Chapter 1

Flow in Open Channels

10.1 Introduction

- **Free surface**: The interface between the air and the upper layer of water.

- Ocean waves, river currents, and overland flow of rainfall are examples of free-surface flow that occur in nature.

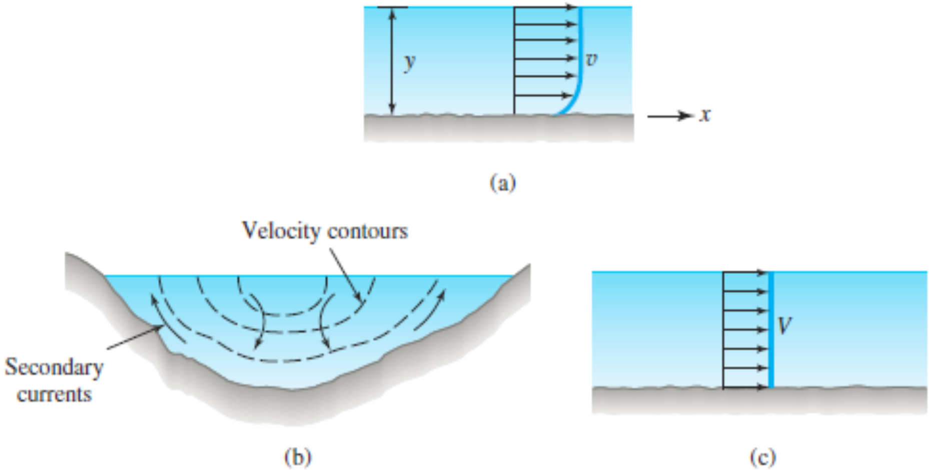


Figure 10.1 Free-surface flow: (a) centerline velocity distribution; (b) cross section; (c) one-dimensional model.

10.2 Open-Channel Flows

- In relatively short reaches, called transitions, there is a rapid change in depth and velocity.
- **Rapidly varied flow**: A rapid change in depth and velocity.
- **Gradually varied flow**: In extensive reaches of a channel, the velocity and depth change in a slow manner.

10.2 Open-Channel Flows

Table 10.1 Combinations of One-Dimensional Free-Surface Flows

Type of flow	*Average velocity*	*Depth*
Steady, uniform	$V = \text{const.}$	$y = \text{const.}$
Steady, nonuniform	$V = V(x)$	$y = y(x)$
Unsteady, uniform	$V = V(t)$	$y = y(t)$
Unsteady, nonuniform	$V = V(x, t)$	$y = y(x, t)$

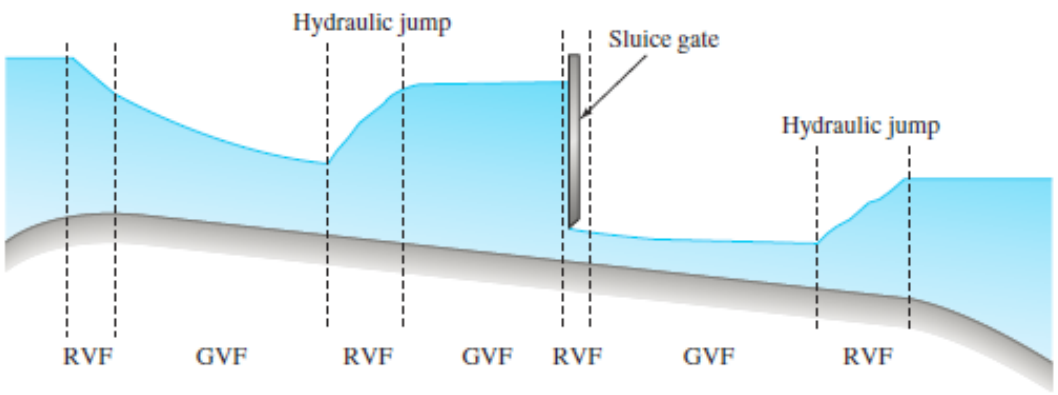


Figure 10.2 Steady nonuniform flow in a channel.

10.2 Open-Channel Flows

Froude number

$$\mathrm{Fr} = \frac{V}{\sqrt{gL}}$$

For example, if Fr > 1, the flow possesses a relatively high velocity and shallow depth; on the other hand, when Fr < 1, the velocity is relatively low and the depth is relatively deep.

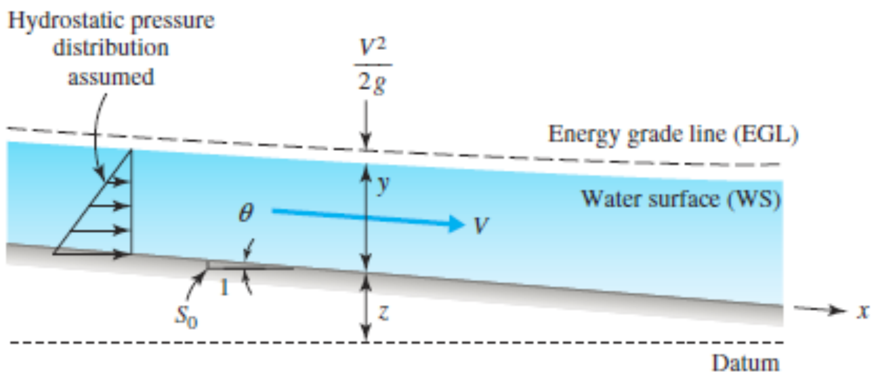


Figure 10.3 Reach of open-channel flow.

10.3 Uniform Flow

Channel cross sections can be considered *regular* or *irregular*.

Regular section: Section whose shape does not vary along the length of the channel.

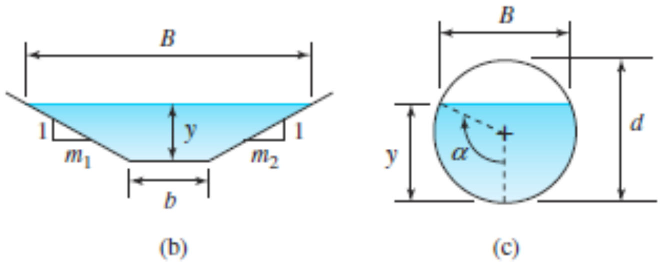


Figure 10.4 Representative regular cross sections: (a) rectangular; (b) trapezoidal; (c) circular.

10.3 Uniform Flow

Wetted perimeter: The length of the line of contact between the liquid and the channel.

Hydraulic radius: The area divided by the wetted perimeter.

For a rectangular channel:

$$A = by$$

$$P = b + 2y$$

$$R = \frac{A}{P} = \frac{by}{b + 2y}$$

For a trapezoidal section:

$$A = by + \frac{1}{2}y^2(m_1 + m_2)$$

$$P = b + y(\sqrt{1 + m_1^2} + \sqrt{1 + m_2^2})$$

$$B = b + y(m_1 + m_2)$$

10.3 Uniform Flow

For a circular cross section:

$$A = \frac{d^2}{4}(\alpha - \sin\alpha\cos\alpha)$$

$$P = \alpha d$$

$$B = d\sin\alpha$$

$$\alpha = \cos^{-1}\left(1 - 2\frac{y}{d}\right)$$

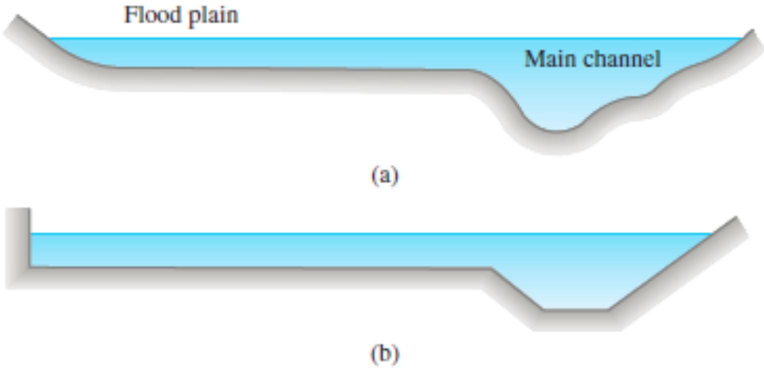


Figure 10.5 Generalized section representation: (a) actual cross section; (b) composite cross section.

10.3 Uniform Flow

Equation for Uniform Flow:

Uniform flow occurs in a channel when the depth and velocity do not vary along its length.

Chezy-Manning equation:

$$Q = \frac{c_1}{n} A R^{2/3} \sqrt{S_0}$$

The depth associated with uniform flow is designated y_0; it is called either *uniform depth* or *normal depth*. Uniform flow rarely occurs in rivers because of the irregularity of the geometry.

10.3 Uniform Flow

Water is flowing at a rate of 4.5 m^3/s in a trapezoidal channel (Figure 10.4b) whose bottom width is 2.4 m and side slopes are 1 vertical to 2 horizontal. Compute y_0 if $n = 0.012$ and $S_0 = 0.0001$.

