OPEN CHANNEL

Energy concepts

INTRODUCTION

The energy head at any position along the channel is the sum of the vertical distance measured from a horizontal datum, z, the depth of flow, y, and the kinetic energy head $\frac{V^2}{2g}$. That sum defines the energy line and is termed the total energy, H.

$$H = z + y + \frac{V^2}{2g}$$

Losses will occur for a real fluid between any two sections of the channel. The total energy will not remain constant. The energy balance is given by this relation.

$$H_1 = H_2 + h_L$$

where; h_L is the head loss.

The only manner in which energy can be added to an open channel flow system is for mechanical pumping or lifting of the liquid to take place.

SPECIFIC ENERGY

It is convenient in open channel flow to measure the energy relative to the bottom of the channel. It provides a useful means to analyze complex flow situations.

Specific energy is designated as E:

It is the sum of the flow depth, y, and kinetic energy head, $\frac{V^2}{2g}$.

$$E = y + \frac{V^2}{2g}$$

RECTANGULAR section:

For a rectangular section, the specific energy can be expressed as a function of the depth, y.

The specific discharge, q, is defined as the total discharge divided by the channel width.

$$q = \frac{Q}{b} = V \cdot y$$

The specific energy for a rectangular channel can thus be put in the form:

$$E = y + \frac{q^2}{2gy^2}$$

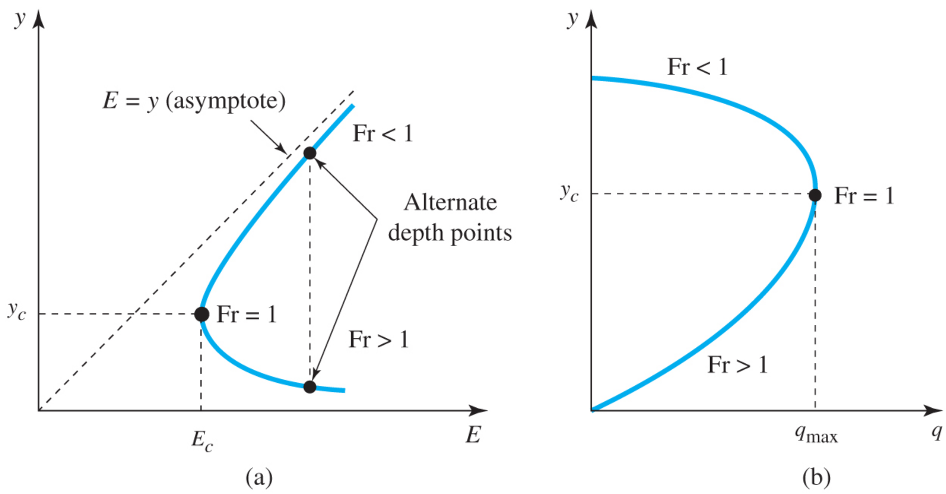

Figure 10.6 Variation of specific energy and specific discharge with depth: (a) E versus y for constant q; (b) q versus y for constant E.

Figure 10.6 is showing E-y relation.

It is found that a specific discharge requires at least a minimum energy.

This minimum energy is referred to as a critical energy, E_c.

The corresponding depth, y_c, is called the critical depth.

If the specific energy is greater than E_c, two depths are possible; those depths are referred to as alternate depths.

Maximum unit discharge, q_{max}, occurs at critical depth.

The critical depth, y_c can be evaluated by setting the derivative of E with respect to y equal to zero.

$$E = y + \frac{q^2}{2gy^2}$$

$$\frac{dE}{dy} = 1 - \frac{q^2}{gy^3} = 0$$

Masukkan $q = Vy$

$$0 = 1 - \frac{V^2}{gy} = 1 - Fr^2$$

Note: Froude number in a rectangular channel is:

$$Fr = \frac{V}{\sqrt{gy}} = \frac{q}{\sqrt{gy^3}}$$

At critical depth, y_c , Froude number is equal to one.

At $y = y_c \implies Fr = 1$

$$Fr = 1 = \frac{q}{\sqrt{g(y_c)^3}}$$

$$y_c = \left(\frac{q^2}{g}\right)^{\frac{1}{3}}$$

Critical flow conditions, E_c , can conveniently be expressed by :

$$E_c = y_c + \frac{q^2}{2g(y_2)^2}$$

$$= \frac{3}{2} y_c$$

On the *E*-*y* curve;

$Fr < 1$	Flow is relatively slow (tranquil)	Subcritical flow
$Fr > 1$	Flow is relatively rapid (shooting)	Supercritical flow

GENERALIZED CROSS SECTION

For a generalized section, the specific energy is written in terms of the total discharge, Q , and the cross-sectional area, A.

$$E = y + \frac{Q^2}{2gA^2}$$

The minimum energy condition is obtained by differentiating it with respect to y :

$$\frac{dE}{dy} = 0 = 1 - \frac{Q^2}{gA^3} \cdot \frac{dA}{dy}$$

The corresponding change in area is:

$$dA = B \cdot dy$$

Thus setting above equation equal to zero :

$$0 = 1 - \frac{Q^2 B}{gA^3}$$

Froude number can written as:

$$Fr = \sqrt{\frac{Q^2 B}{g A^3}} = \frac{Q/A}{\sqrt{gA/B}} = \frac{V}{\sqrt{gA/B}}$$

The ratio A/B is termed the hydraulic depth.

