OPEN CHANNEL

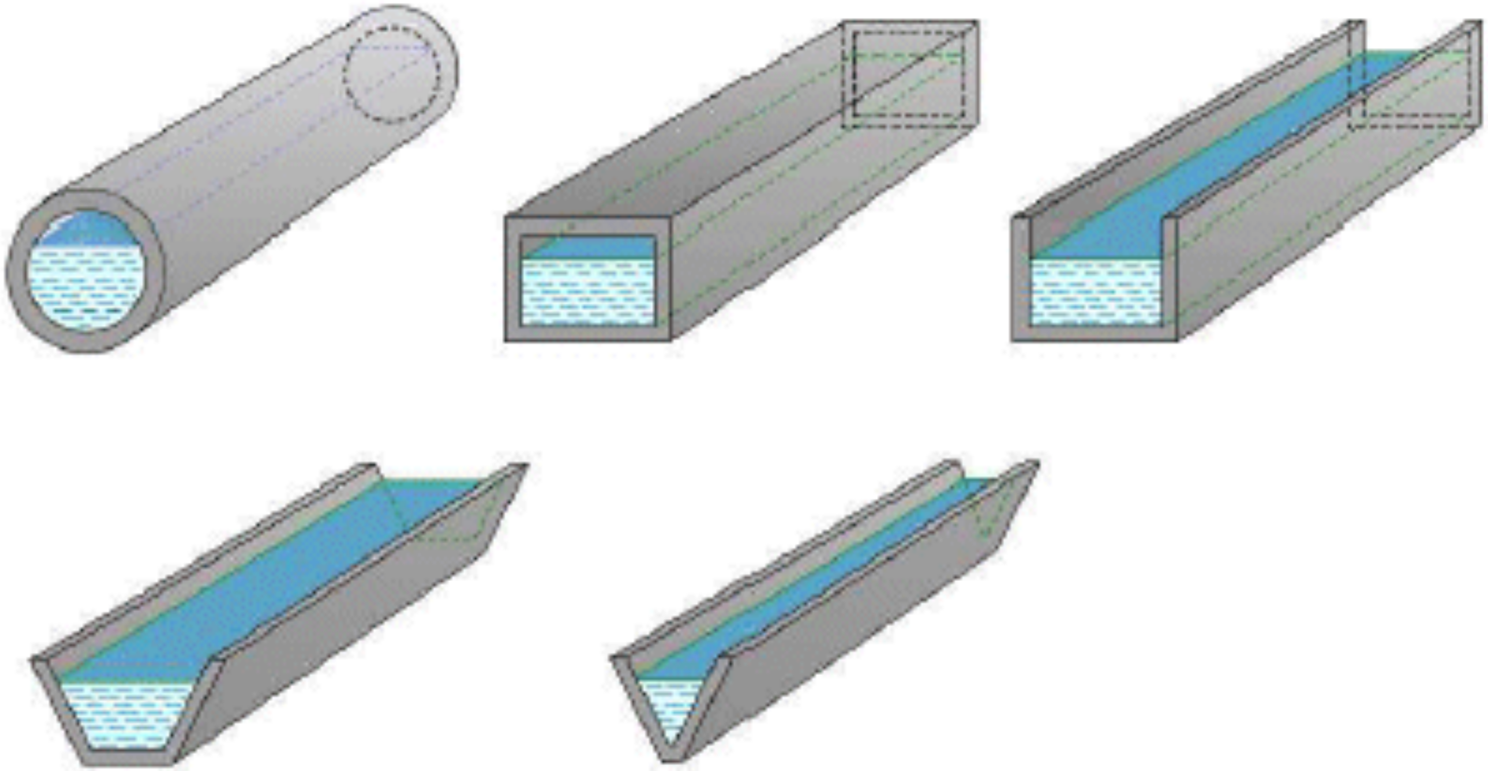
OPEN CHANNEL FLOW

Open channel flow is a flow of liquid, basically water in a conduit with a free surface. The open channel flows are driven by gravity alone, and the pressure gradient at the atmospheric interface is negligible.

Closed duct flows are full of fluid, have no free surface within, and are driven by a pressure gradient along the duct axis.

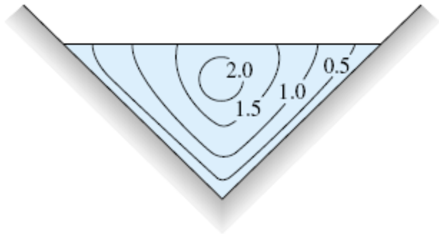
That is a surface on which pressure is equal to local atmospheric pressure.

Open channel flows are characterized by the presence of a liquid-gas interface called the free surface. Some are natural flows such as rivers, creeks and floods, some are human made systems such as fresh water aquaducts, irrigation, sewers and drainage ditches.

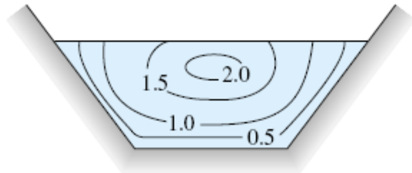


An open channel always has two sides and a bottom, where the flow satisfies the no-slip condition. Therefore even a straight channel has a three-dimensional velocity distribution. Some measurements of straight channel velocity contours are shown below.

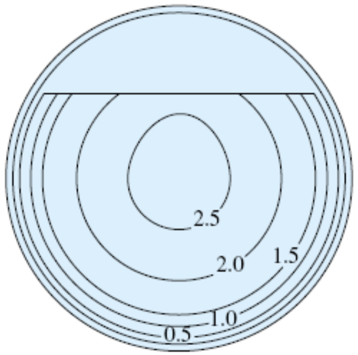
The profiles are quite complex, with maximum velocity typically occurring in the midplane about 20% below the surface.



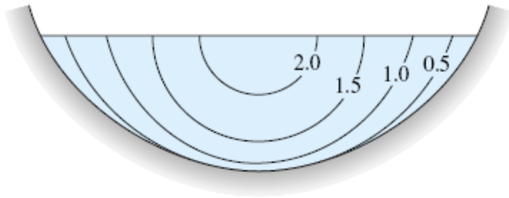
Triangular channel



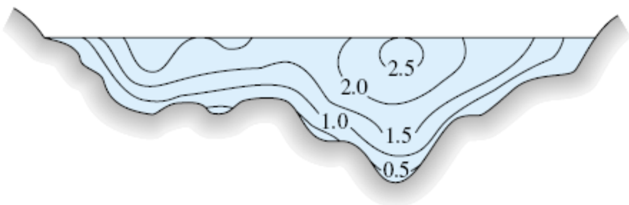
Trapezoidal channel



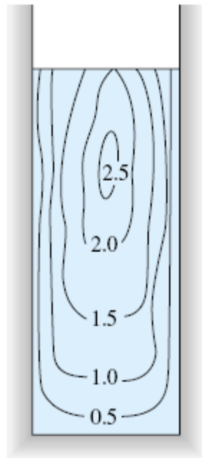
Pipe



Shallow ditch



Natural irregular channel



Narrow rectangular section

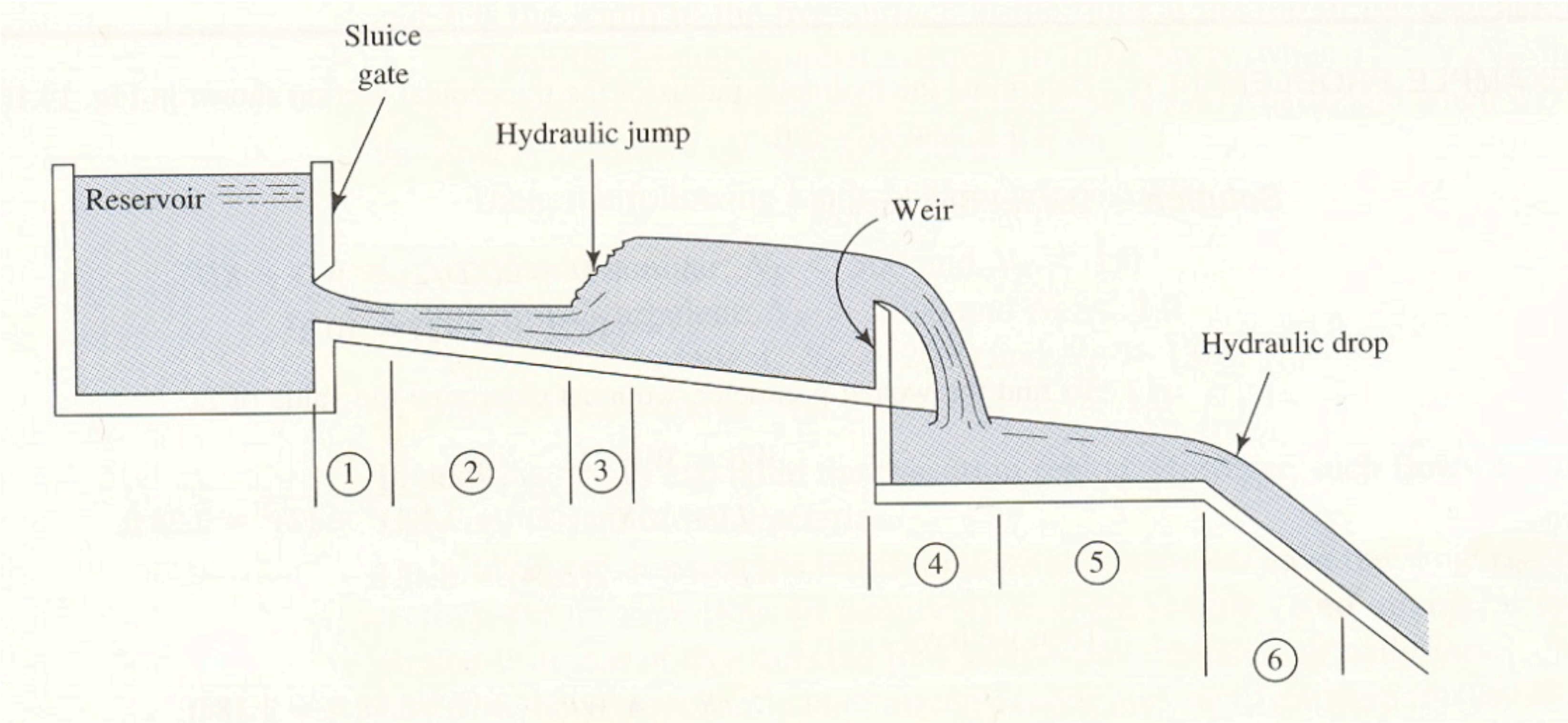
COMPARISON OF OPEN CHANNEL FLOW AND PIPE FLOW

OPEN CHANNEL FLOW	PIPE FLOW
Open channel flow must have a free surface.	No free surface in pipe flow.
A free surface is subject to atmospheric pressure.	No direct atmospheric pressure, hydraulic pressure only.
Flow area is determined by the geometry of the channel plus the level of free surface, which is likely to change along the flow direction and with as well as time.	Flow area is fixed by the pipe dimensions. The cross section of a pipe is usually circular.
The cross section may be of any from circular to irregular form of natural streams, which may change along the flow direction and as well as with time.	The cross section of a pipe is usually circular.
The depth of flow, discharge and the slopes of channel bottom and of the free surface are interdependent.	No such dependence.

TYPES OF OPEN CHANNEL FLOWS

The most common method of classifying open channel flow is by the rate of change of the free surface depth. The simplest and most widely analyzed case in uniform flow.