Solution

Given geometrical data are $b = 2.4$ m and $m_1 = m_2 = 2$. Rearrange Eq. 10.3.13, noting that $R = A/P$ and $c_1 = 1$:

$$\frac{A^{5/3}}{P^{2/3}} = \frac{nQ}{\sqrt{S_0}}$$

Substituting in the known data and trapezoidal geometry, one has

$$\frac{\left[2.4y_0 + \frac{1}{2}y_0^2(2+2)\right]^{5/3}}{[2.4 + y_0(2\sqrt{1+2^2})]^{2/3}} = \frac{0.012 \times 4.5}{\sqrt{0.0001}}$$

Solving for y_0, either by trial-and-error or by use of computational software, yields $y_0 = \underline{1.28 \text{ m}}$.

10.3 Uniform Flow

Uniform flow occasionally occurs in a 5-m-diameter circular concrete conduit (Figure 10.4c), but the depth of flow can vary. The Manning coefficient is $n = 0.013$, and the channel slope is $S_0 = 0.0005$. (a) Calculate the discharge for $y_0 = 3$ m, (b) Plot the discharge-depth curve.

Solution

(a) First, use Eq. 10.3.10 to find the angle α:

$$\alpha = \cos^{-1}(1 - 2y_0/d)$$
$$= \cos^{-1}(1 - 2 \times 3/5) = 101.54° \text{ or } 101.54 \times \pi/180 = 1.772 \text{ rad}$$

Using Eqs. 10.3.7 and 10.3.8, the area and wetted perimeter are

$$A = \frac{d^2}{4}(\alpha - \sin\alpha \cos\alpha) = \frac{5^2}{4}(1.772 - \sin 101.54° \cos 101.54°) = 12.3 \text{ m}^2$$
$$p = \alpha d = 1.772 \times 5 = 8.86 \text{ m}$$

The hydraulic radius is then

$$R = \frac{A}{P} = \frac{12.3}{8.86} = 1.388 \text{ m}$$

Finally, the discharge, when $y = 3$ m, is

$$Q = \frac{1}{n}AR^{2/3}S^{1/2} = \frac{1}{0.013} \times 12.3 \times 1.388^{2/3} \times 0.0005^{0.5} = \underline{26.3 \text{ m}^3/\text{s}}$$

Mathcad is employed to generate the curve. Note that the solution is generalized, so that any diameter, Manning coefficient, or channel slope can be entered into the algorithm. Equations 10.3.7 and 10.3.8 are used to define the area and wetted perimeter, respectively. Either SI or English units can be employed by properly defining the parameter c_1. In this problem a value of 1.0 is used. The MATLAB solution is shown in Figure E10.2 and in Appendix E, Figure E.1. Use y rather than y_0.

Input diameter, Manning coefficient, and channel slope:

$d := 5 \quad n := 0.013 \quad S_0 := 0.0005 \quad c_1 := 1.0$

10.3 Uniform Flow

Define geometric functions:

$$\alpha(y) := \operatorname{acos}\left(1 - 2\cdot\frac{y}{d}\right)$$

$$A(y) := \frac{d^2}{4}\cdot(\alpha(y) - \sin(\alpha(y))\cdot\cos(\alpha(y)))$$

$$P(y) := \alpha(y)\cdot d$$

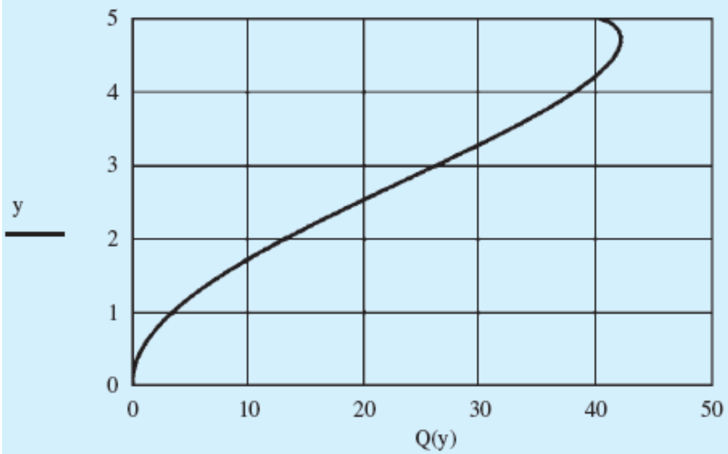
$$R(y) := \frac{A(y)}{P(y)}$$

Define discharge function, (i.e., Manning's equation):

$$Q(y) := \frac{c_1}{n}\cdot A(y)\cdot R(y)^{\frac{2}{3}}\cdot\sqrt{S_0}$$

Plot depth versus discharge:

$$y := 0, 0.01 .. d$$



10.3 Uniform Flow

Most Efficient Section

$$\tau_0 P = \gamma A S_0$$

$$P = cA^{5/2}$$

$$P = b + \frac{2A}{b}$$

$$b + \frac{2A}{b} = cA^{5/2}$$

$$1 + \frac{2}{b}\frac{dA}{db} - \frac{2A}{b^2} = \frac{5}{2}cA^{3/2}\frac{dA}{db}$$

$$\frac{2A}{b^2} = 1$$

$$b = 2y_0$$

10.4 Energy Concepts

Total energy: The sum of the vertical distance to the channel bottom measured from a horizontal datum, the depth of flow, and the kinetic energy head.

$$H = z + y + \frac{V^2}{2g}$$

Energy is actually an energy head.

$$H_1 = H_2 + h_L$$

h_L is the head loss.

10.4 Energy Concepts

Specific Energy

Specific energy: Measurement of energy relative to the bottom of the channel.

$$E = y + \frac{V^2}{2g}$$

Specific discharge: The total discharge divided by the channel width (valid only for a rectangular channel).

$$q = \frac{Q}{b} = Vy$$

$$E = y + \frac{q^2}{2gy^2}$$

10.4 Energy Concepts

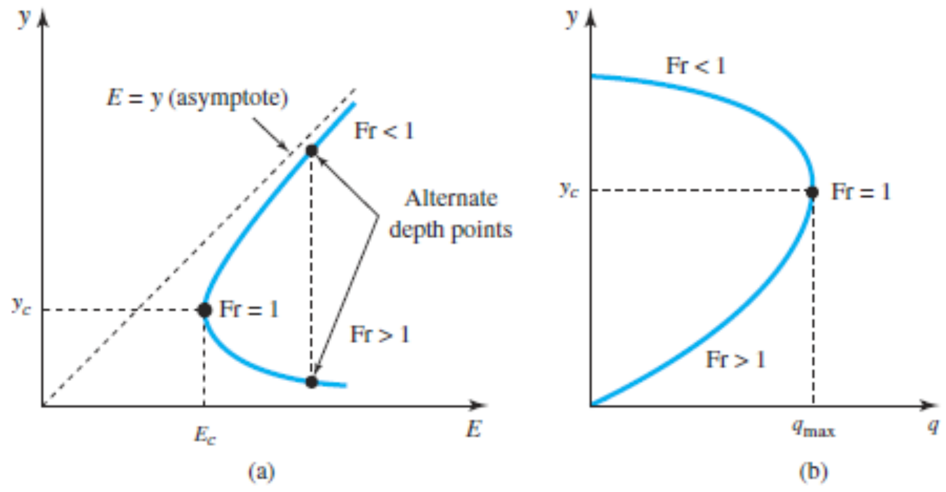


Figure 10.6 Variation of specific energy and specific discharge with depth: (a) E versus y for constant q; (b) q versus y for constant E.

Critical depth: The depth for which specific energy is a minimum.

10.4 Energy Concepts

Alternate depths: The two depths of flow that are possible for a given specific energy and discharge.

$$q = \sqrt{2gy^2(E - y)}$$

$$\mathrm{Fr} = \frac{V}{\sqrt{gy}} = \frac{q}{\sqrt{gy^3}}$$

$$y = y_c = \left(\frac{q^2}{g}\right)^{1/3}$$

$$E_c = \frac{3}{2}y_c$$

For a generalized section, the specific energy is written in terms of the total discharge *Q* and the cross-sectional area *A* as

$$E = y + \frac{Q^2}{2gA^2}$$

10.4 Energy Concepts

Water is flowing in a triangular channel with $m_1 = m_2 = 1.0$ at a discharge of $Q = 3\ \text{m}^3/\text{s}$. If the water depth is 2.5 m, determine the specific energy, Froude number, hydraulic depth, and alternate depth.

Solution

Recognizing that $b = 0$, the flow area and top width are computed from Eqs. 10.3.4 and 10.3.6 as follows:

$$A = \frac{1}{2} y^2 (m_1 + m_2)$$

$$= \frac{1}{2} \times 2.5^2 \times (1 + 1) = 6.25\ \text{m}^2$$

$$B = (m_1 + m_2) y$$

$$= (1 + 1) \times 2.5 = 5.0\ \text{m}$$

Using Eqs. 10.4.12 and 10.4.15, E and Fr are found to be

$$E = y + \frac{Q^2}{2gA^2}$$

$$= 2.5 + \frac{3^2}{2 \times 9.81 \times 6.25^2} = \underline{2.51\ \text{m}}$$

$$\text{Fr} = \sqrt{\frac{Q^2 B}{gA^3}}$$

$$= \sqrt{\frac{3^2 \times 5}{9.81 \times 6.25^3}} = \underline{0.137}$$

The hydraulic depth is

$$\frac{A}{B} = \frac{6.25}{5.0} = \underline{1.25\ \text{m}}$$

The alternate depth is calculated using the energy equation. Recognizing that $A = y^2$, we have

$$2.51 = y + \frac{3^2}{2 \times 9.81 \times (y^2)^2}$$

$$= y + \frac{0.459}{y^4}$$

A trial-and-error solution provides $y = \underline{0.71}\ \text{m}$.

