Chapter 5
Pipe Network Analysis

5.1 Introduction

Water distribution network analysis provides the basis for the design of new systems and the extension of existing systems. Design criteria are that specified minimum flow rates and pressure heads must be attained at the outflow points of the network. The flow and pressure distributions across a network are affected by the arrangement and sizes of the pipes and the distribution of the outflows. Since a change of diameter in one pipe length will affect the flow and pressure distribution everywhere, network design is not an explicit process. Optimal design methods almost invariably incorporate the hydraulic analysis of the system in which the pipe diameters are systematically altered (see, for example, Featherstone and El Jumailly, 1983).

Pipe network analysis involves the determination of the pipe flow rates and pressure heads which satisfy the continuity and energy conservation equations. These may be stated as follows:

(i) *Continuity*: The algebraic sum of the flow rates in the pipes meeting at a junction, together with any external flows, is zero:

$$\sum_{I=1}^{I=\mathrm{NP}(J)} Q_{IJ} - F_J = 0, \quad J = 1, \mathrm{NJ} \tag{5.1}$$

in which Q_{IJ} is the flow rate in pipe IJ at junction J, NP(J) the number of pipes meeting at junction J, F_J the external flow rate (outflow) at J and NJ the total number of junctions in the network.

(ii) *Energy conservation*: The algebraic sum of the head losses in the pipes, together with any heads generated by inline booster pumps, around any closed loop formed by pipes is zero.

$$\sum_{J=1}^{J=\mathrm{NP}(I)} h_{\mathrm{L},IJ} - H_{\mathrm{m},IJ} = 0, \quad I = 1, \mathrm{NL} \tag{5.2}$$

in which $h_{\mathrm{L},IJ}$ is the head loss in pipe J of loop I and $H_{\mathrm{m},IJ}$ is the manometric head generated by a pump in line IJ.

When the equation relating energy losses to pipe flow rate is introduced into Equations 5.1 or 5.2, systems of non-linear equations are produced. No method exists for the direct solution of such sets of equations and all methods of pipe network analysis are iterative. Pipe network analysis is therefore ideally suited for computer application but simple networks can be analysed with the aid of a calculator.

The earliest systematic method of network analysis, due to Professor Hardy-Cross, known as the head balance or 'loop' method is applicable to systems in which the pipes form closed loops. Assumed pipe flow rates, complying with the continuity requirement, Equation 5.1, are successively adjusted, loop by loop, until in every loop Equation 5.2 is satisfied within a specified small tolerance. In a similar later method, due to Cornish, assumed junction head elevations are systematically adjusted until Equation 5.1 is satisfied at every junction within a small tolerance; it is applicable to both open- and closed-loop networks. These methods are amenable to desk calculation but can also be programmed for computer analysis. However convergence is slow since the hydraulic parameter is adjusted at one element (either loop or junction) at a time. In later methods systems of simultaneous linear equations, derived from Equations 5.1 and 5.2 and the head loss–flow rate relationship, are formed, enabling corrections to the hydraulic parameters (flows or heads) to be made over the whole network simultaneously. Convergence is much more rapid but since a number of simultaneous linear equations, depending on the size of the network, have to be solved, these methods are only realistically applicable to computer evaluation.

The majority of the worked examples in this chapter illustrate the use of Equations 5.1 and 5.2 in systems which can be analysed by desk calculation using either the head balance or quantity balance methods. In addition to friction losses, the effect of local losses and booster pumps is shown. The networks illustrated have been analysed by computer but the intermediate steps in the computations have been reproduced, enabling the reader to follow the process as though it were by desk calculation; the numbers have been rounded to an appropriate number of decimal places. An example showing the gradient method is also given.

5.2 The head balance method ('loop' method)

This method is applicable to closed-loop pipe networks. It is probably more widely applied to this type of network than is the quantity balance method. The head balance method was originally devised by Professor Hardy-Cross and is often referred to as the Hardy-Cross method. Figure 5.1 represents the main pipes in a water distribution network.

The outflows from the system are generally assumed to occur at the nodes (junctions); this assumption results in uniform flows in the pipelines, which simplifies the analysis.

For a given pipe system with known junction outflows, the head balance method is an iterative procedure based on initially estimated flows in the pipes. At each junction these flows must satisfy the continuity criterion.

The head balance criterion is that the algebraic sum of the head losses around any closed loop is zero; the sign convention that clockwise flows (and the associated head losses) are positive is adopted.