A/B is equal to y for a rectangular channel.

CHANNEL GEOMETRY

A regular section is one whose shape does not vary along the length of the channel.

An irregular section will have changes in its geometry.

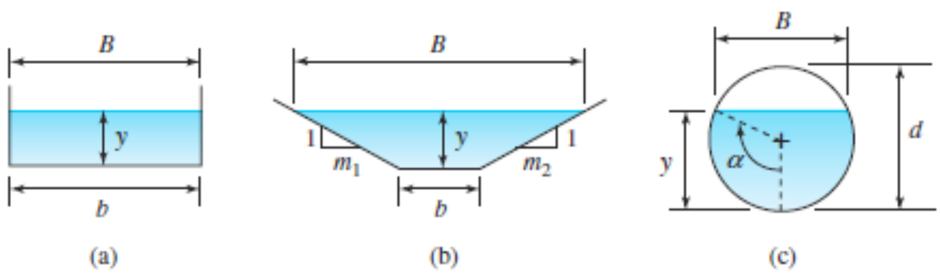

Figure 10.4 Representative regular cross sections: (a) rectangular; (b) trapezoidal; (c) circular.

For trapezoidal:
($P =$ $WP =$ wetted perimeter)
($B =$ free surface width)

$$A = by + \frac{1}{2}y^2(m_1 + m_2)$$

$$P = b + y(\sqrt{1 + m_1^2} + \sqrt{1 + m_2^2})$$

$$B = b + y(m_1 + m_2)$$

For circular cross section:

$$A = \frac{d^2}{4}(\alpha - \sin\alpha\cos\alpha)$$

$$P = \alpha d$$

$$B = d\sin\alpha$$

$$\alpha = \cos^{-1}\left(1 - 2\frac{y}{d}\right)$$

EXAMPLE 1

Water is flowing in a triangular channel with $m_1 = m_2 = 1.0$ at a discharge of $Q = 3\ \text{m}^3/\text{s}$. If the water depth is 2.5 m, determine the specific energy, Froude number, hydraulic depth, and alternate depth.

Solution

Recognizing that $b = 0$, the flow area and top width are computed from Eqs. 10.3.4 and 10.3.6 as follows:

$$
\begin{aligned}
A &= \frac{1}{2} y^2 (m_1 + m_2) \\
&= \frac{1}{2} \times 2.5^2 \times (1 + 1) = 6.25\ \text{m}^2 \\
B &= (m_1 + m_2) y \\
&= (1 + 1) \times 2.5 = 5.0\ \text{m}
\end{aligned}
$$

Using Eqs. 10.4.12 and 10.4.15, E and Fr are found to be

$$
\begin{aligned}
E &= y + \frac{Q^2}{2gA^2} \\
&= 2.5 + \frac{3^2}{2 \times 9.81 \times 6.25^2} = \underline{2.51\ \text{m}}
\end{aligned}
$$

$$
\begin{aligned}
\text{Fr} &= \sqrt{\frac{Q^2 B}{g A^3}} \\
&= \sqrt{\frac{3^2 \times 5}{9.81 \times 6.25^3}} = \underline{0.137}
\end{aligned}
$$

The hydraulic depth is

$$
\frac{A}{B} = \frac{6.25}{5.0} = \underline{1.25\ \text{m}}
$$

The alternate depth is calculated using the energy equation. Recognizing that $A = y^2$, we have

$$
\begin{aligned}
2.51 &= y + \frac{3^2}{2 \times 9.81 \times (y^2)^2} \\
&= y + \frac{0.459}{y^4}
\end{aligned}
$$

A trial-and-error solution provides $y = \underline{0.71\,\text{m}}$.

EXAMPLE 2

A rectangular channel 3 m wide is conveying water at a depth $y_1 = 1.55$ m and velocity $V_1 = 1.83$ m/s. The flow enters a transition region as shown in Figure E10.4a, in which the bottom elevation is raised by $h = 0.20$ m. Determine the depth and velocity in the transition, and the value of h for choking to occur.