10.4 Energy Concepts

Use of the Energy Equation in Transitions

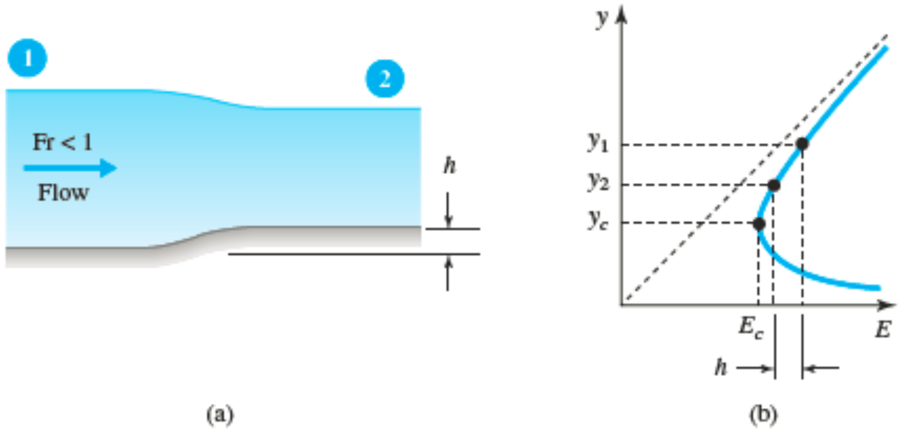


Figure 10.7 Channel constriction: (a) raised channel bottom; (b) specific energy diagram.

The condition of choked flow or a choking condition implies that minimum specific energy exists within the transition.

10.4 Energy Concepts

A rectangular channel 3 m wide is conveying water at a depth $y_1 = 1.55$ m and velocity $V_1 = 1.83$ m/s. The flow enters a transition region as shown in Figure E10.4a, in which the bottom elevation is raised by $h = 0.20$ m. Determine the depth and velocity in the transition, and the value of h for choking to occur.

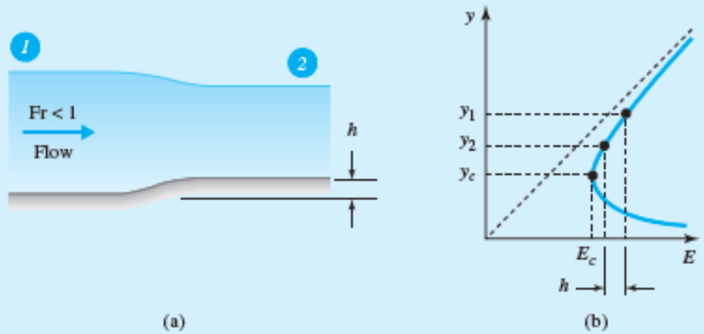


Figure E10.4

Solution

Use Eq. 10.4.4 to find the specific discharge to be

$$q = V_1 y_1$$
$$= 1.83 \times 1.55 = 2.84 \text{ m}^2/\text{s}$$

The Froude number at location 1 is

$$\mathrm{Fr} = \frac{V_1}{\sqrt{g y_1}}$$
$$= \frac{1.83}{\sqrt{9.81 \times 1.55}} = 0.47$$

which is less than unity. Hence, the flow at location 1 is subcritical. The specific energy at location 1 is found, using Eq. 10.4.3, to be

10.4 Energy Concepts

$$E_1 = y_1 + \frac{V_1^2}{2g}$$
$$= 1.55 + \frac{1.83^2}{2 \times 9.81} = 1.72 \text{ m}$$

The specific energy at location 2 is found, using Eq. 10.4.16, to be

$$E_2 = E_1 - h$$
$$= 1.72 - 0.20 = 1.52 \text{ m}$$

If $E_2 > E_c$, it is possible to find the depth y_2. Therefore, E_c is calculated first. From Eqs. 10.4.10 and 10.4.11 the critical conditions are

$$y_c = \left(\frac{q^2}{g}\right)^{1/3} = \left(\frac{2.84^2}{9.81}\right)^{1/3} = 0.94 \text{ m}$$
$$E_c = \frac{3y_c}{2} = 3 \times \frac{0.94}{2} = 1.41 \text{ m}$$

Hence, since $E_2 > E_c$, we can proceed with calculating y_2. The depth y_2 can be evaluated by substituting known values into Eq. 10.4.5:

$$1.52 = y_2 + \frac{2.84^2}{2 \times 9.81 \times y_2^2}$$

The solution is

$$y_2 = \underline{1.26 \text{ m}}$$
$$\therefore V_2 = \frac{q}{y_2}$$
$$= \frac{2.84}{1.26} = \underline{2.25 \text{ m/s}}$$

The flow at location 2 is subcritical since there is no way in which the flow can become supercritical in the transition with the given geometry.

The value of h for critical flow to appear at location 2 is determined by setting $E_2 = E_c$ in Eq. 10.4.16:

$$h = E_1 - E_c = 1.72 - 1.40 = \underline{0.31 \text{ m}}$$

10.4 Energy Concepts

Water flows freely from a reservoir into a trapezoidal channel with bottom width $b = 5.0$ m and side slope parameters $m_1 = m_2 = 2.0$. The elevation of the water surface in the reservoir is 2.3 m above the entrance crest. Assuming negligible losses in the transition and a negligible velocity in the reservoir upstream of the entrance, find the critical depth at the transition and the discharge into the channel.

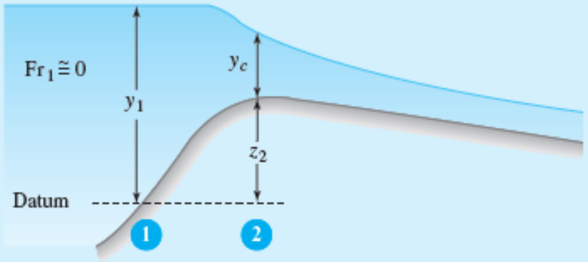


Figure E10.5

Solution

The total energy at location 1 in Figure E10.5 is y_1 since the kinetic energy in the reservoir is negligible ($V_1 \approx 0$). Equating the total energies at locations 1 and 2 gives

$$y_1 = E_2 + z_2$$

Since critical conditions occur at location 2, Eqs. 10.4.12 and 10.4.14 can be combined to eliminate the discharge, with the result

$$E_2 = y_c + \frac{A}{2B}$$

Elimination of E_2 in the two equations yields the expression

$$y_1 - z_2 = y_c + \frac{A}{2B} = y_c + \frac{by_c + \frac{1}{2}(m_1 + m_2)y_c^2}{2[b + (m_1 + m_2)y_c]}$$

or, with the given data, the expression becomes

$$2.3 = y_c + \frac{5y_c + \frac{1}{2}(2+2)y_c^2}{2[5 + (2+2)y_c]}$$

The relation above is a quadratic in y_c. The positive root is chosen, which is

$$y_c = \underline{1.70 \text{ m}}$$

Subsequently, one can find that $A = 14.28 \text{ m}^2$ and $B = 11.80$ m. Use Eq. 10.4.14 to find the discharge to be

$$Q = \sqrt{\frac{gA^3}{B}}$$
$$= \sqrt{\frac{9.8 \times 14.3^3}{11.8}} = \underline{49.3 \text{ m}^3/\text{s}}$$

10.4 Energy Concepts

Energy losses in expansions and contractions are known to be relatively small when the flow is subcritical.

$$h_L = K_e\left(\frac{V_1^2}{2g} - \frac{V_2^2}{2g}\right)$$

$$h_L = K_c\left(\frac{V_2^2}{2g} - \frac{V_1^2}{2g}\right)$$

Flow Measurement

Weir: A device placed in a channel that forces the flow through an opening or aperture, often designed to measure the discharge.

Flow will converge and accelerate to a critical condition near the crest of the weir.

10.4 Energy Concepts

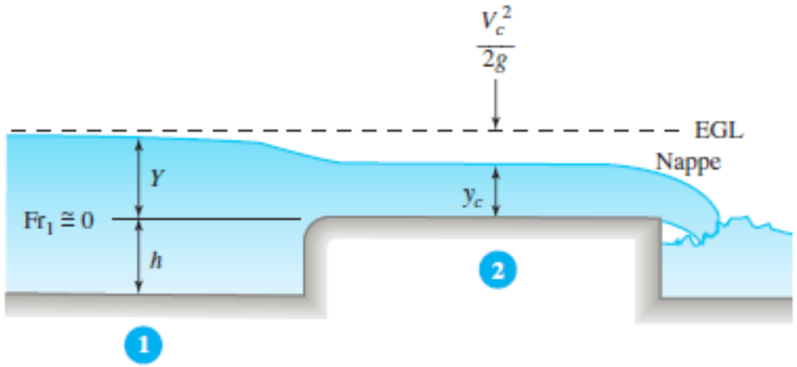


Figure 10.9 Broad-crested weir.

