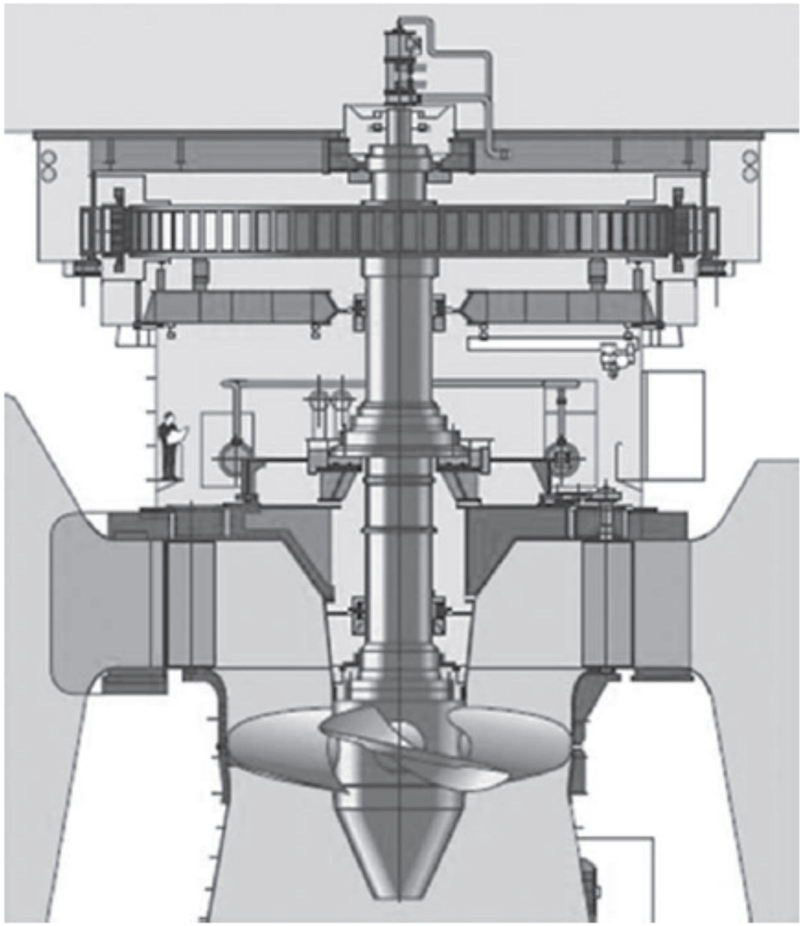


Figure 12.24 Performance curve for a prototype Francis turbine: $D = 1000$ mm, $\omega = 37.7$ rad/s, $\Omega_T = 1.063$, $C_H = 0.23$. (Courtesy of Gilbert Gilkes and Gordon, Ltd.)

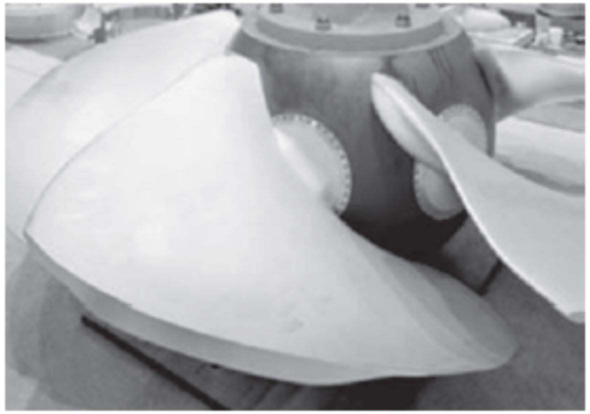
12.5 Turbines



(a)



(b)



(c)