Type of flow	Description
Steady flow	When discharge (Q) does not change with time.
Uniform flow	When depth of fluid does not change for a selected length or section of the channel.
Uniform steady flow	When discharge does not change with time and depth remains constant for a selected section. Cross section should remain unchanged, referred to as a prismatic channel.
Varied steady flow	When depth changes but discharge remains the same.
Varied unsteady flow	When both depth and discharge change along a channel length of interest.
Rapidly varying flow	Depth change is rapid.
Gradually varying flow	Depth change is gradual.



1. Rapidly varying flow
2. Gradually varying flow
3. Hydraulic jump
4. Weir and waterfall
5. Gradually varying
6. Hydraulic drop due to change in channel slope

PARAMETER USED IN OPEN CHANNEL FLOW ANALYSIS

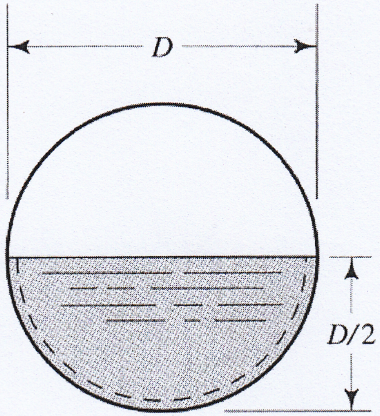
Hydraulic radius, R_h of open channel flow

R_h is a ratio of flow cross sectional area, $\underline{A}$ and wetted perimeter (WP)

$$R_h = \frac{A}{WP}$$

$$D_h = 4R_h$$

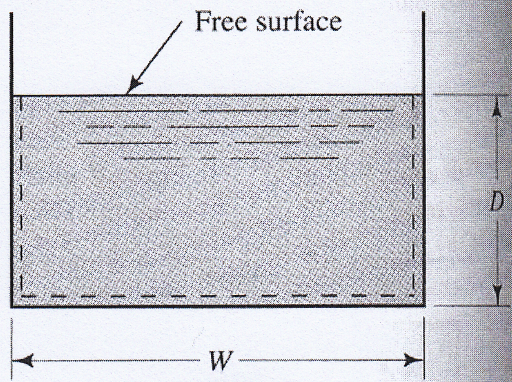
R_h	:	Hydraulic radius
D_h	:	Hydraulic diameter
A	:	Flow cross sectional area
WP	:	Wetted perimeter



$A = \pi D^2/8$

$WP = \pi D/2$

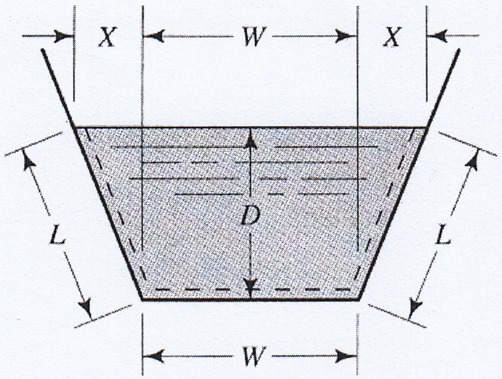
(a) Circular pipe
running half full



$A = WD$

$WP = W + 2D$

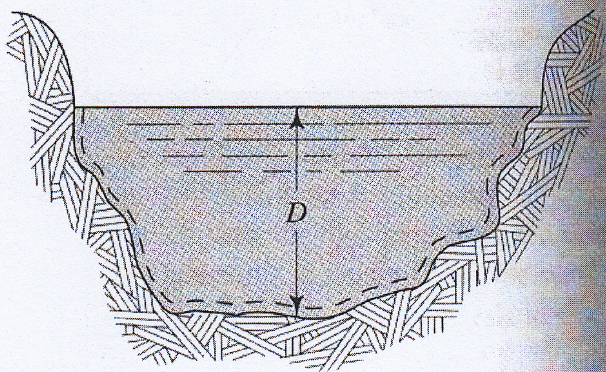
(b) Rectangular
channel



$A = WD + XD$

$WP = W + 2L$

(c) Trapezoidal
channel



A and WP irregular

(d) Natural channel

REYNOLDS NUMBER FOR OPEN CHANNEL FLOW.

$$Re = \frac{\rho V R_h}{\mu} = \frac{V R_h}{\nu}$$

$Re < 500$ – laminar flow

$Re > 2000$ – turbulent flow

Reynolds number for pipe flow

$$Re = \frac{\rho V D}{\mu} = \frac{V D}{\upsilon}$$

$Re < 2000$ – laminar flow

$Re > 4000$ – turbulent flow

Froude number, *Fr*

The Froude number (*Fr*) is a dimensionless number defined as the ratio of channel velocity to the speed of propagation of a small disturbance wave in the channel.

For a rectangular or very wide constant depth channel, Froude number can be defined as :

$$Fr = \frac{\text{Flow velocity}}{\text{Surface wave speed}} = \frac{V}{\sqrt{g y_h}}$$

V = Velocity
g = gravity
y_h = Hydraulic depth

$$y_h = \frac{A}{T}$$

A = Area
T = Top width of the channel

$\mathrm{Fr} < 1.0$ → Sub-critical flow

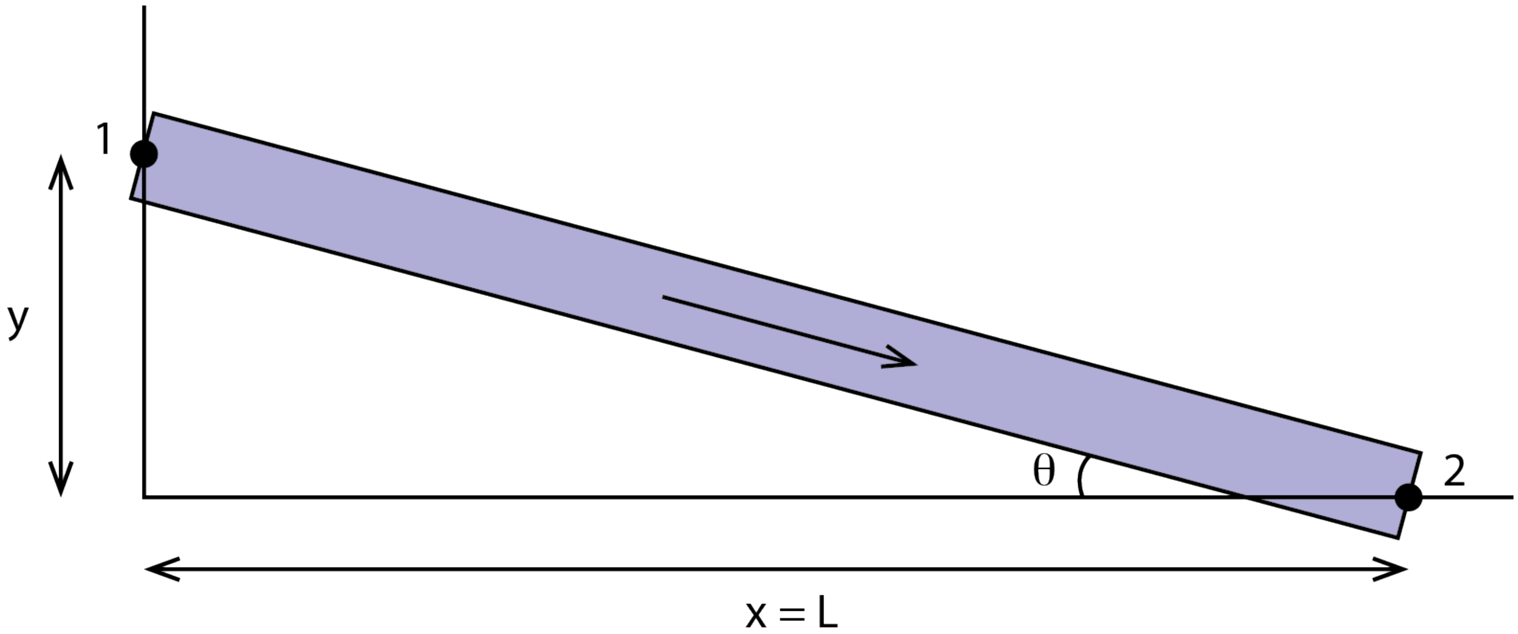
$\mathrm{Fr} = 1.0$ or when $V = \sqrt{g y_h}$ → Critical flow

$\mathrm{Fr} > 1.0$ → Super-critical flow

A combination of both numbers is used to describe channel flow conditions.

THE CHEZY FORMULA

Uniform flow can occur in long straight runs of constant slope and constant channel cross section. The water depth is constant at $y = y_0$, and the velocity is constant at $V = V_0$. The slope be $S_0 = \tan\theta$, where θ is the angle the bottom makes with the horizontal, considered positive for downhill flow. From Bernoulli equation, the head loss becomes:



From Bernoulli equation:

$$0 + 0 + z_1 = 0 + 0 + z_2 + h_f$$

$$h_f = z_1 - z_2 = y = (\tan\theta)x = S_0 L$$

where L is the horizontal distance between section 1 and 2.

Head loss from Darcy-Weisbach is:

$$h_f = f\frac{L}{D}\frac{V^2}{2g}$$

$$V_0 = \left(\frac{8g}{f}\right)^{\frac{1}{2}} \cdot R_h{}^{\frac{1}{2}} \cdot S_0{}^{\frac{1}{2}}$$

For a given shape and bottom roughness, the quantity $\left(\frac{8g}{f}\right)^{\frac{1}{2}}$ is constant and can be denoted by C.

$$C = \left(\frac{8g}{f}\right)^{\frac{1}{2}}$$

Finally, the velocity V_0 can be expressed as :

$$V_0 = \left(\frac{8g}{f}\right)^{\frac{1}{2}} \cdot R_h{}^{\frac{1}{2}} \cdot S_0{}^{\frac{1}{2}} = C \cdot (R_h S_0)^{\frac{1}{2}}$$

$$Q = AV = A \cdot C \cdot (R_h S_0)^{\frac{1}{2}}$$

These are called Chezy formula, first developed by the French engineer Antoine Chezy in conjuction with his experiments on the Saine River and the Courpalet Canal in 1769.

The quantity C, called the Chezy coefficient, varies about 60 $ft^{1/2}/s$ for small rough channels to 160 $ft^{1/2}/s$ for large smooth channels (30 $m^{1/2}/s$ to 90 $m^{1/2}/s$ in SI units).

EXAMPLE 1

A straight rectangular channel is 6 ft wide and 3 ft deep and laid on a slope of 2°. The friction factor is 0.022.

SOLUTION 1

$$C = \sqrt{\frac{8g}{f}} = \sqrt{\frac{(8)(32.2)}{0.022}} = 108 \ \ (ft^{\frac{1}{2}}s)$$

$$R_h = \frac{A}{WP} = \frac{18}{3+6+3} = 1.5 \ \ \ (ft)$$

$$S_0 = \tan 2°$$

$$Q = A \cdot C \cdot (R_h S_0)^{\frac{1}{2}} = (18)(108)(1.5 \times \tan 2°)^{\frac{1}{2}} = 450 \ \ (ft^3/s)$$

For SI units, it can be used directly.

Uniform steady flow and Manning's equation

When discharge remain the same and depth does not change, then we have uniform steady flow.

In this condition, the surface of water is parallel to the bed of the channel. To make sure water (liquid) flow inside the channel, it must have certain angle of inclination, or the channel's slope.

The slope of the channel (S) can be expressed as :-

- An angle = 1 degree
- As percent = 1%
- As fraction = 0.01 or 1 in 100

Manning's equation is used to estimate the velocity of flow in a channel.