The head loss along a single pipe is

$$h = KQ^2$$

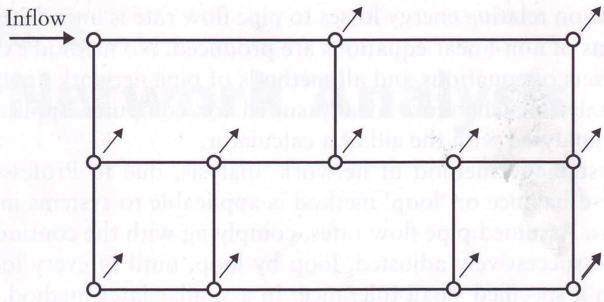

Figure 5.1 Closed-loop pipe network.

If the flow is estimated with an error ΔQ,

$$h = K(Q + \Delta Q)^2 = K[Q^2 + 2Q\Delta Q + \Delta Q^2]$$

Neglecting ΔQ^2 and assuming ΔQ to be small,

$$h = K(Q^2 + 2Q\Delta Q)$$

Now round a closed loop $\sum h = 0$ and ΔQ is the same for each pipe to maintain continuity.

$$\sum h = \sum KQ^2 + 2\Delta Q \sum KQ = 0$$

$$\Rightarrow \Delta Q = -\frac{\sum KQ^2}{2\sum KQ} = -\frac{\sum KQ^2}{2\sum KQ^2/Q}$$

which may be written as $\Delta Q = -\frac{\sum h}{2\sum h/Q}$, where h is the head loss in a pipe based on the estimated flow Q.

5.3 The quantity balance method ('nodal' method)

Figure 5.2 shows a branched-type pipe system delivering water from the impounding reservoir A to the service reservoirs B, C and D. F is a known direct outflow from the node J.

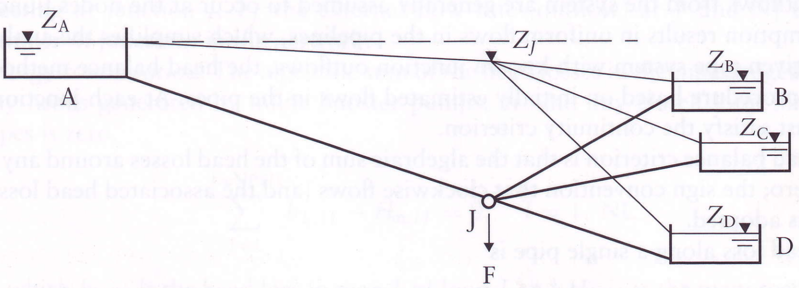

Figure 5.2 Branched-type pipe network.

If Z_J is the true elevation of the pressure head at J, the head loss along each pipe can be expressed in terms of the difference between Z_J and the pressure head elevation at the other end.

For example: $h_{L,AJ} = Z_A - Z_J$.

Expressing the head loss in the form $h = KQ^2$, N such equations can be written as (where N is the number of pipes)

$$\begin{bmatrix} Z_A - Z_J \\ Z_B - Z_J \\ \vdots \\ Z_I - Z_J \end{bmatrix} \begin{matrix} = \\ = \\ \\ = \end{matrix} \begin{bmatrix} (\text{SIGN})\ K_{AJ}(|Q_{AJ}|)^2 \\ (\text{SIGN})\ K_{BJ}(|Q_{BJ}|)^2 \\ \vdots \\ (\text{SIGN})\ K_{IJ}(|Q_{IJ}|)^2 \end{bmatrix} \quad [5.3]$$

and in general, (SIGN) is + or − according to the sign of $(Z_I - Z_J)$. Thus flows towards the junction are positive and flows away from the junction are negative.

K_{IJ} is composed of the friction loss and minor loss coefficients.

The continuity equation for flow rates at J is

$$\sum Q_{IJ} - F = Q_{AJ} + Q_{BJ} + Q_{CJ} + Q_{DJ} - F = 0 \quad [5.4]$$

Examination of Equations 5.3 and 5.4 shows that the correct value of Z_J will result in values of Q_{IJ}, calculated from Equation 5.3, which will satisfy Equation 5.4.

Rearranging Equation 5.3 we have

$$[Q_{IJ}] = \left[(\text{SIGN}) \left(\frac{|Z_I - Z_J|}{K_{IJ}} \right)^{1/2} \right] \quad [5.5]$$

The value of Z_J can be found using an iterative method by making an initial estimate of Z_J, calculating the pipe discharges from Equation 5.5 and testing the continuity condition in Equation 5.4.

If $(\sum Q_{IJ} - E) \neq 0$ (with acceptable limits), a correction ΔZ_J is made to Z_J and the procedure repeated until Equation 5.4 is reasonably satisfied. A systematic correction for ΔZ_J can be developed: expressing the head loss along a pipe as $h = KQ^2$, for a small error in the estimate Z_J, the correction ΔZ_J can be derived as

$$\Delta Z_J = \frac{2(\sum Q_{IJ} - F)}{\sum Q_{IJ}/h_{IJ}}$$

Example 5.7 shows the procedure for networks with multiple unknown junction head elevations.