Figure 12.25 Axial-flow turbine

12.5 Turbines

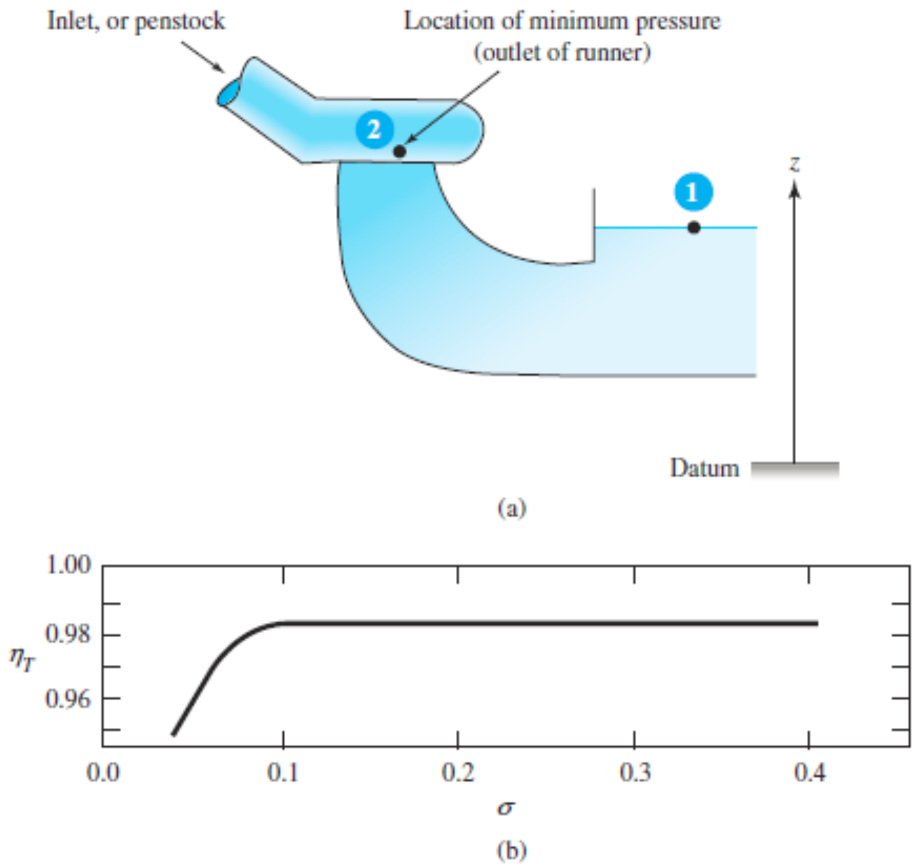
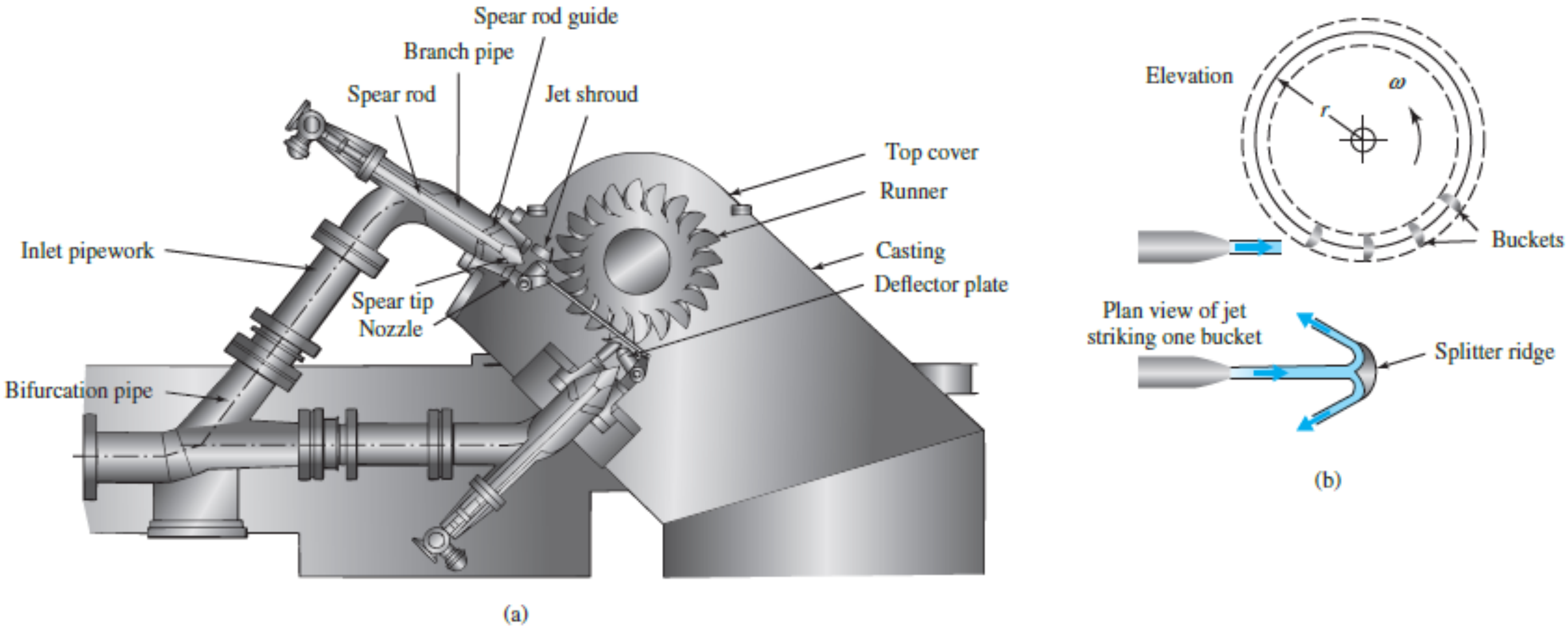


Figure 12.26 Cavitation considerations: (a) schematic; (b) representative cavitation number curve.