The SI units form of Manning's equation:

$$V = \frac{1.0}{n} \cdot {R_h}^{\frac{2}{3}} \cdot S^{\frac{1}{2}}$$

V = Velocity of flow in a channel (m/s)
n = Channel surface roughness. Values developed through experimentation.
R = Hydraulic radius (A/WP) in meter
S = Slope of the channel

The English units form of Manning's equation:

$$V = \frac{1.49}{n} \cdot {R_h}^{\frac{2}{3}} \cdot S^{\frac{1}{2}}$$

V = Velocity of flow in a channel (ft/s)
R = Hydraulic radius (A/WP) in feet

Example of the n values:-

Channel Description	n
Glass, copper, plastic, or other smooth surfaces	0.010
Smooth, unpainted steel, planed wood	0.012
Painted steel or coated cast iron	0.013
Smooth asphalt, common clay drainage tile, trowel-finished concrete, glazed brick	0.013
Uncoated cast iron, black wrought iron pipe, vitrified clay sewer tile	0.014
Brick in cement mortar, float-finished concrete, concrete pipe	0.015
Formed, unfinished concrete, spiral steel pipe	0.017
Smooth earth	0.018
Clean excavated earth	0.022
Corrugated metal storm drain	0.024
Natural channel with stones and weeds	0.030
Natural channel with light brush	0.050
Natural channel with tall grasses and reeds	0.060
Natural channel with heavy brush	0.100

The flowrate of a channel could be determined by :

$$Q = AV = \frac{1.0}{n} \cdot A{R_h}^{\frac{2}{3}} \cdot S^{\frac{1}{2}}$$

where Q is in m^3/s.

For uniform flow, Q is referred to as <u>normal discharge.</u>

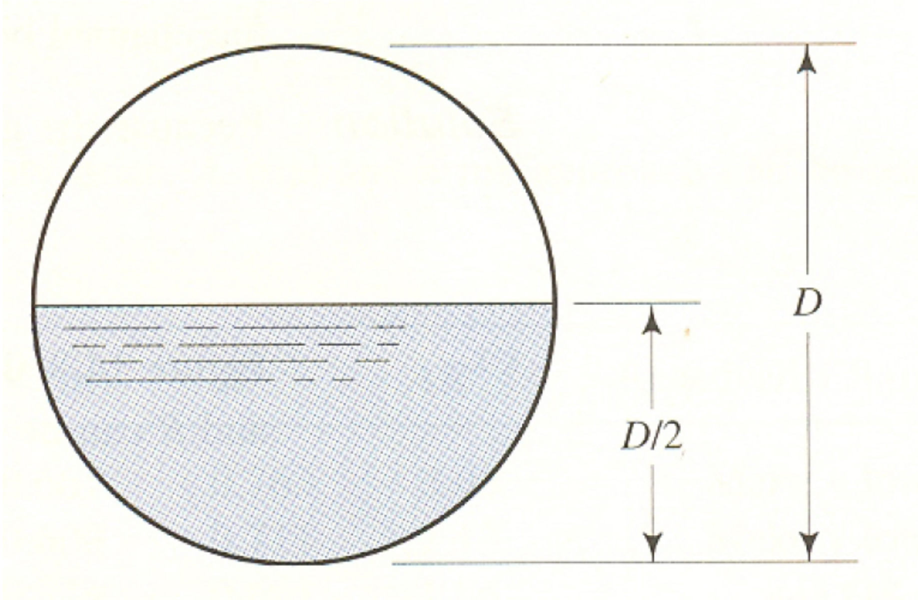
The above equation can also be re-arranged such that :

$$A{R_h}^{\frac{2}{3}} = \frac{nQ}{S^{\frac{1}{2}}}$$

The left-hand side equation is based on channel geometry.
The right-hand side equation is based on flow properties.

Example #01

Determine normal discharge for a 200 mm inside diameter common clay drainage tile running half-full if the slope drops 1m over 1000m.



$$Q = \frac{1.0}{n} \cdot A R_h^{\frac{2}{3}} \cdot S^{\frac{1}{2}}$$

$$n = 0.013$$

$$A = \frac{1}{2} \times \frac{\pi D^2}{4} = 0.0157 \ \ m^2$$

$$WP = \frac{1}{2} \times \pi D = 0.3142 \ \ m$$

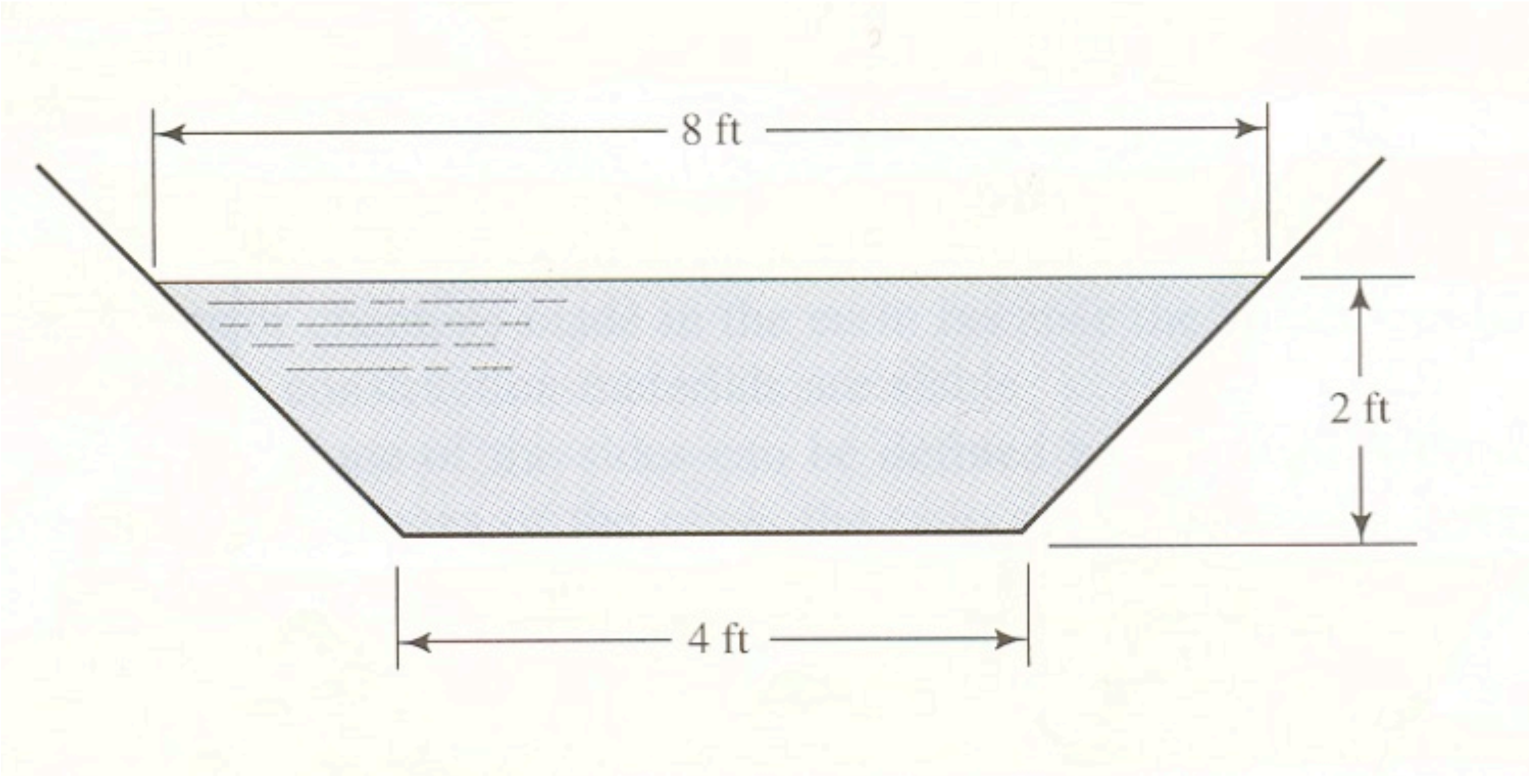
$$R_h = \frac{A}{WP} = \frac{0.0157}{0.3142} = 0.05 \ \ m$$

$$S = 0.001$$

$$Q = \frac{1.0}{n} \cdot A R_h^{\frac{2}{3}} \cdot S^{\frac{1}{2}} = 5.18 \times 10^{-3} \ \ m^3/s$$

Example #02

Calculate slope of channel if normal discharge is 50 ft^3/s. Channel is formed, unfinished concrete.



English unit !!!

$$Q = \frac{1.0}{n} \cdot AR_h^{\frac{2}{3}} \cdot S^{\frac{1}{2}} =$$

$$Q = \frac{1.49}{n} \cdot AR^{\frac{2}{3}} \cdot S^{\frac{1}{2}} \quad \rightarrow \quad S^{\frac{1}{2}} = \frac{Qn}{1.49AR^{\frac{2}{3}}}$$

$A = 12\ \text{ft}^2$

$WP = 9.66\ \text{ft}$

$\mathrm{R} = \frac{\mathrm{A}}{\mathrm{WP}} = 1.24\ \text{ft}$

$n = 0.017$

$S = 0.00169$

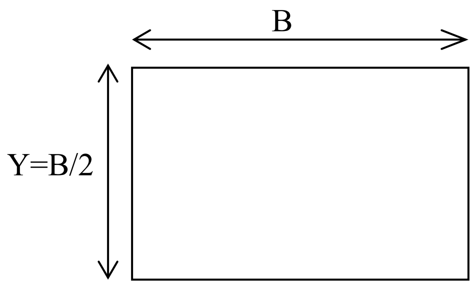
(Channel should drop 1.69 feet for every 1000 feet length.

Example #03

Design a rectangular channel in formed, unfinished concrete with below mention specifications.
Normal flowrate = 5.75 m^3/s
S = 1.2%
Normal depth = half of the width of the channel.

Since we have to design the channel, use this equation.

$$AR^{\frac{2}{3}} = \frac{nQ}{S^{\frac{1}{2}}}$$



$$AR^{\frac{2}{3}} = \frac{nQ}{S^{\frac{1}{2}}} = \frac{0.017 \times 5.75}{(0.012)^{\frac{1}{2}}} = 0.892$$

$$A = BY = B \times \frac{B}{2} = \frac{B^2}{2}$$

$$WP = B + 2Y = 2B$$

$$R = \frac{A}{WP} = \frac{B}{4}$$

$$AR^{\frac{2}{3}} = \left(\frac{B^2}{2}\right)\left(\frac{B}{4}\right)^{\frac{2}{3}} = 0.892$$

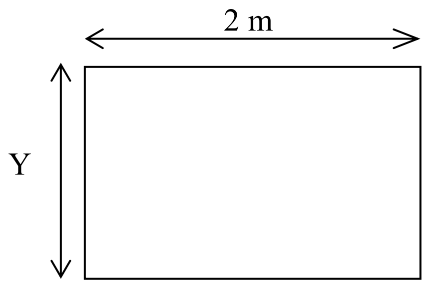
$$B = 1.76 \text{ m}$$

$$Y = 0.88 \text{ m}$$

Example #04

In a rectangular channel as mention in Example #03, the final width was set at 2m and the maximum discharge is 12m³/s. Find the normal depth for this maximum discharge.

$$AR^{\frac{2}{3}} = \frac{nQ}{S^{\frac{1}{2}}}$$



$$AR^{\frac{2}{3}} = \frac{nQ}{S^{\frac{1}{2}}} = \frac{0.017 \times 12}{(0.012)^{\frac{1}{2}}} = 1.862257$$

$$A = 2Y$$

$$WP = 2 + 2Y$$

$$R = \frac{A}{WP} = \frac{2Y}{2+2Y}$$

$$AR^{\frac{2}{3}} = (2Y)\left(\frac{2Y}{2+2Y}\right)^{\frac{2}{3}} = 1.862257$$

Cannot solve directly. Use trial and error method. Can use MS Excel datasheet.