$$Q = \frac{2}{3}\sqrt{\frac{2}{3}g}\, b\, Y^{3/2}$$

10.4 Energy Concepts

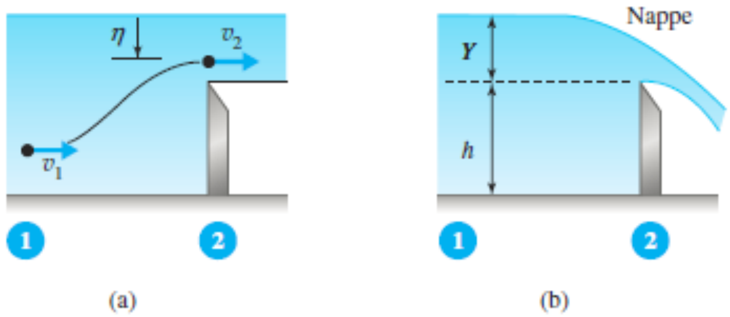


Figure 10.10 Rectangular sharp-crested weir: (a) ideal flow; (b) actual flow.

$$Q = C_d \frac{2}{3}\sqrt{2g}\, b Y^{3/2}$$

10.4 Energy Concepts

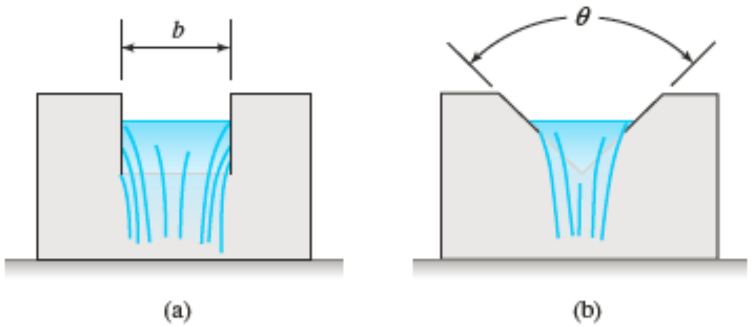


Figure 10.11 Contracted (a) rectangular and (b) V-notch weirs.

$$Q = C_d \frac{8}{15}\sqrt{2g}\left(\tan\frac{\theta}{2}\right)Y^{5/2}$$

10.4 Energy Concepts

Determine the discharge of water over a rectangular sharp-crested weir, $b = 1.25$ m, $Y = 0.35$ m, $h = 1.47$ m, with side walls and with end contractions. If a 90° V-notch weir were to replace the rectangular weir, what would be the required Y for a similar discharge?

Solution

For the rectangular weir, using Eq. 10.4.26, the discharge coefficient is

$$C_d = 0.61 + 0.08\frac{Y}{h} = 0.61 + 0.08 \times \frac{0.35}{1.47} = 0.63$$

Substitute into Eq. 10.4.25 and calculate

$$\begin{aligned} Q &= C_d \frac{2}{3}\sqrt{2g}\, b Y^{3/2} \\ &= 0.63 \times \frac{2}{3} \times \sqrt{2 \times 9.81} \times 1.25 \times 0.35^{3/2} \\ &= 0.48 \text{ m}^3/\text{s} \end{aligned}$$

With end contractions the effective width of the weir is reduced by $0.2Y$, resulting in

$$\begin{aligned} Q &= C_d \frac{2}{3}\sqrt{2g}(b - 0.2Y)\; Y^{3/2} \\ &= 0.63 \times \frac{2}{3} \times \sqrt{2 \times 9.81} \times (1.25 - 0.2 \times 0.35) \times 0.35^{3/2} \\ &= \underline{0.45 \text{ m}^3/\text{s}} \end{aligned}$$

With a discharge of $Q = 0.48$ m³/s, use Eq. 10.4.27 to find Y for the 90° V-notch weir:

$$\begin{aligned} Y &= \left[\frac{Q}{C_a \times \frac{8}{1.5} \times \sqrt{2g}\ \tan(\theta/2)}\right]^{2/5} \\ &= \left[\frac{0.482}{0.58 \times \frac{8}{15} \times \sqrt{2 \times 9.81} \times \tan 45°}\right]^{2/5} = \underline{0.66 \text{ m}} \end{aligned}$$

10.4 Energy Concepts

Parshall flume: An open flume where the throat is constricted to choke the flow to create critical flow followed by a hydraulic jump.

$$Q = 4BH^{1.522B^{0.026}}$$

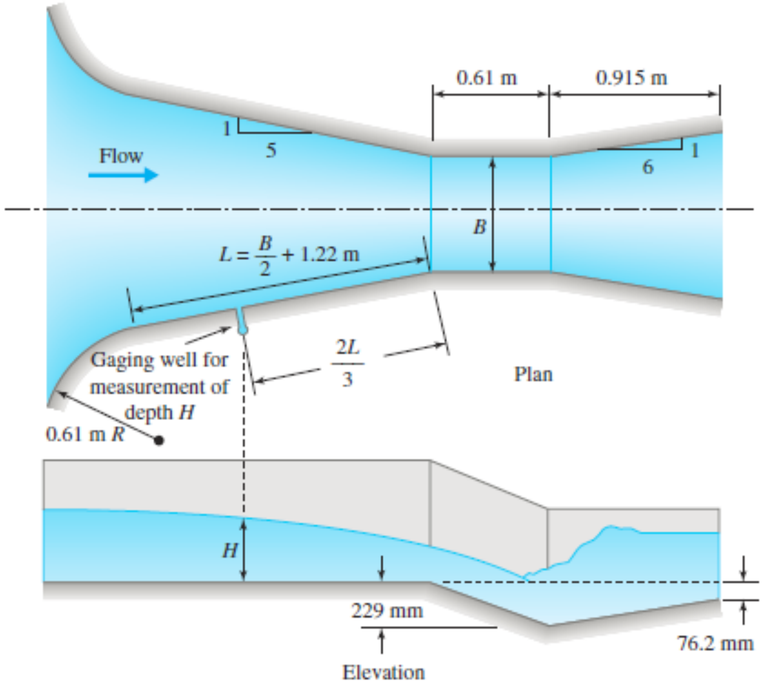


Figure 10.12 Parshall flume. (Based on Henderson, *Open Channel Flow*, 1st, 1966, Pearson Education, Inc., Upper Saddle River, New Jersey.)

10.5 Momentum Concepts

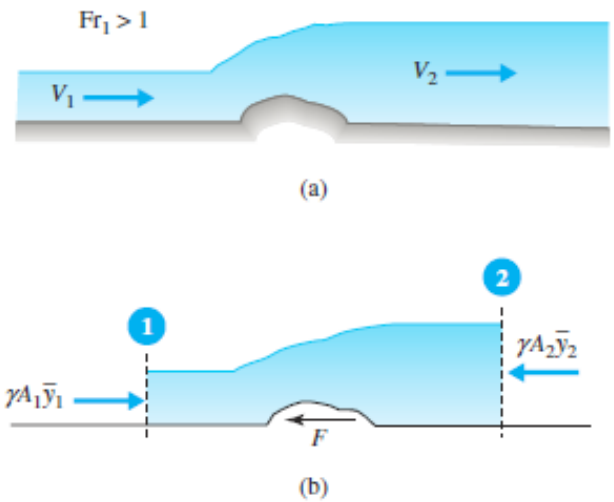


Figure 10.13 Channel flow over an obstacle: (a) idealized flow; (b) control volume.

$$M_1 - M_2 = \frac{F}{\gamma}$$

$$M = A\bar{y} + \frac{Q^2}{gA}$$

$$M = \frac{by^2}{2} + \frac{bq^2}{gy} = b\left(\frac{y^2}{2} + \frac{q^2}{gy}\right)$$

10.5 Momentum Concepts

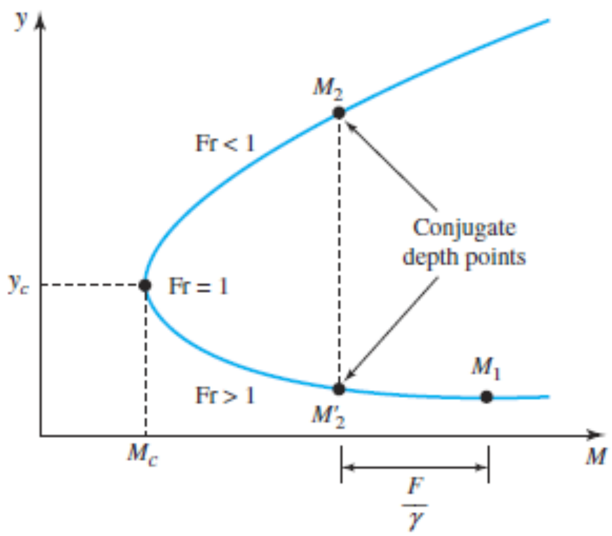


Figure 10.14 Variation of the momentum function with depth.

$$\frac{dM}{dy} = A - \frac{BQ^2}{gA^2} = 0 \qquad Q^2B = gA^3$$

10.5 Momentum Concepts

In a rectangular 5-m-wide channel, water is discharging at 14.0 m^3/s (Figure E10.7). Find the force exerted on the sluice gate when $y_1 = 2$ m and $y_2 = 0.5$ m.

