MECHANICS OF FLUID II
ASSIGNMENT 1
SUBMISSION DATE: NO LATER THAN 27 JUNE 2024
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Q1.

A two-dimensional flow is generated between a fixed and moving vertical plate. The left plate is fixed, and the right plate moves with a velocity U. The distance between the plate is δ. The flow is steady and fully developed, that is no change in x-direction. The fluid is incompressible, and the pressure is constant. Use the Navier-Stokes equations to find the velocity distribution between the plate.

$$\frac{u}{U} = \frac{y}{\delta} + \frac{g\delta^2}{2\upsilon U}\left(1 - \frac{y^2}{\delta^2}\right)$$

Q2.

a) State the difference between rotational and irrotational flow.

b) If a stream function is given by the expression $\psi = 2x^2 - y^3$, prove that the flow is rotational or irrotational. At point P(3, 1), determine the velocity components u and v.

[u = -6 m/s , v = -12 m/s]

Q3.

a) Prove that the circulation Γ can be written as $\Gamma = 2\pi\omega r^2$, where ω is an angular velocity and r is radius.

b) A circulation of 1.0 m^2/s is superimposed on uniform flow about a cylinder having 1 m diameter. The velocity of the uniform water stream is 5 m/s. Determine:
   i) the location of the stagnation points [S(0.5, 1.824°]
   ii) lift force per unit length of cylinder [5000 N/m]
   iii) maximum pressure [max = -40.768 kPa]
   iv) minimum pressure [min = 12.5 kPa]
   v) sketch the stream lines.

Q4.

a) If velocity distribution within the laminar boundary layer on a flat plate is assumed to

$$\frac{u}{U} = \frac{3}{2}\frac{y}{\delta} - \frac{1}{2}\frac{y^3}{\delta^3}$$

Determine:
   i) the boundary layer thickness δ in terms of Reynolds number, and
   ii) the skin friction coefficient C_f in terms of Reynolds number.

b) A smooth flat plate 2 m × 2 m is exposed to a wind velocity of 100 km/hr. If laminar boundary layer exists up to a value of Recritical = 5×10^5, determine:
   i) the maximum distance from the leading edge up to which laminar boundary layer exists,
   ii) the total drag force.

Take density of air is 1.164 kg/m^3 and kinematic viscosity is 1.6×10^{-5} m^2/s.

Q5.

a) Explain briefly the importance of Mach number.

b) Determine the Mach number when an aeroplane is flying at 800 km/hr through still air having pressure of 90 kPa and temperature 25°C.Take k = 1.4 and R = 287 J/kgK. At the stagnation point of the plane, i.e., at the nose of the plane, calculate:

   i) pressure,

   ii) temperature, and

   iii) density.

Q6.

a) Explain on the importance of boundary layer theory. List two (2) solutions in reducing the boundary layer effect in an engineering application.

b) Power law velocity profile of a turbulent and incompressible flow with zero pressure gradient is approximated as

$$\frac{u}{U} = \left(\frac{y}{\delta}\right)^{\frac{1}{9}}$$

Derive the boundary layer thickness, δ and the skin friction coefficient, C_f expression in the function of Reynolds number. Use the empirical relation for the shear stress of

$$\tau_o = 0.0233\rho U^2 \left(\frac{v}{U\delta}\right)^{\frac{1}{4}}$$

where

u =local velocity

U =free stream velocity

y =vertical distance

δ =boundary layer thickness

ρ =density of fluid

v =kinematic viscosity of fluid

c) An airplane wing of 2 m chord length and 11 m span flies at 800 km/hr in air at 30°C (ρ =1.17 kg/m^3, μ =1.86 × 10^{-5} N.s/m^2). Assume that the resistance of the wing surfaces is a flat plate. Determine:

   i) the total drag force on the wing [2.95 kN]

   ii) the power required to overcome the drag [655.55 kW]

Q7.

a) Which two (2) of the following statements are true?
   i) Shock wave only occurs in supersonic flow.
   ii) The Mach number downstream of a normal shock wave can be supersonic.
   iii) The static pressure increases across a normal shock wave.

b) A normal shock wave exits in a Mach 2 stream of air having a static temperature and pressure of 280 K and 200 kPa. By using Table Q7, at downstream of the shock wave, determine:
   i) the Mach number [$M_2 = 0.5774$]
   ii) the pressure [$P_2 = 900$ kPa]
   iii) the temperature [$T_2 = 472.5$ K]

c) State the shape of the nose of a supersonic jet and justify your answer.

Q8. A centrifugal pump spins at 700 rpm and its relative outlet velocity, v_{r2} is 5 m/s. Given that the dimension of the pump as shown in Table Q8.

Table Q8

Parameter	Symbol	Value
Inlet blade width	b_1	40 mm
Outlet blade width	b_2	20 mm
Inlet radius	r_1	30 mm
Outlet radius	r_2	100 mm
Angle of absolute velocity at inlet	α_1	40°
Impeller blade angle at inlet	β_1	50°
Impeller blade angle at outlet	β_2	35°
relative velocity of fluid at outle	v_{r2}	5 m/s

a) Draw the velocity triangles for both the inlet and outlet sections, and ensure they are labelled correctly.

b) Calculate the volume flow rate [$Q = 0.036$ m^3/s]

c) Calculate the theoretical pump head by using Eq. Q8. [$H_t = 2.42$ m]

$$H_t = \frac{\omega^2 r_2^2}{g} - \frac{\omega \cot \beta_2}{2\pi b_2 g} Q \qquad \text{(Eq. Q8)}$$

d) Calculate the absolute inlet velocity, V_1 [$V_1 = 7.42$ m/s]

e) Calculate the pressure rise [$\Delta P = 41.94$ kPa]