From Excel,

The normal depth must be 1.348 meter.

From MS Excel worksheet:

	A	B	C	D	E
1	Y	(2Y)*(2Y/2+2y)^2/3		Y	(2Y)*(2Y/2+2y)^2/3
2	1	1.25992105		1.341	1.849880536
3	1.01	1.276737399		1.342	1.851652965
4	1.02	1.293588171		1.343	1.853425602
5	1.03	1.310472801		1.344	1.855198447
6	1.04	1.327390738		1.345	1.856971499
7	1.05	1.344341442		1.346	1.858744759
8	1.06	1.361324386		1.347	1.860518226
9	1.07	1.378339053		1.348	1.8622919
10	1.08	1.39538494		1.349	1.86406578
11	1.09	1.412461552		1.35	1.865839867
12	1.1	1.429568405		1.351	1.86761416
13	1.11	1.446705026		1.352	1.869388658
14	1.12	1.463870951		1.353	1.871163362
15	1.13	1.481065728		1.354	1.872938271
16	1.14	1.498288912		1.355	1.874713384
17	1.15	1.515540069		1.356	1.876488703
18	1.16	1.532818771		1.357	1.878264225
19	1.17	1.550124603		1.358	1.880039952
20	1.18	1.567457156		1.359	1.881815882
21	1.19	1.584816028		1.36	1.883592016
22	1.2	1.602200828		1.361	1.885368353
23	1.21	1.619611172		1.362	1.887144893
24	1.22	1.637046682		1.363	1.888921635
25	1.23	1.654506989		1.364	1.89069858
26	1.24	1.671991731		1.365	1.892475727
27	1.25	1.689500554		1.366	1.894253076
28	1.26	1.70703311		1.367	1.896030626
29	1.27	1.724589056		1.368	1.897808377
30	1.28	1.742168059		1.369	1.899586329
31	1.29	1.75976979		1.37	1.901364482
32	1.3	1.777393928		1.371	1.903142835
33	1.31	1.795040156		1.372	1.904921389
34	1.32	1.812708166		1.373	1.906700142
35	1.33	1.830397652		1.374	1.908479095
36	1.34	1.848108316		1.375	1.910258247
37	1.35	1.865839867		1.376	1.912037598
38	1.36	1.883592016		1.377	1.913817148
39	1.37	1.901364482		1.378	1.915596897
40	1.38	1.919156988		1.379	1.917376843
41	1.39	1.936969261		1.38	1.919156988
42	1.4	1.954801037		1.381	1.92093733
43	1.41	1.972652051		1.382	1.92271787
44	1.42	1.990522048		1.383	1.924498606
45					

CONVEYANCE AND MOST EFFICIENT CHANNEL SHAPES

$$Q = \frac{1.0}{n} \cdot AR^{\frac{2}{3}} \cdot S^{\frac{1}{2}}$$

Other than the S term, all other terms are related to channel cross section and its features.

These terms together are referred to as the Conveyance (K) of the channel.

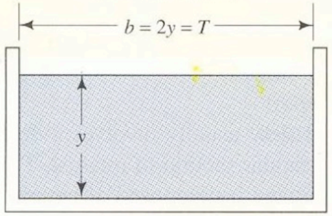
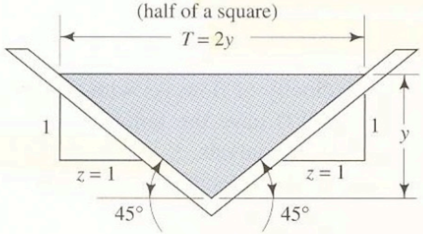
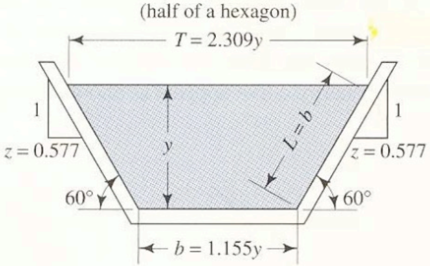
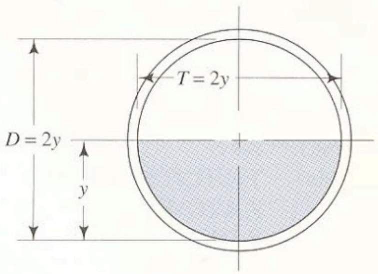
$$K = \frac{1.0}{n} \cdot AR^{\frac{2}{3}}$$

Then,

$$Q = K \cdot S^{\frac{1}{2}}$$

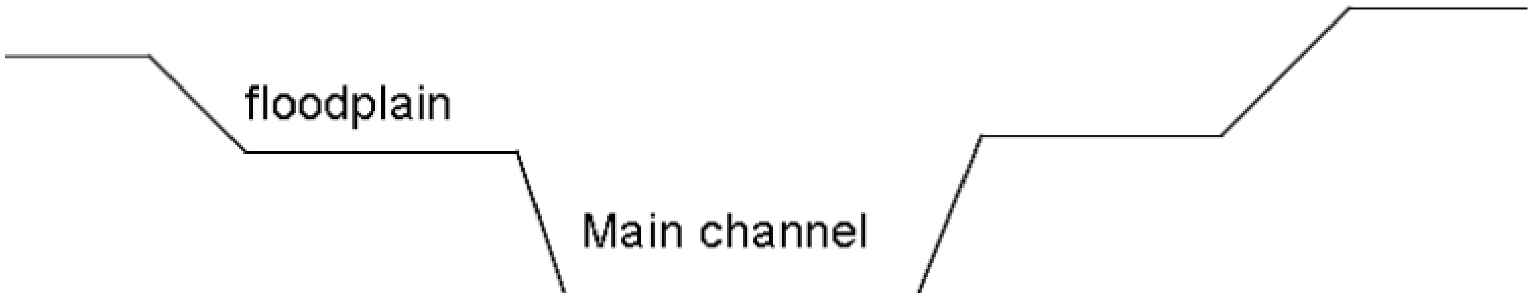
$$R = \frac{A}{WP}$$

K is maximum when wetted perimeter (WP) is the least for a given area. This is also the most efficient cross section for conveying flow.
For circular section, half full flow is the most efficient.

Section	Area A	Wetted Perimeter WP	Hydraulic Radius R
Rectangle (half of a square) 	$2.0y^2$	$4y$	$y/2$
Triangle (half of a square) 	y^2	$2.83y$	$0.354y$
Trapezoid (half of a hexagon) 	$1.73y^2$	$3.46y$	$y/2$
Semicircle 	$\frac{1}{2}\pi y^2$	πy	$y/2$

COMPOUND SECTIONS

It is occurs when channel shape changes with flow depth. It is a typical idea in natural stream sections during flooding.



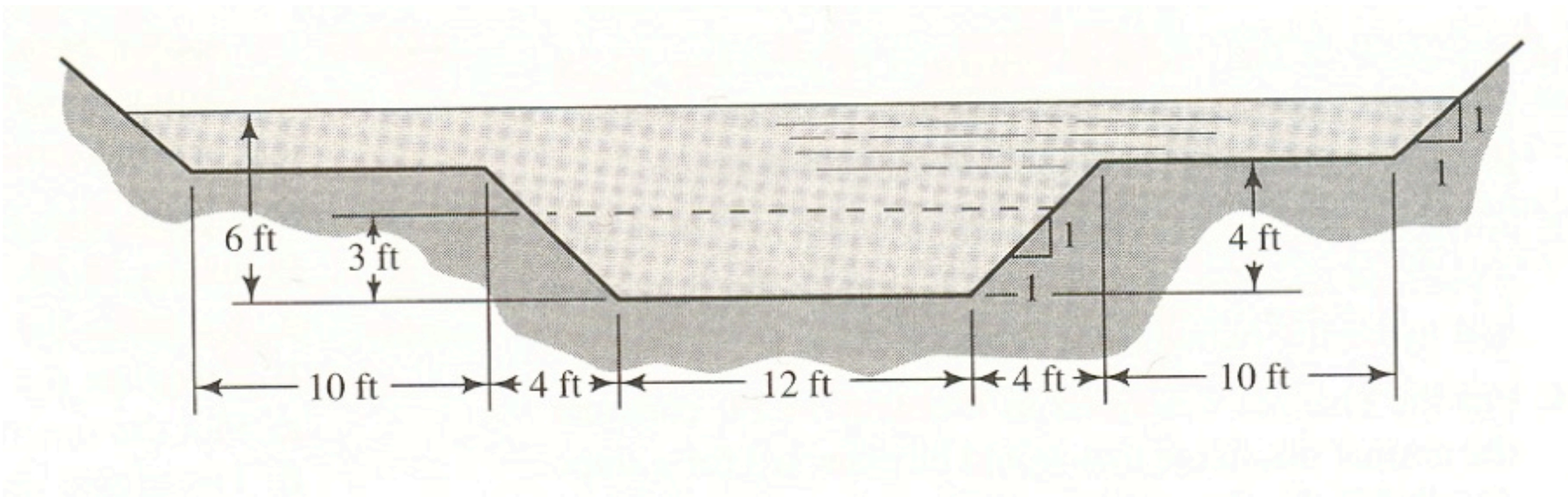
At normal condition, water flows in the main channel. During floods, water spills over the flood plain.

We need to know the flowrate, Q at various depths or vice-versa. So that we could design channels or determine channel safety for various flood magnitudes.

Example #05

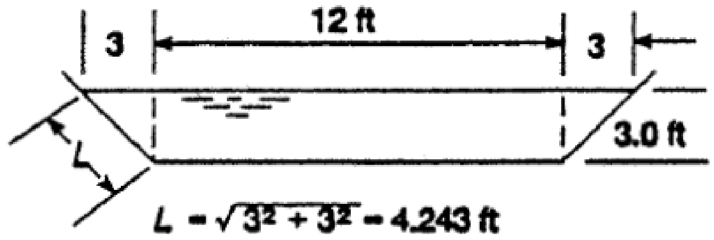
Channel type: Natural channel with levees.
Slope: 0.00015

Determine the normal discharge, Q for depth 3 ft and 6 ft.



a. **Depth = 3.0 ft:**

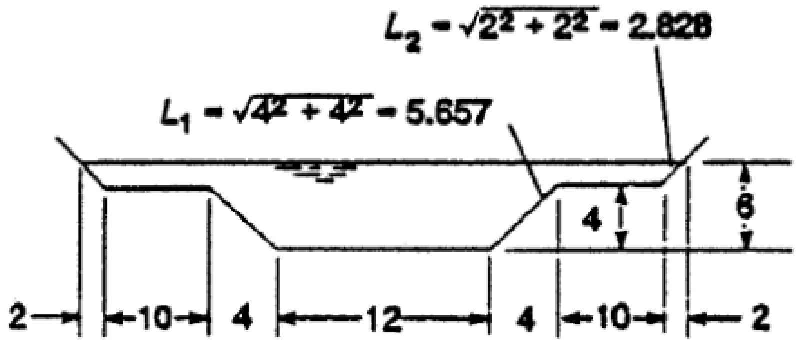
$$A = (3)(12) + 2\left[\frac{1}{2}(3)(3)\right] = 45 \text{ ft}^2$$



$$WP = 12 + 2(4.243) = 20.485 \text{ ft}$$
$$R = A/WP = 45/20.485 = 2.197 \text{ ft}$$
$$Q = \frac{1.49}{0.04}(45)(2.197)^{2/3}(0.00015)^{1/2} = \mathbf{34.7\ ft^3/s}$$

b. **Depth = 6.0 ft:**

$$A = (4)(12) + 2\left[\frac{1}{2}(4)(4)\right]$$
$$+ (2)(40) + 2\left[\frac{1}{2}(2)(2)\right]$$



$$A = 148 \text{ ft}^2$$
$$WP = 2(2.828) + 2(10) + 2(5.657)$$
$$+ 12 = 48.97 \text{ ft}$$
$$R = A/WP = 148/48.97 = 3.022 \text{ ft}$$
$$Q = \frac{1.49}{0.04}(148)(3.022)^{2/3}(0.00015)^{1/2} = \mathbf{141.1\ ft^3/s}$$