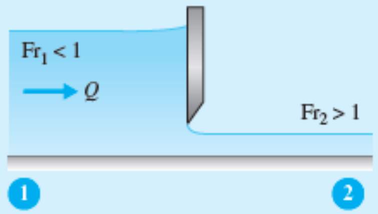


Figure E10.7

Solution

Using Eq. 10.5.3, the momentum functions at 1 and 2 are

$$M_1 = A_1\bar{y}_1 + \frac{Q^2}{gA_1}$$

$$= 5 \times 2 \times 1 + \frac{(14)^2}{9.81 \times 5 \times 2} = 12.0 \text{ m}^3$$

$$M_2 = A_2\bar{y}_2 + \frac{Q^2}{gA_2}$$

$$= 5 \times 0.5 \times 0.25 + \frac{(14)^2}{9.81 \times 5 \times 0.5} = 8.62 \text{ m}^3$$

The resultant force acting on the fluid control volume is determined, using Eq. 10.5.2, to be

$$F = \gamma(M_1 - M_2)$$

$$= 9800 \times (12.0 - 8.62) = \underline{33\,100 \text{ N}}$$

Hence the force on the gate acts in the downstream sense with a magnitude of 33.1 kN.

10.5 Momentum Concepts

Hydraulic Jump

A phenomenon where fluid flowing at a supercritical state will undergo a transition to a subcritical state.

$$\frac{y_2}{y_1} = \frac{1}{2}\left(\sqrt{1 + 8\mathrm{Fr}_1^2} - 1\right)$$

$$h_j = \frac{(y_2 - y_1)^3}{4y_1y_2}$$

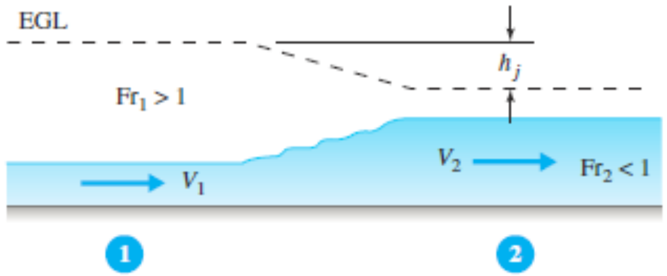
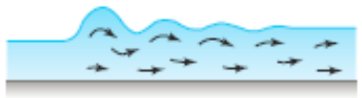
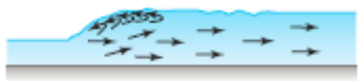
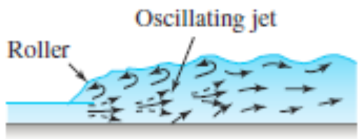


Figure 10.15 Idealized hydraulic jump.

10.5 Momentum Concepts

Table 10.2 Hydraulic Jumps in Horizontal Rectangular Channels

Upstream Fr	*Type*	*Description*	
1.0–1.7	Undular	Ruffled or undular water surface; surface rollers form near Fr = 1.7	
1.7–2.5	Weak	Prevailing smooth flow; low energy loss	
2.5–4.5	Oscillating	Intermittent jets from bottom to surface, causing persistent downstream waves	
4.5–9.0	Steady	Stable and well-balanced; energy dissipation contained in main body of jump	
>9.0	Strong	Effective, but with rough, wavy surface downstream	

Source: Adapted with permission from Chow, 1959. (Based on Chow, 1959)

10.5 Momentum Concepts

A hydraulic jump is situated in a 4-m-wide rectangular channel. The discharge in the channel is 7.5 m³/s, and the depth upstream of the jump is 0.20 m. Determine the depth downstream of the jump, the upstream and downstream Froude numbers, and the rate of energy dissipated by the jump.

Solution

Find the unit discharge and upstream Froude number:

$$q = \frac{Q}{b}$$
$$= \frac{7.5}{4} = 1.88\ \text{m}^2/\text{s}$$
$$\text{Fr}_1 = \frac{q}{\sqrt{g y_1^3}}$$
$$= \frac{1.88}{\sqrt{9.81 \times 0.20^3}} = \underline{6.71}$$

The downstream depth is computed, using Eq. 10.5.9, to be

$$y_2 = \frac{y_1}{2}\left(\sqrt{1 + 8\text{Fr}_1^2} - 1\right)$$
$$= \frac{0.20}{2}\left(\sqrt{1 + 8 \times 6.71^2} - 1\right) = \underline{1.80\ \text{m}}$$

The downstream Froude number is

$$\text{Fr}_2 = \frac{q}{\sqrt{g y_2^3}}$$
$$= \frac{1.88}{\sqrt{9.81 \times 1.80^3}} = \underline{0.25}$$

The head loss in the jump is given by Eq. 10.5.11:

$$h_j = \frac{(y_2 - y_1)^3}{4 y_1 y_2}$$
$$= \frac{(1.80 - 0.20)^3}{4 \times 0.20 \times 1.80} = 2.84\ \text{m}$$

Hence, the rate of energy dissipation in the jump is (see Eq. 4.5.25)

$$\gamma Q h_j = 9800 \times 7.5 \times 2.84 = 2.09 \times 10^5\ \text{W} \quad \text{or} \quad \underline{209\ \text{kW}}$$

10.5 Momentum Concepts

A translating hydraulic jump is a positive surge wave, maintaining a stable front as it propagates into an undisturbed region.

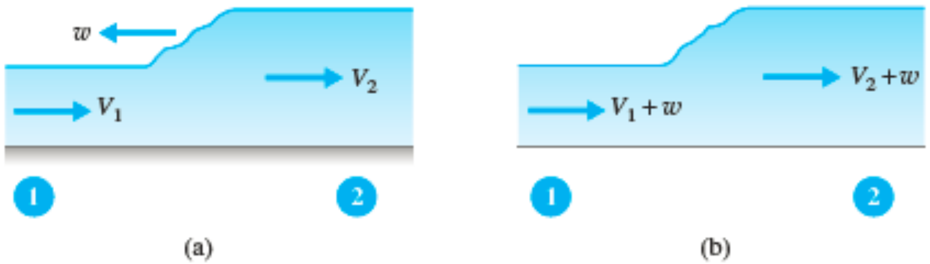


Figure 10.16 Translating hydraulic jump: (a) front moving upstream; (b) front appears stationary by superposition.

10.5 Momentum Concepts

Drag on Submerged Objects

$$F = C_D A \rho \frac{V^2}{2}$$

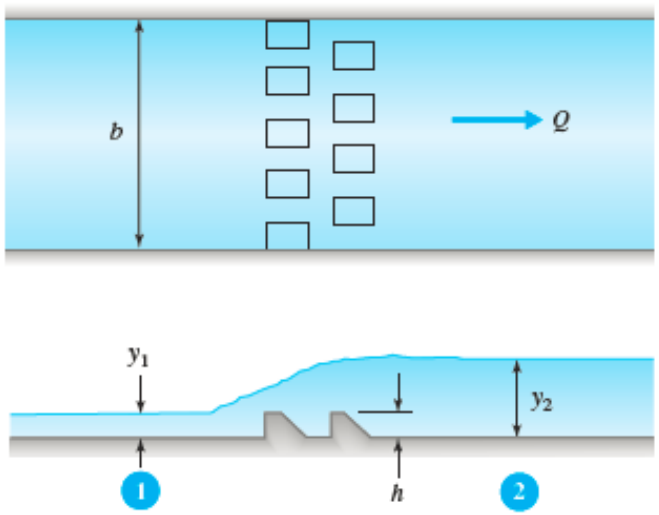


Figure 10.17 Stilling basin with baffle blocks.

10.5 Momentum Concepts

In the flow situation presented in Example 10.8, a series of baffle blocks is placed in the channel as shown in Figure E10.10. Laboratory experimentation has shown that the arrangement has an effective drag coefficient of 0.25, provided that the blocks are submerged in the flow. If the blocks are 0.15 m high, and if the discharge, upstream depth and width remain the same as in Example 10.8, determine the depth downstream of the jump and the rate of energy dissipated by the jump. The total width of the blocks is the channel width.

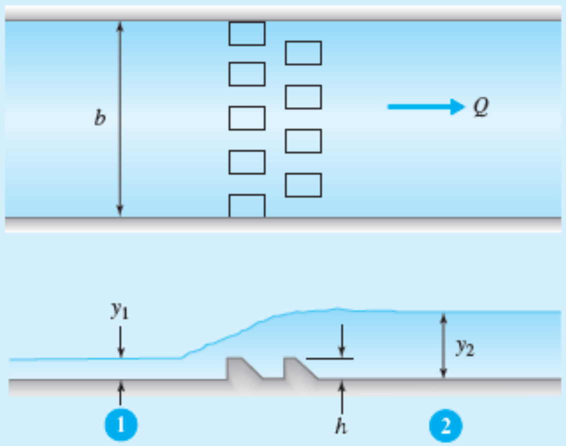


Figure E10.10

Solution

It is necessary to use Eq. 10.5.2 since obstacles (i.e., the baffle blocks) are placed within the control volume. The upstream velocity is

$$V_1 = \frac{Q}{A_1}$$

$$= \frac{7.5}{4 \times 0.2} = 9.83 \text{ m/s}$$

10.5 Momentum Concepts

The force F due to the presence of the baffle blocks is computed using Eq. 10.5.14:

$$F = C_D A \rho \frac{V_1^2}{2}$$

$$= 0.25 \times (4 \times 0.15) \times 1000 \times \frac{9.38^2}{2} = 6600 \text{ N}$$