Q9.

a) State two (2) main differences between the reaction turbine and the impulse turbine.

b) A Pelton wheel is developed to generate a power of 5.5 MW under the head of 150 m at a speed of 180 rpm. Given that the maximum efficiency of the turbine is 85%, the velocity coefficient and the speed factor are 0.95 and 0.42, respectively.
   i) Sketch the velocity diagram for the wheel's jet, and label all the velocity components
   ii) Calculate the required flow rate [$Q = 4.4$ m^3/s]

c) Refer to Q9(b), if the engineer designed the wheel diameter to be 10 times bigger than the jet diameter, determine:
   i) the wheel diameter [$D_w = 2.42$ m]
   ii) the diameter of each jet [$D_{jet} = 0.242$ m]
   iii) the number of jets required [2 jets required]

Appendix

Von Karman integral equation

$$\tau_0 = \frac{d}{dx}\int_0^{\delta} \rho u (U_\infty - u) dy$$

Momentum thickness

$$\theta = \int_0^{\delta} \frac{u}{U_\infty}\left(1 - \frac{u}{U_\infty}\right) dy$$

Isentropic Flow and Normal Shock Wave

$$\frac{T_0}{P} = 1 + \frac{k-1}{2} M^2$$

$$\frac{P_0}{P} = \left(1 + \frac{k-1}{2} M^2\right)^{\frac{k}{(k-1)}}$$

$$\frac{\rho_0}{\rho} = \left(1 + \frac{k-1}{2} M^2\right)^{\frac{1}{(k-1)}}$$

$$M_2^2 = \frac{M_1^2 + \frac{2}{(k-1)}}{\frac{2k}{k-1} M_1^2 - 1}$$

$$\frac{P_2}{P_1} = \frac{2k}{k+1} M_1^2 - \frac{k-1}{k+1}$$

$$\frac{P_3}{P_2} = \left[\frac{1 + \frac{1}{2}(k-1)M_2^2}{1 + \frac{1}{2}(k-1)M_3^2}\right]^{\frac{k}{k-1}}$$

Turbomachinery

$$Power = \rho g Q H$$

Table Q7

Normal Shock Wave Table

k = 1.4

M_1	M_2	P_2/P_1	ρ_2/ρ_1	T_2/T_1	P_{02}/P_{01}	P_1/P_{02}
1.70	0.6405	3.2050	2.1977	1.4583	0.8557	0.2368
1.71	0.6380	3.2448	2.2141	1.4655	0.8516	0.2343
1.72	0.6355	3.2848	2.2304	1.4727	0.8474	0.2320
1.73	0.6330	3.3251	2.2467	1.4800	0.8431	0.2296
1.74	0.6305	3.3655	2.2629	1.4873	0.8389	0.2273
1.75	0.6281	3.4063	2.2791	1.4946	0.8346	0.2251
1.76	0.6257	3.4472	2.2952	1.5019	0.8302	0.2228
1.77	0.6234	3.4884	2.3113	1.5093	0.8259	0.2206
1.78	0.6210	3.5298	2.3273	1.5167	0.8215	0.2184
1.79	0.6188	3.5715	2.3433	1.5241	0.8171	0.2163
1.80	0.6165	3.6133	2.3592	1.5316	0.8127	0.2142
1.81	0.6143	3.6555	2.3751	1.5391	0.8082	0.2121
1.82	0.6121	3.6978	2.3909	1.5466	0.8038	0.2100
1.83	0.6099	3.7404	2.4067	1.5541	0.7993	0.2080
1.84	0.6078	3.7832	2.4224	1.5617	0.7948	0.2060
1.85	0.6057	3.8263	2.4381	1.5693	0.7902	0.2040
1.86	0.6036	3.8695	2.4537	1.5770	0.7857	0.2020
1.87	0.6016	3.9131	2.4693	1.5847	0.7811	0.2001
1.88	0.5996	3.9568	2.4848	1.5924	0.7765	0.1982
1.89	0.5976	4.0008	2.5003	1.6001	0.7720	0.1963
1.90	0.5956	4.0450	2.5157	1.6079	0.7674	0.1945
1.91	0.5937	4.0895	2.5310	1.6157	0.7627	0.1927
1.92	0.5918	4.1341	2.5463	1.6236	0.7581	0.1909
1.93	0.5899	4.1791	2.5616	1.6314	0.7535	0.1891
1.94	0.5880	4.2242	2.5767	1.6394	0.7488	0.1873
1.95	0.5862	4.2696	2.5919	1.6473	0.7442	0.1856
1.96	0.5844	4.3152	2.6069	1.6553	0.7395	0.1839
1.97	0.5826	4.3611	2.6220	1.6633	0.7349	0.1822
1.98	0.5808	4.4071	2.6369	1.6713	0.7302	0.1806
1.99	0.5791	4.4535	2.6518	1.6794	0.7255	0.1789
2.00	0.5774	4.5000	2.6667	1.6875	0.7209	0.1773
2.01	0.5757	4.5468	2.6815	1.6956	0.7162	0.1757
2.02	0.5740	4.5938	2.6962	1.7038	0.7115	0.1741
2.03	0.5723	4.6411	2.7109	1.7120	0.7069	0.1726
2.04	0.5707	4.6885	2.7255	1.7203	0.7022	0.1710