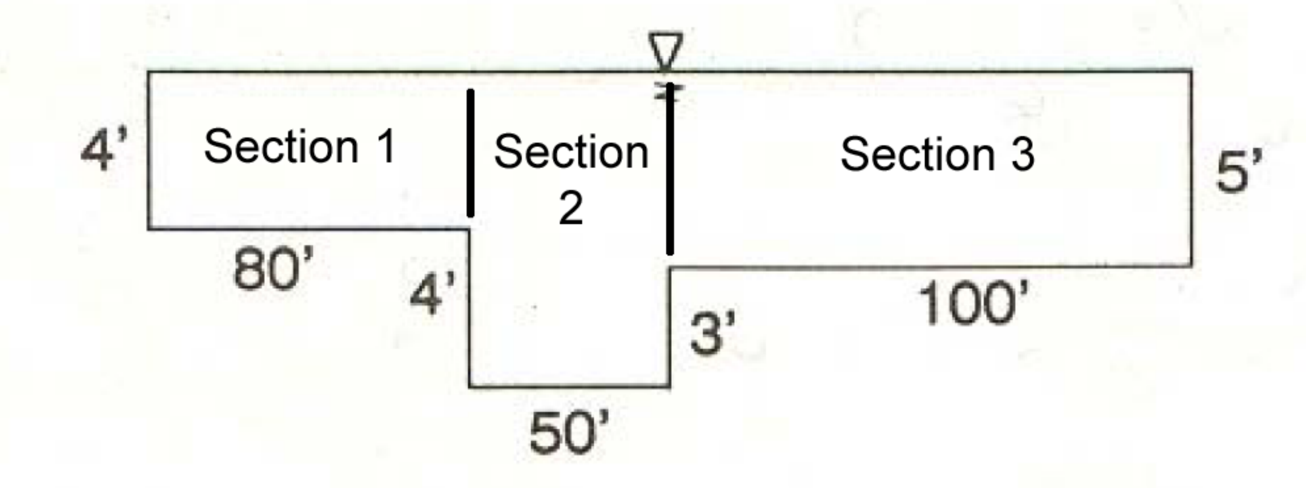
COMPOUND SECTION

This is more realistic situation, where the channel roughness (value of n) may be different for floodplain than the main channel.

In this case, we need to determine velocity for each sub-section, and then sum up the discharges for the sections.

Example #06

Slope: 0.5%
n for bank = 0.06
n for main channel = 0.03
Calculate discharge for depth of 8 feet?



$$A_1 = 80 \times 4 = 320 \quad , \quad A_2 = 50 \times 8 = 400 \quad , \quad A_3 = 100 \times 5 = 500$$

$$WP_1 = 80 + 4 = 84 \quad , \quad WP_2 = 4 + 50 + 3 = 57 \quad , \quad WP_3 = 100 + 5 = 105$$

$$V_i = \frac{1.49}{n_i}\left(\frac{A_i}{WP_i}\right)^{\frac{2}{3}} S^{\frac{1}{2}}$$

$$Q = \sum_{i=1}^{n} A_i V_i = (1.49)(0.005)^{\frac{1}{2}}\left[\frac{(320/84)^{\frac{2}{3}}(320)}{0.06} + \frac{(400/57)^{\frac{2}{3}}(400)}{0.03} + \frac{(500/57)^{\frac{2}{3}}(400)}{0.06}\right] = 9010 \text{ ft}^2/\text{s}$$

ENERGY PRINCIPLES FOR OPEN CHANNEL FLOW

Energy at particular point in the channel is potential energy and kinetic energy.

Specific energy:

$$E = y + \frac{V^2}{2g} = y + \frac{Q^2}{2gA^2}$$

$y =$ Depth of flow
$V =$ Velocity
$Q =$ Discharge
$A =$ Cross sectional flow area

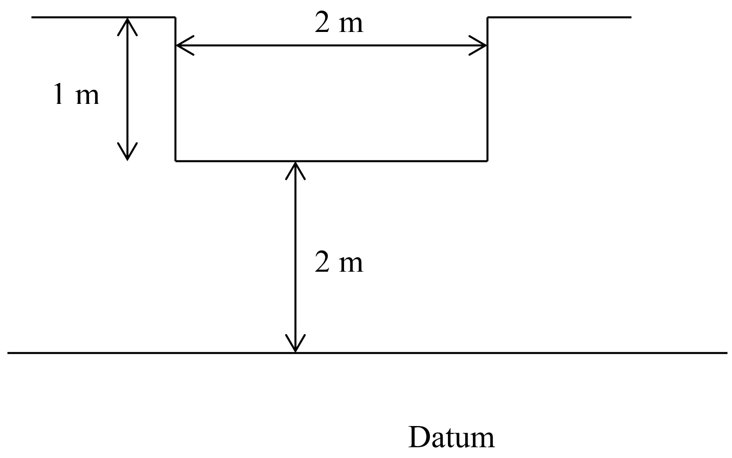
Total energy:

$$E = y + z + \frac{V^2}{2g} = y + z + \frac{Q^2}{2gA^2}$$

$z =$ Height of the channel bottom from the datum

Example:

Rectangular channel width = 2 m
Depth = 1 m
Q = 4.0 m3/s
Height above datum = 2 m



Determine the specific energy and total energy.

Specific energy:

$$E = y + \frac{V^2}{2g} = 1 + \frac{4^2}{(2)(9.81)(2)^2} = 1.20 \text{ m}$$

Total energy:

E = datum height + specific energy = 2.0 + 1.2 = 3.2 m

Specific energy diagram

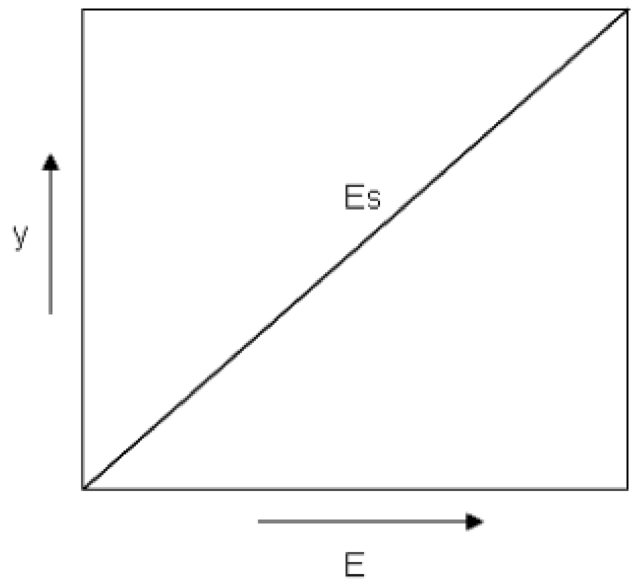
The specific energy can be plotted graphically as a function of depth of flow.

$$E = y + \frac{V^2}{2g} = y + \frac{Q^2}{2gA^2}$$

y = Static energy, E_s (potential energy)

$\frac{Q^2}{2gA^2}$ = Kinetic energy, E_k

Relationship between y and static energy, E_s



Relationship between y and kinetic energy, E_k

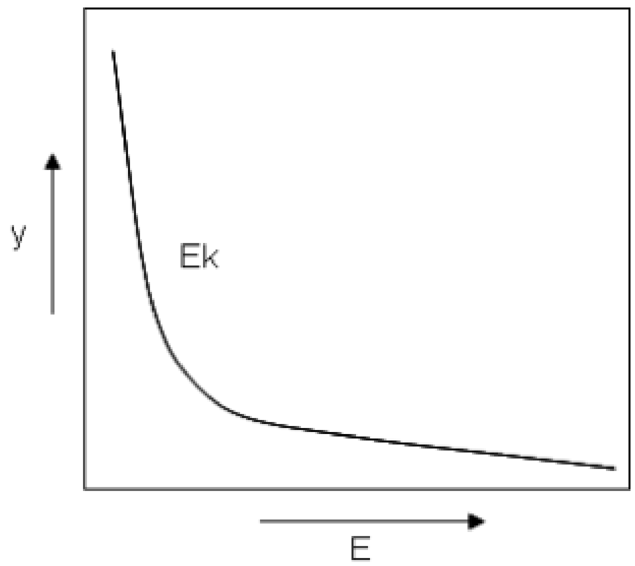
$$E_k = \frac{Q^2}{2gA^2}$$

For a rectangular channel;

Substitute Q with specific discharge (discharge per unit width), $q = \frac{Q}{B}$

Substitute area, A with, $A = B \cdot y$

$$E_k = \frac{Q^2}{2gA^2} = \frac{q^2}{2gy^2}$$



EXAMPLE

A rectangular channel, width is 4 m, flowrate is 12 m^3/s and depth of flow is 2.5 m.

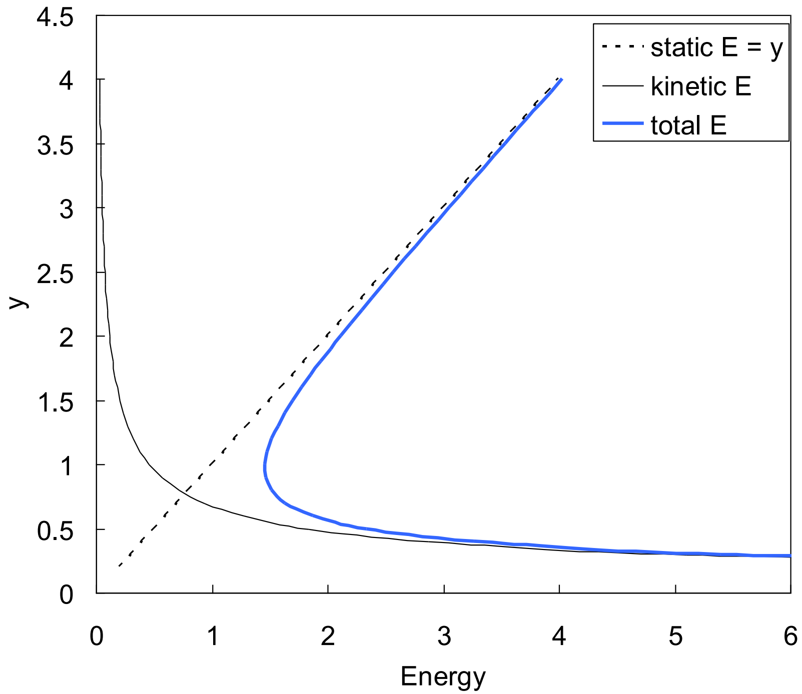
Draw specific energy diagram
Find critical and alternate depth?