Note that the frontal area is the width of the channel multiplied by the height of the blocks. Substituting known conditions into Eq. 10.5.2, making use of Eq. 10.5.4 which defines M for a rectangular channel, and noting that $q = 7.5/4 = 1.88 \text{ m}^2/\text{s}$, we find

$$b\left(\frac{y_1^2}{2} + \frac{q^2}{gy_1}\right) - b\left(\frac{y_2^2}{2} + \frac{q^2}{gy_2}\right) = \frac{F}{\gamma}$$

$$4\left(\frac{0.2^2}{2} + \frac{1.88^2}{9.81 \times 0.2}\right) - 4\left(\frac{y_2^2}{2} + \frac{1.88^2}{9.81 y_2}\right) = \frac{6600}{9800}$$

The relation reduces to

$$y_2^2 + \frac{0.721}{y_2} = 3.31$$

The trial-and-error solution for y_2 is $\underline{1.70 \text{ m}}$. The change in specific energy between locations 1 and 2 is

$$E_1 - E_2 = y_1 + \frac{q^2}{2gy_1^2} - \left(y_2 + \frac{q^2}{2gy_2^2}\right)$$

$$= 0.2 + \frac{1.88^2}{2 \times 9.81 \times 0.2^2} - \left(1.70 + \frac{1.88^2}{2 \times 9.81 \times 1.70^2}\right)$$

$$= 2.94 \text{ m}$$

The rate of energy dissipation, therefore, is

$$\gamma Q(E_1 - E_2) = 9800 \times 7.5 \times 2.94$$

$$= 2.16 \times 10^5 \text{ W} \quad \text{or} \quad \underline{216 \text{ kW}}$$

10.5 Momentum Concepts

Numerical Solution of the Momentum Equation

Consider a trapezoidal channel with conditions known at location 1 upstream of the jump.

$$M_2 - M_1 + \frac{F}{\gamma} = 0$$

$$\frac{y_2^2}{6}(2my_2 + 3b) + \frac{Q^2}{g(by_2 + my_2^2)} - M_1 + \frac{F}{\gamma} = 0$$

10.6 Nonuniform Gradually Varied Flow

Differential Equation for Gradually Varied Flow

$$\frac{dy}{dx} = \frac{S_0 - S}{1 - \mathrm{Fr}^2}$$

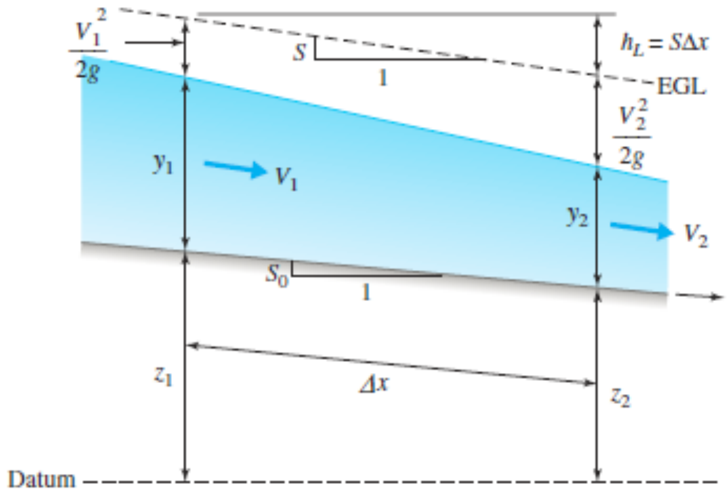
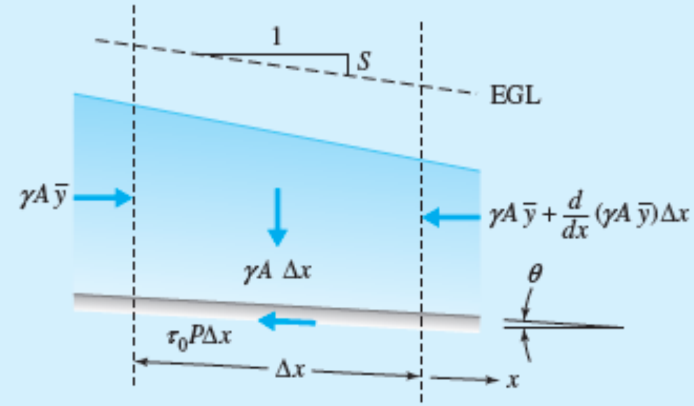


Figure 10.18 Nonuniform gradually varied flow.

10.6 Nonuniform Gradually Varied Flow

Using an appropriate control volume for gradually varied flow, show that the slope S of the energy grade line is equivalent to $\tau_0/\gamma R$.



10.6 Nonuniform Gradually Varied Flow

Solution

The control volume is shown in Figure E10.12. The resultant force acting on the control volume is due to the incremental change in hydrostatic pressure $[\gamma d(A\bar{y})/dx]\Delta x$, the component of weight in the x-direction $\gamma A \sin\theta\, \Delta x$, and the resistance term $\tau_0 P \Delta x$. Using the momentum equation

$$\Sigma F_x = \dot{m}(V_{2x} - V_{1x})$$

with $V_{2x} - V_{1x} = (dV/dx)\,\Delta x$ results in

$$-\gamma \frac{d}{dx}(A\bar{y})\,\Delta x + \gamma A \sin\theta\, \Delta x - \tau_0 P\, \Delta x = \rho V A \frac{dV}{dx}\Delta x$$

This relation can be simplified by noting that

$$\frac{d(A\bar{y})}{dx} = \frac{d(A\bar{y})}{dy}\frac{dy}{dx} = A\frac{dy}{dx}$$

and $P = A/R$. Substitute and divide the equation by $\gamma A \Delta x$, the weight of the control volume, and find that

$$-\frac{dy}{dx} + \sin\theta - \frac{\tau_0}{\gamma R} = \frac{V}{g}\frac{dV}{dx}$$

Since $\sin\theta \simeq S_0$ for small θ, the equation above can be rearranged in the form

$$\frac{\tau_0}{\gamma R} - S_0 = -\frac{dy}{dx} - \frac{V}{g}\frac{dV}{dx}$$
$$= -\frac{d}{dx}\left(y + \frac{V^2}{2g}\right)$$

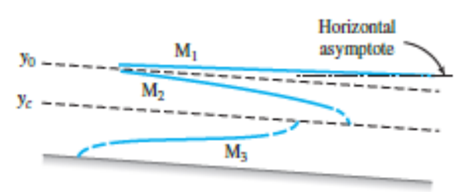
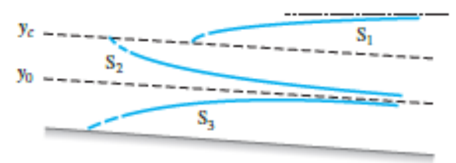
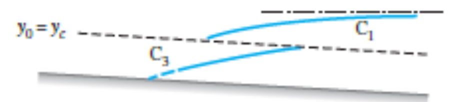
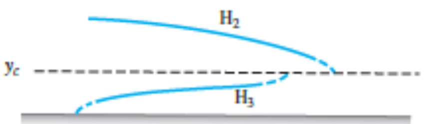
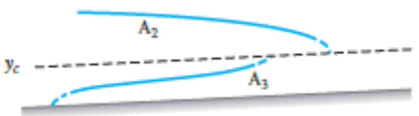
Upon comparison with Eq. 10.6.2, it is seen that the right-hand side is equivalent to $S - S_0$, and consequently,

$$\frac{\tau_0}{\gamma R} - S_0 = S - S_0$$

or

$$S = \frac{\tau_0}{\gamma R}$$

10.6 Nonuniform Gradually Varied Flow

Channel slope	Profile type	Depth range	Fr	$\frac{dy}{dx}$	$\frac{dE}{dx}$	
Mild $S_0 < S_c$ $y_0 > y_c$	M_1	$y > y_0 > y_c$	< 1	> 0	> 0	
	M_2	$y_0 > y > y_c$	< 1	< 0	< 0	
	M_3	$y_0 > y_c > y$	> 1	> 0	< 0	
Steep $S_0 > S_c$ $y_0 < y_c$	S_1	$y > y_c > y_0$	< 1	> 0	> 0	
	S_2	$y_c > y > y_0$	> 1	< 0	> 0	
	S_3	$y_c > y_0 > y$	> 1	> 0	< 0	
Critical $S_0 = S_c$ $y_0 = y_c$	C_1	$y > y_c$ or y_0	< 1	> 0	> 0	
	C_3	y_c or $y_0 > y$	> 1	> 0	< 0	
Horizontal $S_0 = 0$ $y_0 \to \infty$	H_2	$y > y_c$	< 1	< 0	< 0	
	H_3	$y_c > y$	> 1	> 0	< 0	
Adverse $S_0 < 0$ y_0 undefined	A_2	$y > y_c$	< 1	< 0	< 0	
	A_3	$y_c > y$	> 1	> 0	< 0	

10.6 Nonuniform Gradually Varied Flow

By assuming a wide rectangular channel, develop the right-hand side of Eq. 10.6.4 to show how dy/dx varies with y.