12.5 Turbines

Figure 12.27 Impulse turbine (Pelton type)



12.5 Turbines

$$\eta_T = 2\phi(C_v - \phi)(1 - \cos\beta_2)$$

$$\phi = \frac{r\omega}{\sqrt{2gH_T}}$$

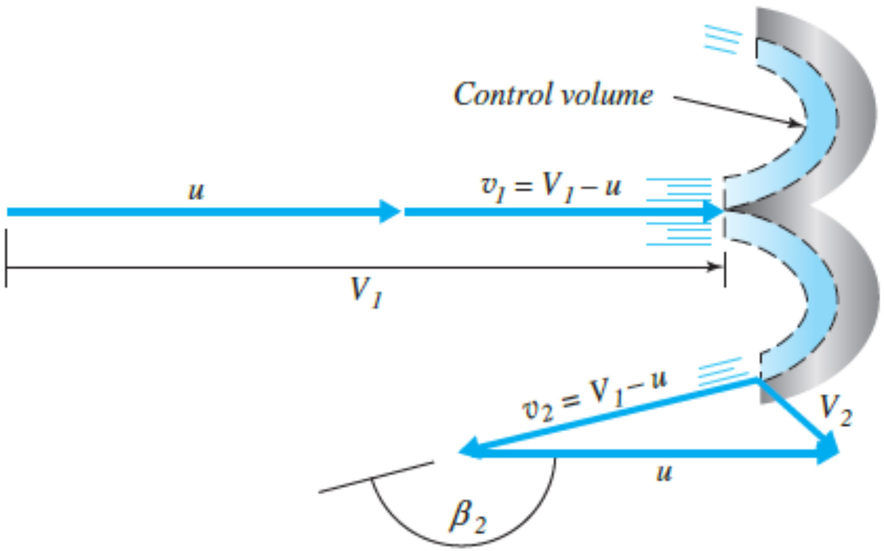


Figure 12.28 Velocity vector diagram for Pelton bucket.

12.5 Turbines

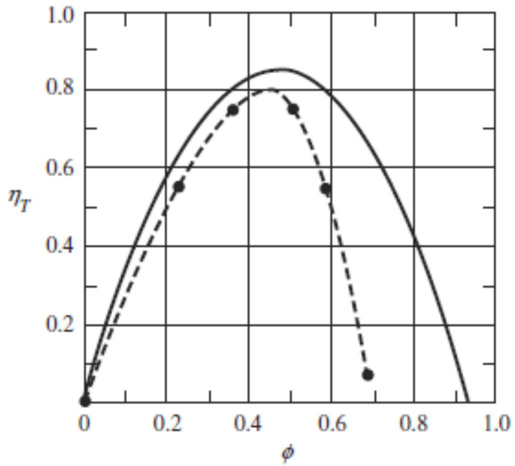


Figure 12.29 Speed factor versus efficiency for a laboratory-scale Pelton turbine: sc
Eq. 12.5.14 ($C_v = 0.94$, $\beta_2 = 168°$); dashed line, experimental data.