Evaluation of K_{IJ}:

$$K_{IJ} = \frac{\lambda L}{2gDA^2} + \frac{C_m}{2gA^2} \ (= K_f + K_m)$$

where C_m is the sum of the minor loss coefficients. λ can be obtained from the Moody chart using an initially assumed value of velocity in the pipe (say 1 m/s). A closer approximation to the velocity is obtained when the discharge is calculated. For automatic computer analysis Equation 5.5 should be replaced by the Darcy–Colebrook–White combination:

$$Q = -2A\sqrt{2gD\frac{h_f}{L}}\,\log\left(\frac{k}{3.7D} + \frac{2.51\nu}{D\sqrt{2gDh_f/L}}\right) \qquad [5.6]$$

For each pipe, $h_{f,IJ}$ (friction head loss) is initialised to $Z_I - Z_J$, Q_{IJ} calculated from Equation 5.6 and $h_{f,IJ}$ re-evaluated from $h_{f,IJ} = (Z_I - Z_J) - K_m Q_{IJ}^2$. This subroutine follows the procedure of Example 4.2.

5.4 The gradient method

In addition to Equations 5.1–5.6, the gradient method needs the following vector and matrix definitions:

NT = number of pipelines in the network

NN = number of unknown piezometric head nodes

[A12] = 'connectivity matrix' associated with each one of the nodes. Its dimension is NT × NN with only two non-zero elements in the ith row:

−1 in the column corresponding to the initial node of pipe i

1 in the column corresponding to the final node of pipe i

NS = number of fixed head nodes

[A10] = topologic matrix: pipe to node for the NS fixed head nodes. Its dimension is NT × NS with a −1 value in rows corresponding to pipelines connected to fixed head nodes

Thus, the head loss in each pipe between two nodes is

$$[\mathrm{A11}][\mathrm{Q}] + [\mathrm{A12}][\mathrm{H}] = -[\mathrm{A10}][\mathrm{H_0}] \qquad [5.7]$$

where

[A11] = diagonal matrix of NT × NT dimension, defined as

$$[\mathrm{A11}] = \begin{bmatrix} \alpha_1 Q_1^{(n_1-1)} + \beta_1 + \frac{\gamma_1}{Q_1} & 0 & \cdots & 0 \\ 0 & \alpha_2 Q_2^{(n_2-1)} + \beta_2 + \frac{\gamma_2}{Q_2} & \cdots & 0 \\ \vdots & \vdots & \vdots\vdots\vdots & \vdots \\ 0 & 0 & \cdots & \alpha_{\mathrm{NT}} Q_{\mathrm{NT}}^{(n_{\mathrm{NT}}-1)} + \beta_{\mathrm{NT}} + \frac{\gamma_{\mathrm{NT}}}{Q_{\mathrm{NT}}} \end{bmatrix} \qquad [5.8]$$

[Q] = discharge vector with NT × 1 dimension

[H] = unknown piezometric head vector with NN × 1 dimension

[$\mathrm{H_0}$] = fixed piezometric head vector with NS × 1 dimension

Equation 5.7 is an energy conservation equation. The continuity equation for all nodes in the network is

$$[\mathrm{A21}][\mathrm{Q}] = [\mathrm{q}] \qquad [5.9]$$

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where [A21] is the transpose matrix of [A12] and [q] water consumption and water supply vector in each node with NN × 1 dimension.

In matrix form, Equations 5.7 and 5.9 are

$$\begin{bmatrix} [A11] & [A12] \\ [A21] & [0] \end{bmatrix} \begin{bmatrix} [Q] \\ [H] \end{bmatrix} = \begin{bmatrix} -[A10][H_0] \\ [q] \end{bmatrix} \quad [5.10]$$

The upper part is *nonlinear*, which implies that Equation 5.10 must use some *iterative* algorithm for its solution. Gradient method consists of a truncated Taylor expansion. Operating simultaneously on ([Q], [H]) field and applying the gradient operator, we can write

$$\begin{bmatrix} [N][A11]' & [A12] \\ [A21] & [0] \end{bmatrix} \begin{bmatrix} [dQ] \\ [dH] \end{bmatrix} = \begin{bmatrix} [dE] \\ [dq] \end{bmatrix} \quad [5.11]$$

where [N] is the diagonal matrix ($n_1, n_2, \ldots, n_{NT}$) with NT × NT dimension and [A11]′ = NT × NT matrix defined as

$$[A11]' = \begin{bmatrix} \alpha_1 Q_1^{(n_1-1)} & 0 & 0 & \cdots & 0 \\ 0 & \alpha_2 Q_2^{(n_2-1)} & 0 & \cdots & 0 \\ 0 & 0 & \alpha_3 Q_3^{(n_3-1)} & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots\vdots\vdots & \vdots \\ 0 & 0 & 0 & \cdots & \alpha_{NT} Q_{NT}^{(n_{NT}-1)} \end{bmatrix} \quad [5.12]$$