Solution

For a wide rectangular channel, assume that $b \gg y$, so that the wetted perimeter is approximated by $P \simeq b$. The hydraulic radius then becomes $R = A/P - (by)/b = y$. Noting that $Q = qb$, the Chezy-Manning equation, used to evaluate S, simplifies to

$$S = \frac{(qbn)^2}{\left(by^{5/3}\right)^2} = \frac{(qn)^2}{y^{10/3}}$$

It is assumed that in the Chezy-Manning equation $c_1 = 1$. For a rectangular section the square of the Froude number (see Eq. 10.4.9) is

$$\mathrm{Fr}^2 = \frac{q^2}{gy^3}$$

Substituting into Eq. 10.6.4 gives

$$\frac{dy}{dx} = \frac{S_0 - (qn)^2/y^{10/3}}{1 - q^2/(gy)^3}$$

Since $(qn)^2 y_0^{-10/3} = S_0$ and $\mathrm{Fr}_c^2 = q^2/(gy_c^3) = 1$, the relation can be written as

$$\frac{dy}{dx} = S_0 \frac{1 - (y_0/y)^{10/3}}{1 - (y_c/y)^3}$$

This equation can be used as an alternative to Eq. 10.6.4 to evaluate the water surface profiles shown in Table 10.3.

10.6 Nonuniform Gradually Varied Flow

Controls and Critical Flow

A control is a channel feature that establishes a depth-discharge relationship in its vicinity.

$$1 = \frac{1}{2}\left[\sqrt{1 + 8\frac{(V - w)^2}{gy}} - 1\right] \qquad w = V \pm \sqrt{gy} = V \pm c$$

If Fr < 1, the first wave would travel upstream, and the opposite wave would travel downstream at a speed less than *2c*. For Fr > 1 in the channel, since *V* > *c*, both waves are swept downstream.

10.6 Nonuniform Gradually Varied Flow

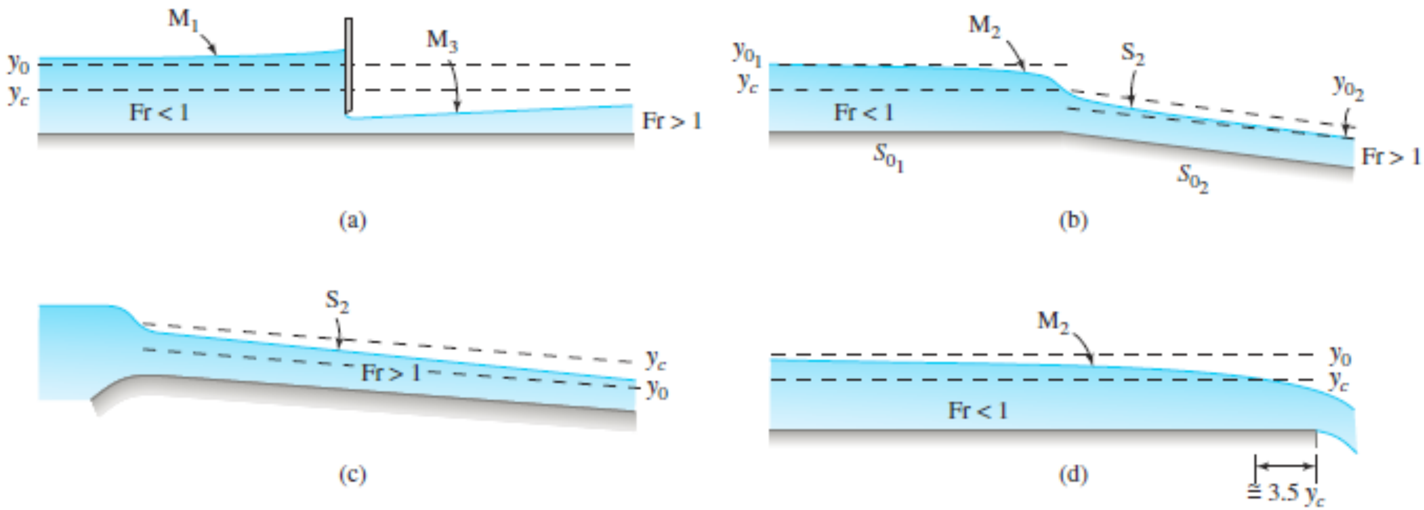


Figure 10.19 Representative controls: (a) sluice gate; (b) change in slope from mild (S_{01}) to steep (S_{02}); (c) entrance to a steep channel; (d) free outfall.

10.6 Nonuniform Gradually Varied Flow

Profile Synthesis

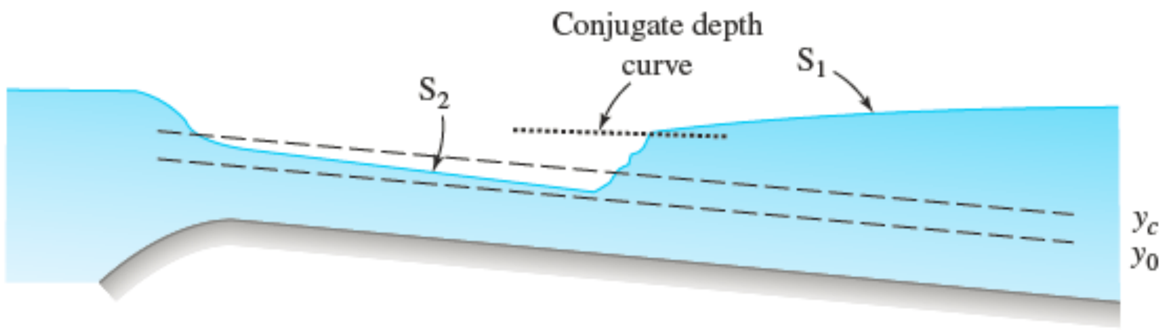


Figure 10.20 Example of profile synthesis.

10.6 Nonuniform Gradually Varied Flow

In a rectangular channel, $b = 3$ m, $n = 0.015$, $S_0 = 0.0005$, and $Q = 5\,\text{m}^3/\text{s}$. At the entrance to the channel, flow issues from a sluice gate at a depth of 0.15 m. The channel is sufficiently long that uniform flow conditions are established away from the entrance region, Figure E10.14a. Find the nature of the water surface profile in the vicinity of the entrance and the depth before the hydraulic jump.

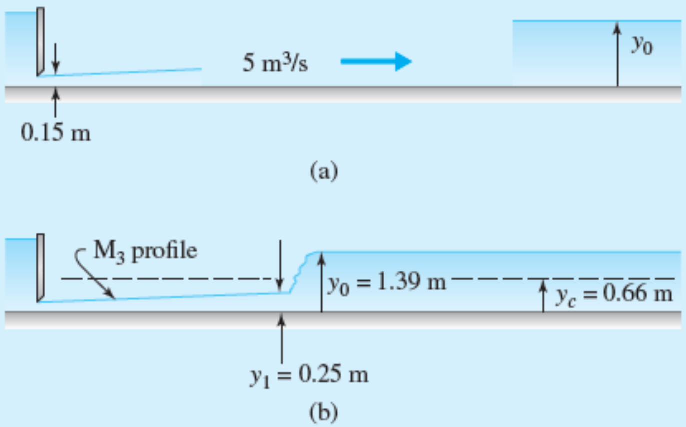


Figure E10.14

Solution

First find y_0 and y_c to determine the type of channel. To find y_0, follow the method shown in Example 10.1. Substitute known data into the Chezy-Manning equation:

$$\frac{3(y_0)^{5/3}}{(3 + 2y_0)^{2/3}} = \frac{0.015 \times 5}{\sqrt{0.0005}} = 3.354$$

10.6 Nonuniform Gradually Varied Flow

Solving gives $y_0 = 1.39$ m. Next, the critical depth is computed to be

$$y_c = \left(\frac{q^2}{g}\right)^{1/3}$$
$$= \left[\frac{(5/3)^2}{9.81}\right]^{1/3} = 0.66 \text{ m}$$

Since $y_0 > y_c$, a mild slope condition exists. The gate is a control and there will be an M_3 profile beginning at the entrance, terminated by a hydraulic jump. Downstream of the jump, the condition of uniform flow acts as a control, so at that location the depth is y_0, and the Froude number is

$$\text{Fr}_0 = \frac{q}{\sqrt{gy_0^3}}$$
$$= \frac{5/3}{\sqrt{9.81 \times 1.39^3}} = 0.325$$

Using Eq. 10.5.10, the depth before the jump is

$$y_1 = \frac{y_0}{2}\left(\sqrt{1 + 8Fr_0^2} - 1\right)$$
$$= \frac{1.39}{2}\left(\sqrt{1 + 8 \times 0.325^2} - 1\right) = \underline{0.25 \text{ m}}$$

The depths y_c and y_1 have been calculated to two significant figures, since the Manning coefficient is known to only two significant figures. Figure E10.14b shows the profile.