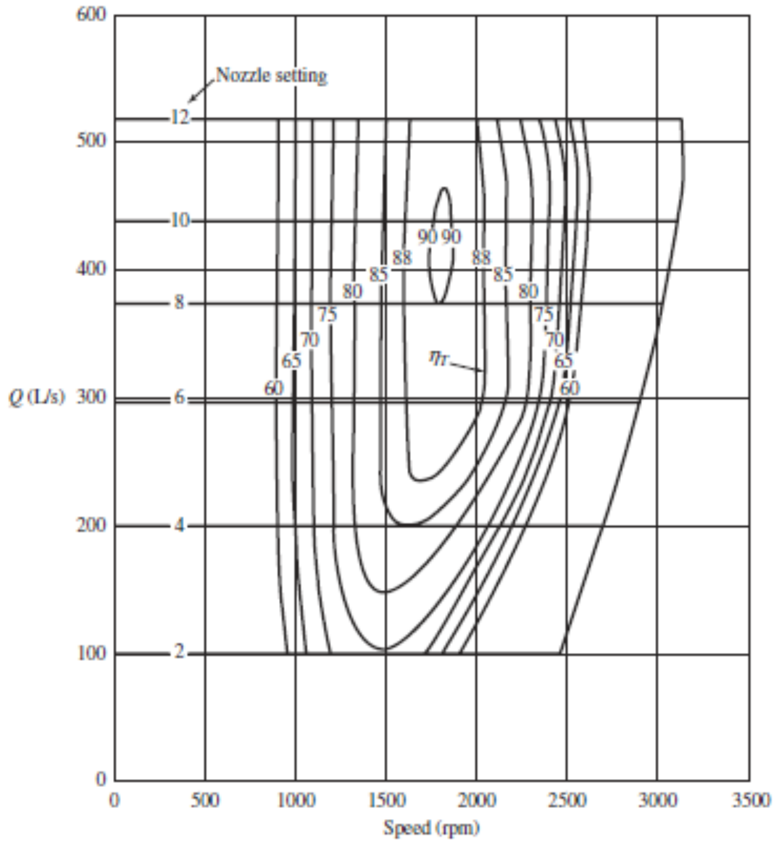


Figure 12.30 Isoefficiency curves for a Pelton turbine: $D = 500$ mm, $H_T = 500$ m.
(Courtesy of Gilbert Gilkes and Gordon, Ltd.)

12.5 Turbines

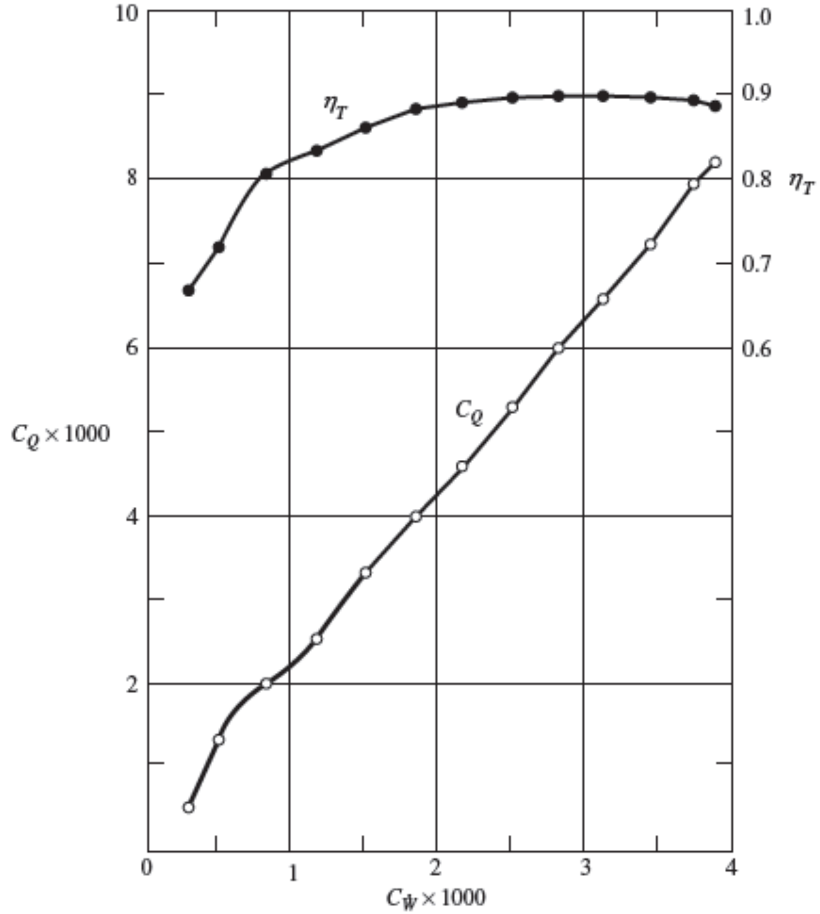


Figure 12.31 Performance and efficiency curves for a prototype Pelton turbine: $D = 1000$ mm, $\omega = 75.4$ rad/s, $\Omega_T = 0.135$, $C_H = 0.52$. (Courtesy of Gilbert Gilkes and Gordon, Ltd.)

12.5 Turbines

A Pelton turbine rotates at an angular speed of 400 rpm, developing 67.5 kW under a head of 60 m of water. The inlet pipe diameter at the base of the single nozzle is 200 mm. The operating conditions are $C_v = 0.97, \phi = 0.46$, and $\eta_T = 0.83$. Determine (a) the volumetric flow rate, (b) the diameter of the jet, (c) the wheel diameter, and (d) the pressure in the inlet pipe at the nozzle base.