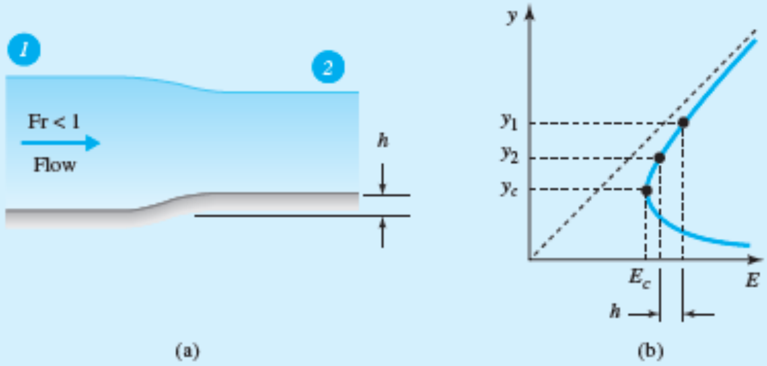

Figure E10.4

Solution

Use Eq. 10.4.4 to find the specific discharge to be

$$q = V_1 y_1$$
$$= 1.83 \times 1.55 = 2.84 \text{ m}^2/\text{s}$$

The Froude number at location 1 is

$$\text{Fr} = \frac{V_1}{\sqrt{g y_1}}$$
$$= \frac{1.83}{\sqrt{9.81 \times 1.55}} = 0.47$$

which is less than unity. Hence, the flow at location 1 is subcritical. The specific energy at location 1 is found, using Eq. 10.4.3, to be

$$E_1 = y_1 + \frac{V_1^2}{2g}$$

$$= 1.55 + \frac{1.83^2}{2 \times 9.81} = 1.72 \text{ m}$$

The specific energy at location 2 is found, using Eq. 10.4.16, to be

$$E_2 = E_1 - h$$
$$= 1.72 - 0.20 = 1.52 \text{ m}$$

If $E_2 > E_c$, it is possible to find the depth y_2. Therefore, E_c is calculated first. From Eqs. 10.4.10 and 10.4.11 the critical conditions are

$$y_c = \left(\frac{q^2}{g}\right)^{1/3} = \left(\frac{2.84^2}{9.81}\right)^{1/3} = 0.94 \text{ m}$$

$$E_c = \frac{3y_c}{2} = 3 \times \frac{0.94}{2} = 1.41 \text{ m}$$

Hence, since $E_2 > E_c$, we can proceed with calculating y_2. The depth y_2 can be evaluated by substituting known values into Eq. 10.4.5:

$$1.52 = y_2 + \frac{2.84^2}{2 \times 9.81 \times y_2^2}$$

The solution is

$$y_2 = \underline{1.26 \text{ m}}$$

$$\therefore V_2 = \frac{q}{y_2}$$

$$= \frac{2.84}{1.26} = \underline{2.25 \text{ m/s}}$$

The flow at location 2 is subcritical since there is no way in which the flow can become supercritical in the transition with the given geometry.

The value of h for critical flow to appear at location 2 is determined by setting $E_2 = E_c$ in Eq. 10.4.16:

$$h = E_1 - E_c = 1.72 - 1.40 = \underline{0.31 \text{ m}}$$

EXAMPLE 3

Water flows freely from a reservoir into a trapezoidal channel with bottom width $b = 5.0$ m and side slope parameters $m_1 = m_2 = 2.0$. The elevation of the water surface in the reservoir is 2.3 m above the entrance crest. Assuming negligible losses in the transition and a negligible velocity in the reservoir upstream of the entrance, find the critical depth at the transition and the discharge into the channel.

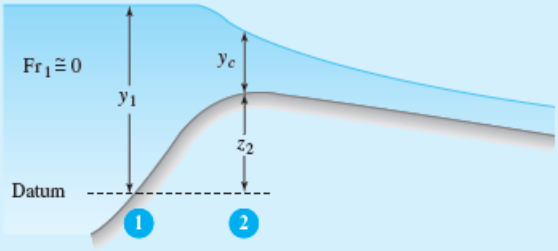

Figure E10.5

Solution

The total energy at location 1 in Figure E10.5 is y_1 since the kinetic energy in the reservoir is negligible ($V_1 \approx 0$). Equating the total energies at locations 1 and 2 gives

$$y_1 = E_2 + z_2$$

Since critical conditions occur at location 2, Eqs. 10.4.12 and 10.4.14 can be combined to eliminate the discharge, with the result

$$E_2 = y_c + \frac{A}{2B}$$

Elimination of E_2 in the two equations yields the expression

$$y_1 - z_2 = y_c + \frac{A}{2B} = y_c + \frac{by_c + \frac{1}{2}(m_1 + m_2)y_c^2}{2[b + (m_1 + m_2)y_c]}$$

or, with the given data, the expression becomes

$$2.3 = y_c + \frac{5y_c + \frac{1}{2}(2 + 2)y_c^2}{2[5 + (2 + 2)y_c]}$$

The relation above is a quadratic in y_c. The positive root is chosen, which is

$$y_c = \underline{1.70 \text{ m}}$$

Subsequently, one can find that $A = 14.28 \text{ m}^2$ and $B = 11.80$ m. Use Eq. 10.4.14 to find the discharge to be

$$Q = \sqrt{\frac{gA^3}{B}}$$

$$= \sqrt{\frac{9.8 \times 14.3^3}{11.8}} = \underline{49.3 \text{ m}^3/\text{s}}$$