10.7 Numerical Analysis of Water Surface Profiles

$$\frac{Q\,n}{c_1 A R^{2/3}\sqrt{S_0}} - 1 = 0$$

$$\frac{Q^2 B}{g A^3} - 1 = 0$$

$$s(y) = \frac{Q^2 n^2}{c_1^2 \left[A(y)\right]^2 \left[R(y)\right]^{4/3}}$$

Standard Step Method

$$\begin{aligned}\frac{dE}{dx} &= \frac{d}{dx}\left(y + \frac{V^2}{2g}\right)\\ &= S_0 - S(y)\end{aligned}$$

10.7 Numerical Analysis of Water Surface Profiles

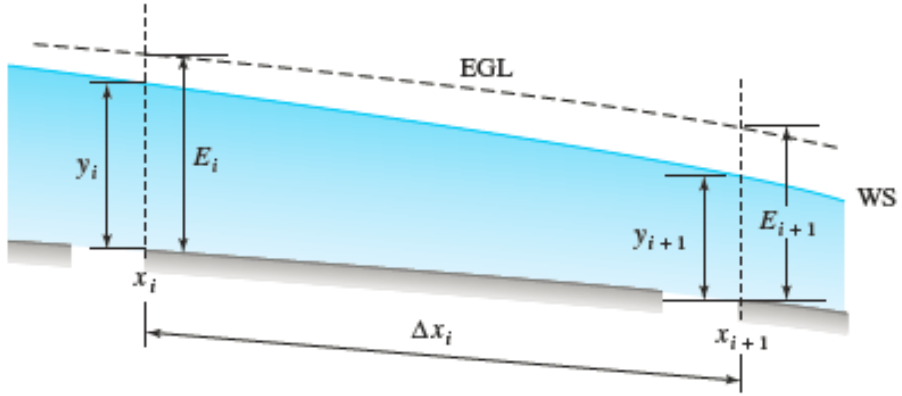


Figure 10.21 Notation for computing gradually varied flow.

$$E_{i+1} - E_i = \int_{x_i}^{x_{i+1}} [S_0 - S(y)]\ dx$$
$$\simeq (x_{i+1} - x_i)[S_0 - S(y_m)]$$

$$x_{i+1} = x_i + \frac{E_{i+1} - E_i}{S_0 - S(y_m)}$$

10.7 Numerical Analysis of Water Surface Profiles

Water is flowing at $Q = 22\ \text{m}^3/\text{s}$ in a long trapezoidal channel, $b = 7.5$ m, $m_1 = m_2 = 2.5$. A free overfall is located at the downstream end of the channel, where $x = 2000$ m. For $n = 0.015$, $S_0 = 0.0006$, find the water surface profile and energy grade line for a distance of approximately 800 m upstream from the free outfall.

Solution

Equations 10.7.1 and 10.7.2 are used to evaluate y_0 and y_c by substituting in known data:

$$\frac{22 \times 0.015 \times \left[7.5 + 2y_0\sqrt{1 + (2.5)^2}\right]^{2/3}}{\left[7.5y_0 + \frac{1}{2}y_0^2(2.5 + 2.5)\right]^{5/3}\sqrt{0.0006}} - 1 = 0$$

$$\frac{(22)^2 \times [7.5 + y_c(2.5 + 2.5)]}{9.8 \times \left[7.5y_c + \frac{1}{2}y_c^2(2.5 + 2.5)\right]^3} - 1 = 0$$

10.7 Numerical Analysis of Water Surface Profiles

The roots of these equations can be found using a routine such as Excel Solver®; the solutions are $y_0 = 1.29$ m and $y_c = 0.86$ m. Hence the channel is a mild type, and control will be close to the free overfall at the downstream end of the channel. Without any serious loss of accuracy, one can assume that critical conditions will exist at the free overfall. Referring to Table 10.3, the profile upstream of the overfall will be of type M_2. An Excel spreadsheet solution is shown in Table E10.15. The upper part shows the values of critical and normal depths found by using Solver. In the residual column are very small numbers that should be close to zero; see the two above equations. The lower part of the table shows the step method solution. Calculations proceed from station 1 to station 5 in a straightforward manner, with arbitrary values of depth selected and placed in the y column. The beginning value of x (2000 m) is placed in the first cell of the x column, and the remaining distances are computed as explained on the previous page. At station 6, different values of y are chosen until the distance is close to the desired value of 1200 m; a depth of 1.27 m results in a distance of 1230 m, which is acceptable. The spreadsheet equations for computing normal and critical depths, and for the step method, are provided in Appendix E. In addition, a MATLAB solution to this problem is shown in Appendix E, Figure E.5.

Table E10.15

	Depth [m]	Residual
Critical	0.865	1.087E-06
Normal	1.292	1.812E-06

Station	y[m]	A[m^2]	V[m/s]	E[m]	y_m[m]	S(y_m)	Δx[m]	x[m]
1	0.865	8.358	2.632	1.218				2000
2	0.950	9.381	2.345	1.230	0.908	2.165E-03	−8	1992
3	1.050	10.631	2.069	1.268	1.000	1.527E-03	−41	1951
4	1.150	11.931	1.844	1.323	1.100	1.081E-03	−114	1837
5	1.250	13.281	1.656	1.390	1.200	7.866E-04	−357	1480
6	1.270	13.557	1.623	1.404	1.260	6.574E-04	−250	1230

10.7 Numerical Analysis of Water Surface Profiles

Numerical Integration Method

A useful integration scheme is the two-point Gauss–Legendre quadrature.

$$x_{i+1} = x_i + \int_{y_i}^{y_{i+1}} \frac{1 - \mathrm{Fr}^2}{S_0 - S} dy$$
$$= x_i + \int_{y_i}^{y_{i+1}} G(y)\, dy$$

$$\int_{y_i}^{y_{i+1}} G(y)\, dy = \frac{y_{i+1} - y_i}{2}\left[G\left(\frac{y_{i+1} + y_i - \sqrt{3}/3(y_{i+1} - y_i)}{2}\right) + G\left(\frac{y_{i+1} + y_i + \sqrt{3}/3(y_{i+1} - y_i)}{2}\right)\right]$$

10.7 Numerical Analysis of Water Surface Profiles

Irregular Channels

$$S = \frac{Q^2}{(\Sigma K_i)^2} \qquad K_i = \left(\frac{c_1 A R^{2/3}}{n}\right)i \qquad \alpha = \frac{(\Sigma A_i)^2}{(\Sigma K_i)^3} \Sigma \left(\frac{K_i^3}{A_i^2}\right)$$

Direct Integration Methods

$$x = \frac{y_0}{S_0}\left[u - F(u,N) + \left(\frac{y_c}{y_0}\right)^M \frac{J}{N} F(v,J)\right]$$

$$F(u,N) = \int_0^u \frac{d\eta}{1-\eta^N}$$

10.7 Numerical Analysis of Water Surface Profiles

A wide rectangular channel conveys a discharge of $q = 3.72\ \text{m}^3/\text{s}$ per meter width on a slope of $S_0 = 0.001$. At a given location the depth is 3 m. Determine the distance upstream where the depth is 2.5 m. The Manning coefficient is 0.025.

Solution

By comparing Eq. 10.7.12 with the result of Example 10.13 for a wide rectangular channel, we find that $N = 3.33$ and $M = 3$. Therefore, one can calculate J to be

$$J = \frac{N}{N - M + 1} = \frac{3.33}{3.33 - 3 + 1} = 2.5$$

Also from Example 10.13, we can determine y_0 in the manner

$$y_0 = \left(\frac{q^2 n^2}{S_0}\right)^{3/10} = \left(\frac{3.72^2 \times 0.025^2}{0.001}\right)^{3/10} = 1.91\ \text{m}$$

Furthermore,

$$y_c = \left(\frac{q^2}{g}\right)^{1/3} = \left(\frac{3.72^2}{9.81}\right)^{1/3} = 1.12\ \text{m}$$

Substitute these values into Eq. 10.7.13 and simplify:

$$x = \frac{1.91}{0.001}\left[u - F(u, 3.33) + \left(\frac{1.12}{1.91}\right)^3 \times \frac{2.5}{3.33} F(v, 2.5)\right]$$

$$= 1910\left[u - F(u, 3.33) + 0.151 F(v, 2.5)\right]$$

10.7 Numerical Analysis of Water Surface Profiles

Since $y_0 > y_c$, and at the downstream location $y > y_0$, the profile is an M_1 curve. At the downstream location where the depth $y = 3$ m,

$$u = \frac{y}{y_0} = \frac{3}{1.91} = 1.57 \quad \text{and} \quad v = u^{N/J} = 1.57^{3.33/2.5} = 1.825$$

From Appendix E, Table E.1, making use of linear interpolation between recorded values we find that

$$F(1.57, 3.33) = 0.166 \quad \text{and} \quad F(1.825, 2.5) = 0.300$$

Considering x as a distance measured from an arbitrary datum, we find that

$$x = 1910(1.57 - 0.166 + 0.151 \times 0.300) = 2763 \text{ m}$$

To determine the distance upstream of the weir where the depth is 2.5 m, we perform the following calculations in a manner analogous to those at the downstream location:

$$u = \frac{2.5}{1.91} = 1.31 \quad \text{and} \quad v = 1.31^{3.33/2.5} = 1.43$$

$$F(1.31, 3.33) = 0.578 \quad \text{and} \quad F(1.43, 2.5) = 0.474$$

$$x = 1910(1.31 - 0.578 + 0.151 \times 0.474) = 1536 \text{ m}$$

Hence, the distance between the two locations, from a depth of 3 m to where the depth is 2.5 m, is $2763 - 1536 = 1227$ m, or approximately 1230 m.

10.8 Summary

Table 10.4 Formulas for Rectangular and General Sections

Section	Fr	y_c	E	M
Rectangular	$\dfrac{q}{\sqrt{gy^3}}$	$\left(\dfrac{q^2}{g}\right)^{1/3}$	$y + \dfrac{q^2}{2gy^2}$	$\dfrac{by^2}{2} + \dfrac{q^2}{gy}$
General	$\dfrac{Q/A}{\sqrt{gA/B}}$	$\dfrac{Q^2B}{gA^3} = 1$	$y + \dfrac{Q^2}{2gA^2}$	$A\bar{y} + \dfrac{Q^2}{gA}$