Solution

(a) The discharge is computed from Eq. 12.5.13 to be

$$Q = \frac{\dot{W}_T}{\gamma H_T \eta_T} = \frac{67\,500}{9810 \times 60 \times 0.83} = \underline{0.138\ \text{m}^3/\text{s}}$$

(b) From Eq. 12.5.12, the velocity of the jet is

$$V_1 = C_v\sqrt{2gH_T} = 0.97\sqrt{2 \times 9.81 \times 60} = 33.3\ \text{m/s}$$

The area of the jet is the discharge divided by V_1, or

$$A_1 = \frac{Q}{V_1} = \frac{0.138}{33.3} = 4.14 \times 10^{-3}\ \text{m}^2$$

Hence the jet diameter D_1 is

$$D_1 = \sqrt{\frac{4}{\pi}A_1} = \sqrt{\frac{4}{\pi} \times 4.14 \times 10^{-3}} = 0.0726\ \text{m} \quad \text{or} \quad \underline{73\ \text{mm}}$$

(c) Use Eq. 12.5.15 to compute the wheel diameter D to be

$$D = 2r = \frac{2\phi}{\omega}\sqrt{2gH_T}$$

$$= \frac{2 \times 0.46}{400 \times \pi/30}\sqrt{2 \times 9.81 \times 60} = 0.754\ \text{m} \quad \text{or} \quad \underline{754\ \text{mm}}$$

(d) The area of the inlet pipe is

$$A = \frac{\pi}{4} \times 0.20^2 = 0.0314\ \text{m}^2$$

The piezometric head just upstream of the nozzle is equal to H_T, so that the pressure at that location is

$$p = \gamma\left(H_T - \frac{Q^2}{2gA^2}\right)$$

$$= 9810\left(60 - \frac{0.138^2}{2 \times 9.81 \times 0.0314^2}\right)$$

$$= 5.79 \times 10^5\ \text{Pa} \quad \text{or} \quad \text{approximately } \underline{580\ \text{kPa}}$$

12.5 Turbines

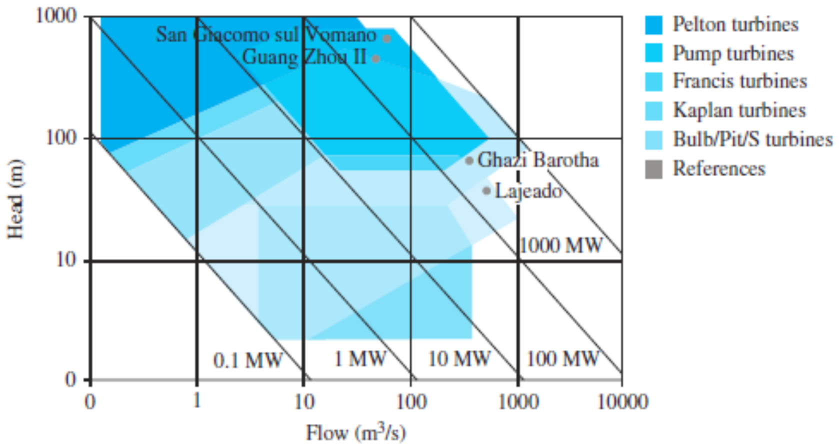


Figure 12.32 Application ranges for hydraulic turbines. (Courtesy of Voith Hydro.)

Figure 12.33 Runner and top cover for pump turbine, Waldeck, Germany. (Courtesy of Voith Hydro.)

12.5 Turbines

A discharge of 2100 m^3/s and a head of 113 m are available for a proposed pumped-storage hydroelectric scheme. Reversible Francis pump/turbines are to be installed; in the turbine mode of operation, $\Omega_T = 2.19$, the rotational speed is 240 rpm, and the efficiency is 80%. Determine the power produced by each unit and the number of units required.

Solution

The power produced by each unit is found using the definition of specific speed, Eq. 12.3.15. Solving for the power, we have

$$\dot{W}_T = \rho\left[\frac{\Omega_T}{\omega}(gH_T)^{5/4}\right]^2$$

$$= 1000\left[\frac{2.19}{240 \times \pi/30}(9.81 \times 113)^{5/4}\right]^2 = \underline{3.11 \times 10^8 \text{ W}}$$

From Eq. 12.5.13, the discharge in each unit is

$$Q = \frac{\dot{W}_T}{\gamma H_T \eta_T}$$

$$= \frac{3.11 \times 10^8}{9800 \times 113 \times 0.8} = 351 \text{ m}^3/\text{s}$$

The required number of units is equal to the available discharge divided by the discharge in each unit, or 2100/351 = 5.98. Hence, *six units* are required.