$$E = y + \frac{Q^2}{2gA^2}$$

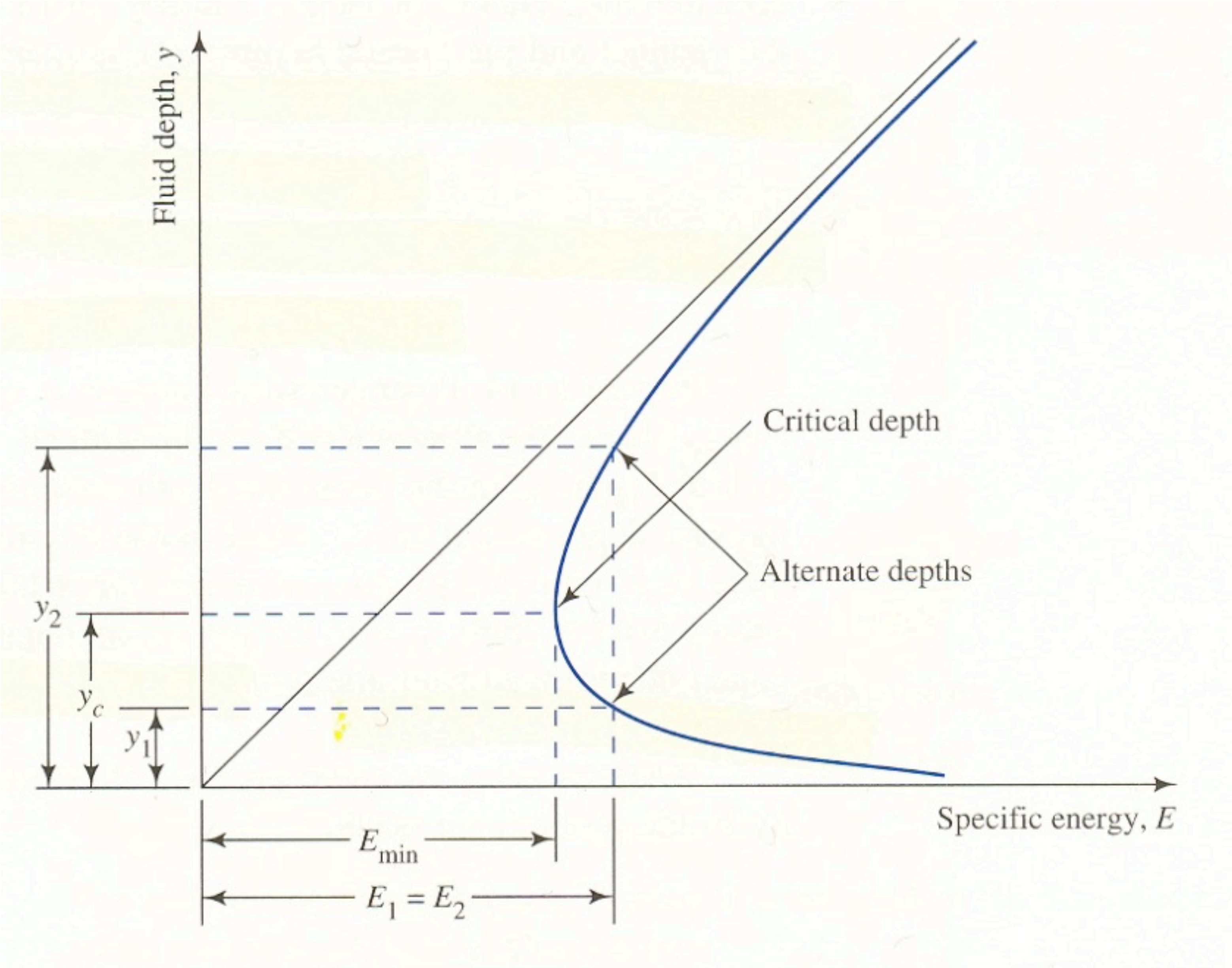
$$E = y + \frac{q^2}{2gy^2}$$

$$q = \frac{Q}{B} = \frac{12}{4} = 3\ \mathrm{m^2/s}$$

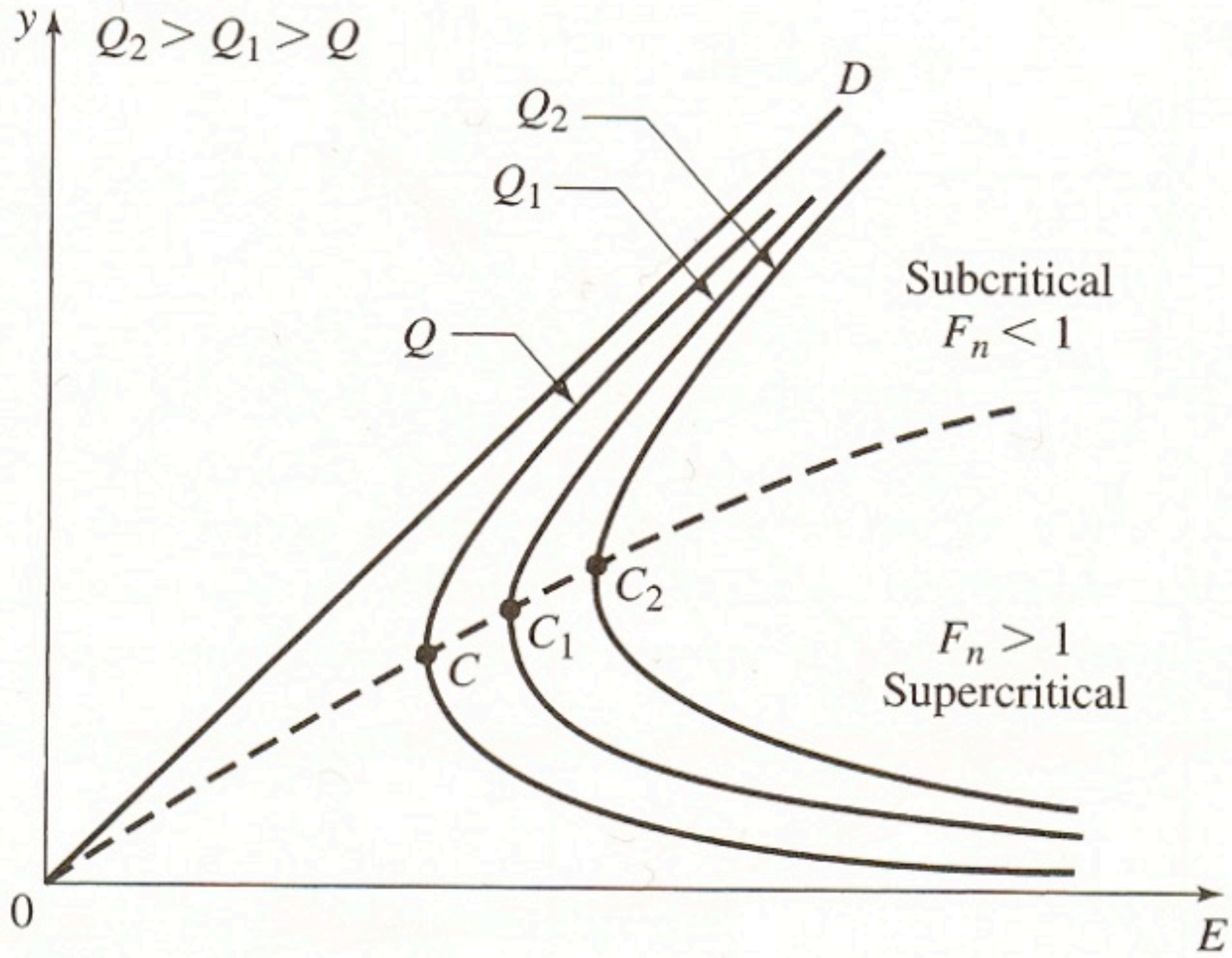
	A	B	C	D	E
1		E=y+(q^2/(2gy^2)		q = 12/4 = 3	
2					
3	**y (depth)**	**static E = y**	**kinetic E**	**total E**	
4	0.2	0.2	11.4679	11.6679	
5	0.3	0.3	5.0968	5.3968	
6	0.4	0.4	2.8670	3.2670	
7	0.5	0.5	1.8349	2.3349	
8	0.6	0.6	1.2742	1.8742	
9	0.7	0.7	0.9362	1.6362	
10	0.8	0.8	0.7167	1.5167	
11	0.9	0.9	0.5663	1.4663	
12	1	1	0.4587	1.4587	
13	1.1	1.1	0.3791	1.4791	
14	1.2	1.2	0.3186	1.5186	
15	1.3	1.3	0.2714	1.5714	
16	1.4	1.4	0.2340	1.6340	
17	1.5	1.5	0.2039	1.7039	
18	1.6	1.6	0.1792	1.7792	
19	1.7	1.7	0.1587	1.8587	
20	1.8	1.8	0.1416	1.9416	
21	1.9	1.9	0.1271	2.0271	
22	2	2	0.1147	2.1147	
23	2.1	2.1	0.1040	2.2040	
24	2.2	2.2	0.0948	2.2948	
25	2.3	2.3	0.0867	2.3867	
26	2.4	2.4	0.0796	2.4796	
27	2.5	2.5	0.0734	2.5734	
28	2.6	2.6	0.0679	2.6679	
29	2.7	2.7	0.0629	2.7629	
30	2.8	2.8	0.0585	2.8585	
31	2.9	2.9	0.0545	2.9545	
32	3	3	0.0510	3.0510	
33	3.1	3.1	0.0477	3.1477	
34	3.2	3.2	0.0448	3.2448	
35	3.3	3.3	0.0421	3.3421	
36	3.4	3.4	0.0397	3.4397	
37	3.5	3.5	0.0374	3.5374	
38	3.6	3.6	0.0354	3.6354	
39	3.7	3.7	0.0335	3.7335	
40	3.8	3.8	0.0318	3.8318	
41	3.9	3.9	0.0302	3.9302	
42	4	4	0.0287	4.0287	
43					



Meaning from the graph:



1. The diagram applies for a given cross section and discharge.
2. As the depth of flow increases, the static energy increases, and the kinetic energy decreases
3. The total energy curve approaches the static energy curve for high depths and the kinetic energy curve for small depths
4. The specific energy is minimum (Emin) for a particular depth – this depth happens to be the critical depth – Depth for which the Froude's number = 1.0. velocity = Vc.
5. Emin – only energy value with a singular depth!
6. Depths less than the critical depths – supercritical flow. Froude Number > 1.0. V > Vc.
7. Depths greater than the critical depths – subcritical flow. Froude Number < 1.0. V < Vc.
8. For all other energy values – there are two depth associated – one greater than the critical depth and one less than the critical depth.
9. The two depths associated with the same energy values are referred to as – Alternate depths
10. As discharge increases, the specific energy curves move to the upper right portion of the chart.



y
$Q_2 > Q_1 > Q$
D
Q_2
Q_1
Q
Subcritical
$F_n < 1$
C_2
C_1
C
$F_n > 1$
Supercritical
0
E

$$E = y + \frac{Q^2}{2gA^2}$$

$$E = y + \frac{q^2}{2gy^2}$$

There is a minimum value of *E* at a certain value of *y* called the *critical depth*.

$$\frac{dE}{dy} = 0 = 1 - \frac{2q^2}{2gy^3}$$

It shows that E_{min} occurs at y_c

$$y_c = \left(\frac{q^2}{g}\right)^{\frac{1}{3}} = \left(\frac{Q^2}{b^2 g}\right)^{\frac{1}{3}}$$

The associated minimum energy is ;

$$E_{\min} = E(y_c) = \frac{3}{2} y_c$$

The depth, y_c corresponds to channel velocity equal to the shallow-water wave propagation speed, C_0.

$$Fr = \frac{V}{\sqrt{gy}}$$

$$q^2 = gy_c^3 = (gy_c)y_c^2 = V_c^2 y_c^2$$

By comparison it follows that the critical channel velocity is:

$$V_c = (g y_c)^{\frac{1}{2}} = C_0$$

$$Fr = 1$$

For $E < E_{\min}$, no solution exists.

For $E > E_{\min}$, two solutions are possible:

(1) Large depth with $V < V_c$, called sub-critical.
(2) Small depth with $V > V_c$, called super-critical.

In sub-critical flow, disturbances can propagate upstream because wave speed $C_0 > V$.

In super-critical flow, waves are swept downstream: Upstream is a zone of silence, and a small obstruction in the flow will create a wedge-shape wave exactly analogous to the Mach waves.

The angle of these waves must be:

$$\mu = \sin^{-1} \frac{C_0}{V} = \sin^{-1} \frac{(g y)^{\frac{1}{2}}}{V}$$

The wave angle and the depth can thus be used as a simple measurement of super-critical flow velocity.

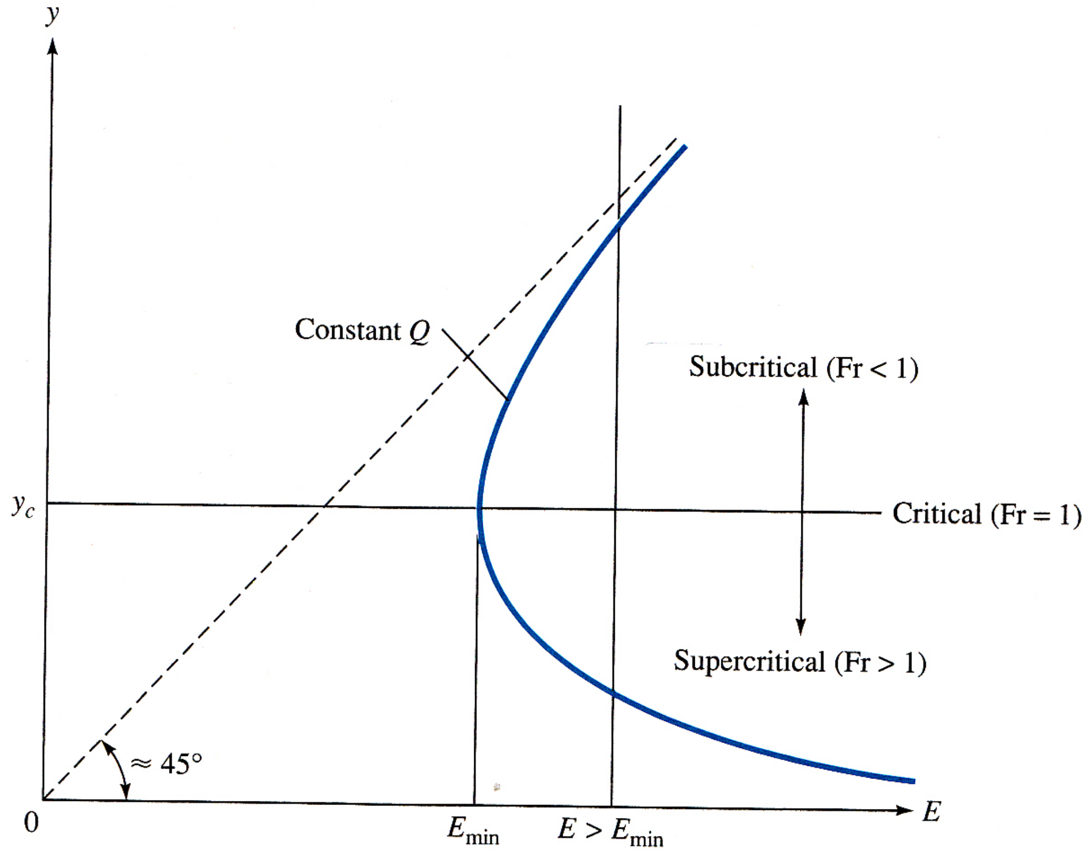


Illustration of a specific energy curve. The curve for each flow rate Q has a minimum energy corresponding to critical flow. For energy greater than minimum, there are two *alternate* flow states, one subcritical and one supercritical.

Critical depth, y_c

$$y_c = \left(\frac{q^2}{g}\right)^{\frac{1}{3}} = \left(\frac{3^2}{9.81}\right)^{\frac{1}{3}} = 0.971\,\text{m}$$

Minimum specific energy, E_{min}

$$E_{\min} = \frac{3}{2} y_c = \frac{3}{2}(0.971) = 1.457\ \text{m}$$

Since given depth was 2.5 m > 0.971 m, the given depth is sub-critical and the other depth should be super-critical.

Now, determining alternate depths (Energy at 2.5 m)

$$E = y + \frac{q^2}{2gy^2} = 2.5 + \frac{3^2}{2 \times 9.81 \times 2.5^2} = 2.57\ \text{m}$$

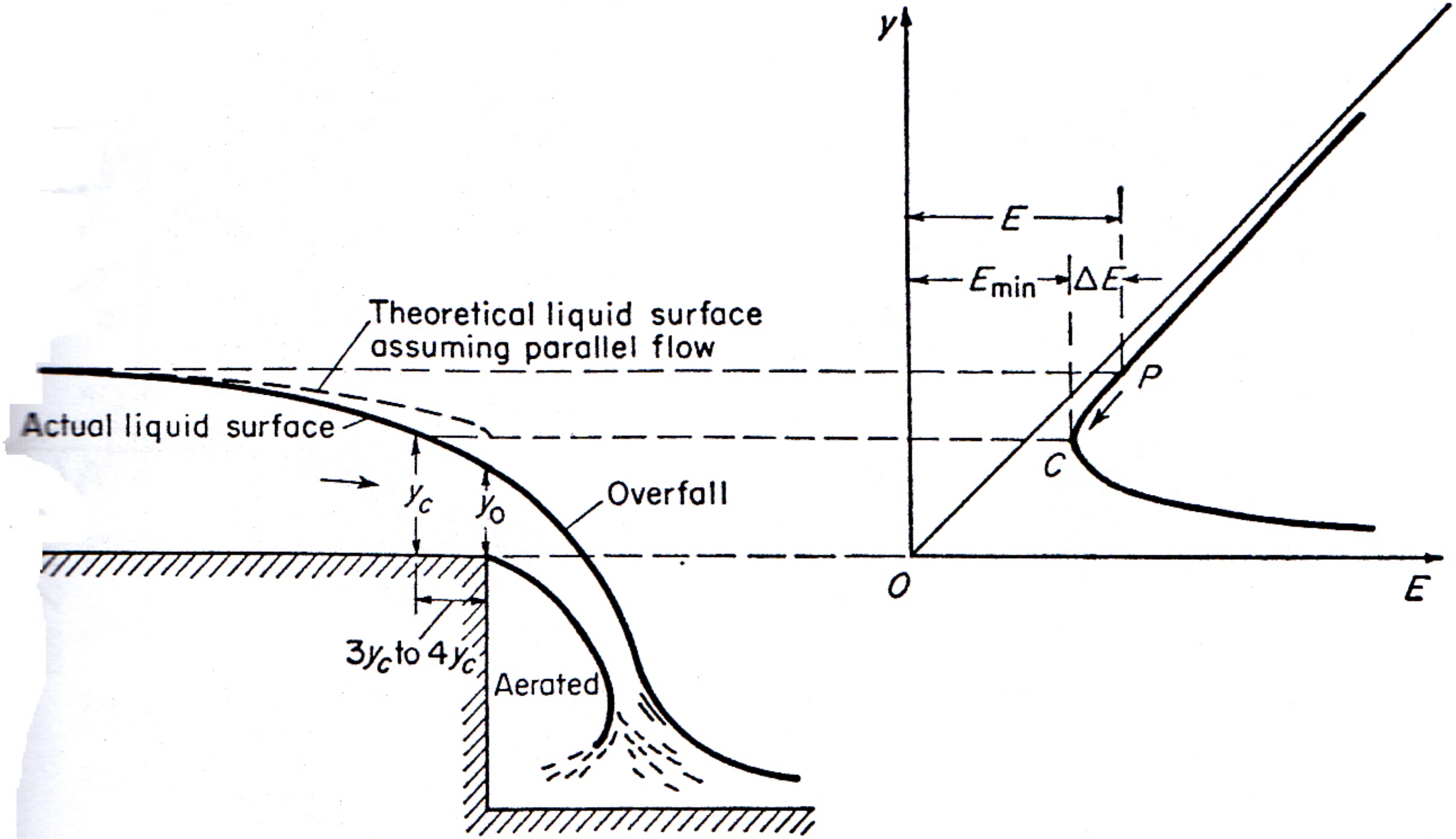
This energy value is the same for the other alternate (super-critical) depth, so;

$$2.57 = y + \frac{3^2}{2 \times 9.81 \times y^2}$$

Determine value of y by trial and error method. (use Excel ok)

	A	B
1	y	E
2	0.43	2.910885
3	0.431	2.900386
4	0.432	2.889967
5	0.433	2.879627
6	0.434	2.869365
7	0.435	2.859181
8	0.436	2.849073
9	0.437	2.839042
10	0.438	2.829086
11	0.439	2.819206
12	0.44	2.809399
13	0.441	2.799665
14	0.442	2.790005
15	0.443	2.780416
16	0.444	2.770899
17	0.445	2.761453
18	0.446	2.752077
19	0.447	2.74277
20	0.448	2.733533
21	0.449	2.724364
22	0.45	2.715262
23	0.451	2.706228
24	0.452	2.69726
25	0.453	2.688358
26	0.454	2.679522
27	0.455	2.67075
28	0.456	2.662042
29	0.457	2.653398
30	0.458	2.644818
31	0.459	2.636299
32	0.46	2.627843
33	0.461	2.619448
34	0.462	2.611115
35	0.463	2.602841
36	0.464	2.594628
37	0.465	2.586473
38	0.466	2.578378
39	0.467	2.570341
40	0.468	2.562362
41	0.469	2.554441
42	0.47	2.546576
43		

super-critical alternate depth, $y = 0.467$ m



A free overfall.

HYDRAULIC JUMP

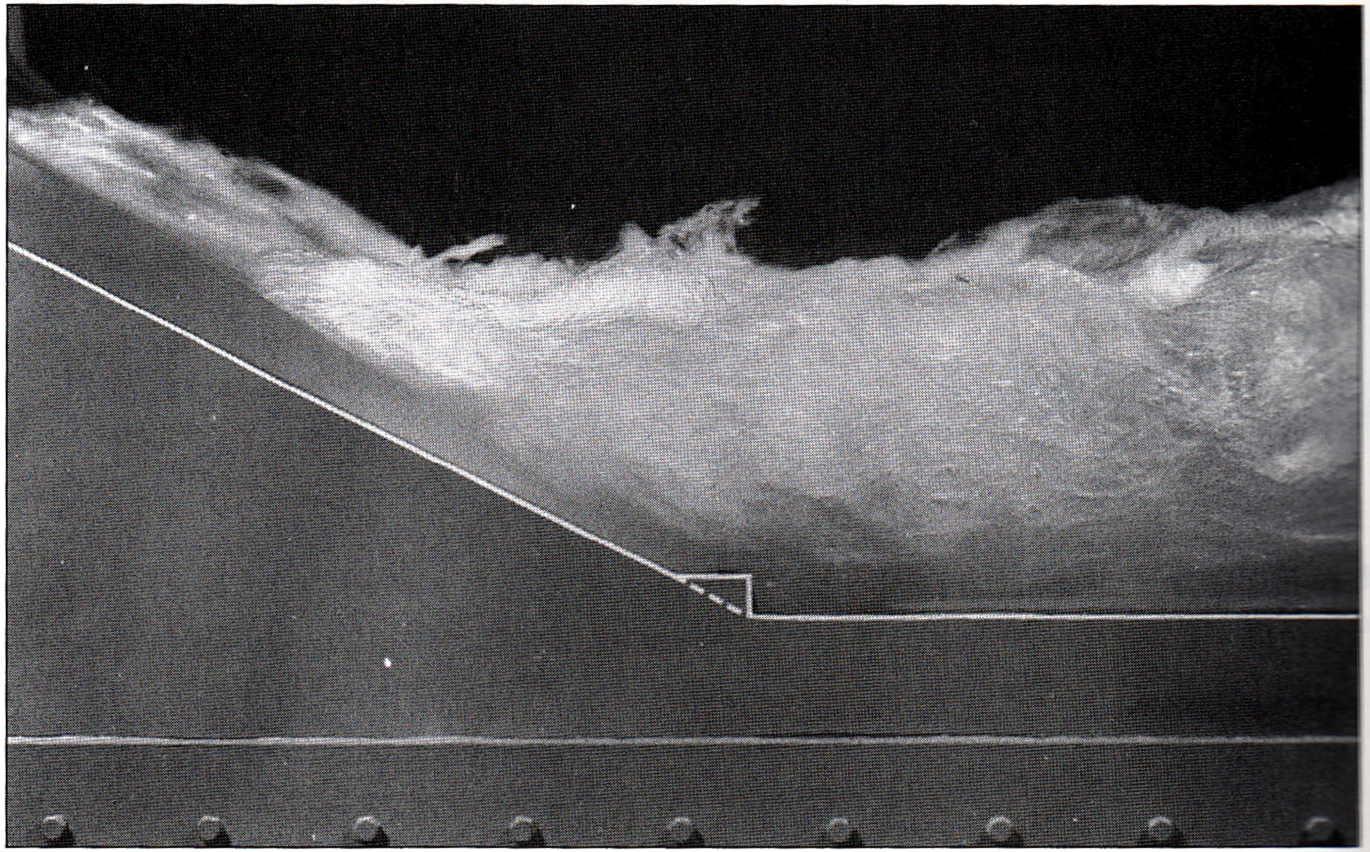
In open channel flow a supercritical flow can change quickly back to a subcritical flow by passing through a hydraulic jump

The upstream flow is fast and shallow.
The downstream flow is slow and deep.

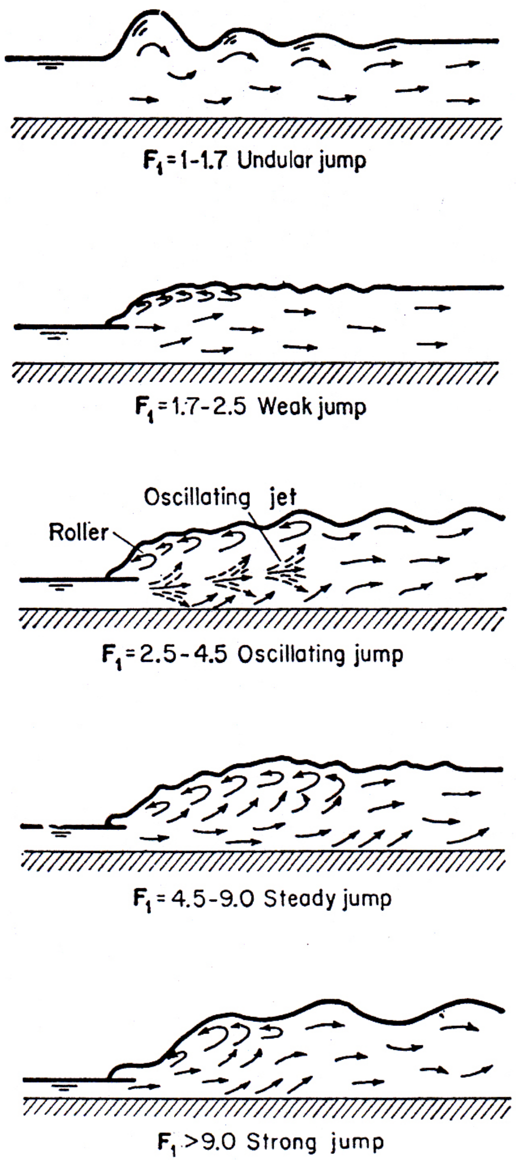
The hydraulic jump is quite thick, ranging in length from 4 to 6 time the downstream depth.

It is very important that such jumps be located on specially designed aprons; otherwise the channel bottom will be badly scoured by the agitation.

Jumps also mixed fluids very effectively and have application to sewage and water treatment designs.

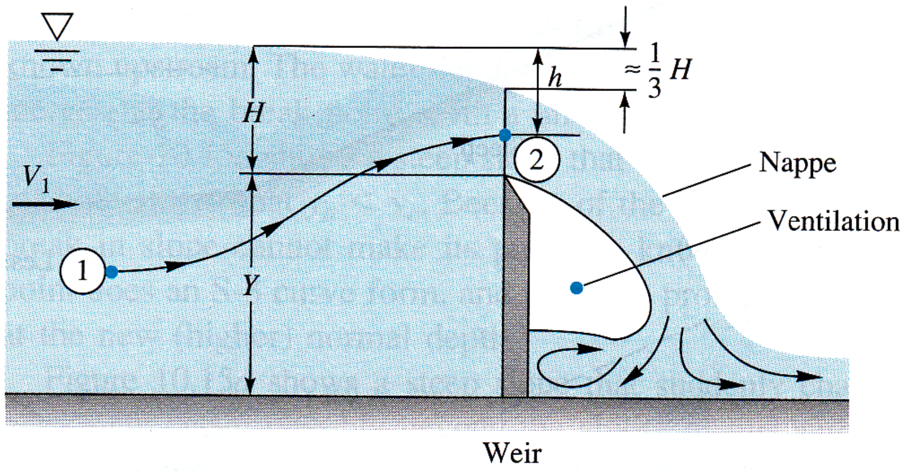


Hydraulic jump formed on a spillway model for the Karnafuli Dam in Bangladesh. *(Courtesy of the St. Anthony Falls Hydraulic Laboratory, University of Minnesota.)*

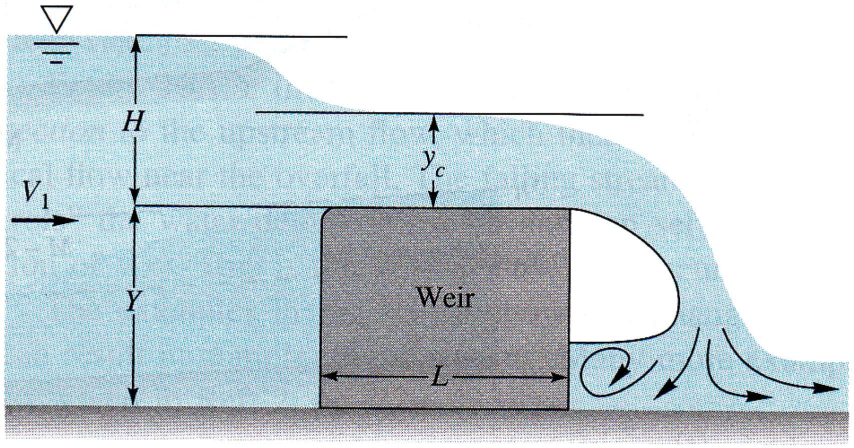


Types of hydraulic jump.

Type of weir



(a)



(b)

Fig. 10.16 Flow over wide, well-ventilated weirs: (*a*) sharp-crested; (*b*) broad-crested.

For sharp-crested weir

Theoretical flow rate, $Q \approx 0.81\left(\frac{2}{3}\right)(2g)^{\frac{1}{2}}(H)^{\frac{3}{2}}$

This formula is functionally correct but the coefficient 0.81 is too high, and should be replaced by an experimentally determined discharge coefficient.

For broad-crested weir

Theoretical flow rate, Theoretical flow rate, $Q = \sqrt{g y_c^3} \approx \frac{1}{\sqrt{3}}\left(\frac{2}{3}\right)(2g)^{\frac{1}{2}}\left(H + \frac{V_1^2}{2g}\right)^{\frac{3}{2}}$

We may neglect the upstream velocity head, $\frac{V_1^2}{2g}$.

The coefficient $\left(\frac{1}{\sqrt{3}}\right)$ is about right but experimental data are preferred.

For all weir, common equation for flow rate is

$$Q = C_d \cdot b \cdot (g)^{\frac{1}{2}}\left(H + \frac{V_1^2}{2g}\right)^{\frac{3}{2}} \approx C_d \cdot b \cdot (g)^{\frac{1}{2}}(H)^{\frac{3}{2}}$$

For wide sharp-crested weir

$$C_d \approx 0.564 + 0.0846\frac{H}{Y}, \quad \text{for } \frac{H}{Y} \le 2$$

For round-nose broad-crested weir

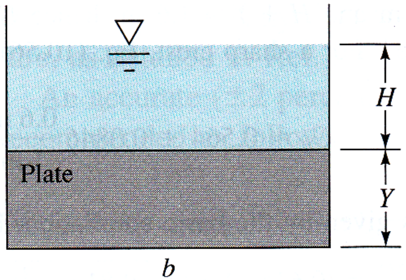
$$C_d \approx 0.544\left(1-\frac{\delta^*/L}{H/L}\right)^{\frac{3}{2}}$$

Where $\dfrac{\delta^*}{L} \approx 0.001 + 0.2\sqrt{\dfrac{\varepsilon}{L}}$

For sharp-nose broad-crested weir

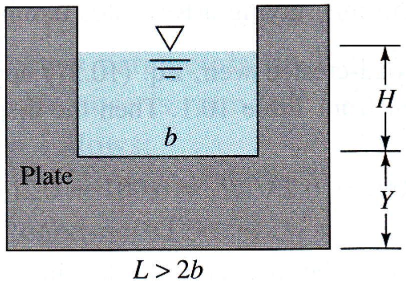
$C_d \approx 0.462$ for $0.08 < \dfrac{H}{L} < 0.33$ and $0.22 < \dfrac{H}{Y} < 0.56$

Thin-plate weirs for flow measurement:



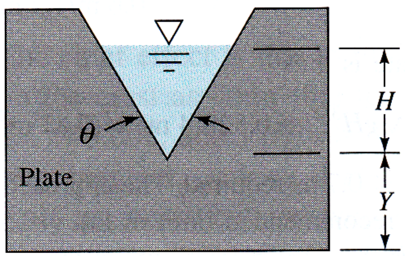
(*a*) Full-width rectangle.

$$Q \approx \left(0.564 + 0.0846\frac{H}{Y}\right) b g^{\frac{1}{2}} H^{\frac{3}{2}}$$



(*b*) Rectangle with side contractions.

$$Q \approx 0.581(b - 0.1H) g^{\frac{1}{2}} H^{\frac{3}{2}} \qquad H < 0.5Y$$



(*c*) V notch.

$$Q \approx 0.44 \tan\frac{\theta}{2} g^{\frac{1}{2}} H^{\frac{5}{2}} \qquad 20° < \theta < 100°$$