In any iteration **i**, [dE] is the energy imbalance in each pipe and [dq] is the discharge imbalance in each node. These are given by

$$[dE] = [A11][Q_i] + [A12][H_i] + [A10][H_0] \quad [5.13]$$

and

$$[dq] = [A21][Q_i] - [q] \quad [5.14]$$

The objective of the gradient method is to solve the system described by Equation 5.11, taking into account that in each iteration

$$[dQ] = [Q_{i+1}] - [Q_i] \quad [5.15]$$

and

$$[dH] - [H_{i+1}] - [H_i] \quad [5.16]$$

Using matrix algebra, it is possible to show that the solution to the system represented by Equation 5.11 is

$$[H_{i+1}] = -\{[A21]([N][A11]')^{-1}[A12]\}^{-1}\{[A21]([N][A11]')^{-1} ([A11][Q_i]) + [A10][H_0] - ([A21][Q_i]) - [q]\} \quad [5.17]$$

$$[Q_{i+1}] = \{[I] - ([N][A11]') - [A11]\}[Q_i] - \{([N][A11]')^{-1}([A12] [H_{i+1}] + [A10][H_0])\} \quad [5.18]$$

The method has the advantage of fast convergence and does not need continuity balancing in each node to begin the process. The method is not suited for hand calculation. Example 5.8 illustrates the methodology.

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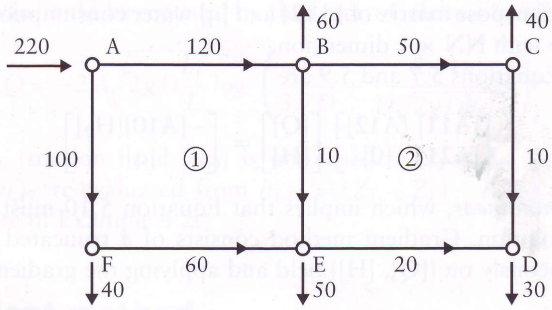

Figure 5.3 Two-loop network.

Worked examples

Example 5.1

Neglecting minor losses in the pipes, determine the flows in the pipes and the pressure heads at the nodes (see Figure 5.3).

Data

Pipe	AB	BC	CD	DE	EF	AF	BE
Length (m)	600	600	200	600	600	200	200
Diameter (mm)	250	150	100	150	150	200	100

Roughness size of all pipes = 0.06 mm
Pressure head elevation at A = 70 m o.d.

Elevation of pipe nodes

Node	A	B	C	D	E	F
Elevation (m o.d.)	30	25	20	20	22	25

Procedure:

1. Identify loops. When using hand calculation the simplest way is to employ adjacent loops, e.g. Loop 1: ABEFA; Loop 2: BCDEB.
2. Allocate estimated flows in the pipes. Only one estimated flow in each loop is required; the remaining flows follow automatically from the continuity condition at the nodes; e.g. since the total required inflow is 220 L/s, if Q_{AB} is estimated at 120 L/s, then $Q_{AF} = 100$ L/s. The initial flows are shown in Figure 5.3.
3. The head loss coefficient $K = \lambda L/2gDA^2$ is evaluated for each pipe, λ being obtained from the λ vs. *Re* diagram (Figure 4.2) corresponding to the flow in the pipe. Alternatively, Barr's equation (Equation 4.12) may be used.

If the Reynolds numbers are fairly high ($\nless 10^5$), it may be possible to proceed with the iterations using the initial λ values, making better estimates as the solution nears convergence.

The calculations proceed in tabular form. Note that Q is written in litres per second simply for convenience; all computations are based on Q in cubic metres per second. However, h/Q could have been expressed in m/(L/s) yielding ΔQ directly in litres per second.

	Pipe	k/D	Q (L/s)	Re ($\times 10^5$)	λ	K	h (m)	$h/Q\left(\frac{\text{m}}{\text{m}^3/\text{s}}\right)$
Loop 1	AB	0.00024	120.00	5.41	0.0157	797.0	11.48	95.64
	BE	0.00060	10.00	1.31	0.0205	33 877.0	3.39	338.77
	EF	0.00040	−60.00	4.51	0.0172	11 229.1	−40.42	673.75
	FA	0.00030	−100.00	5.63	0.0162	336.6	−8.36	83.66
						$\sum$	−33.91	1191.82

$$\Rightarrow \Delta Q = \frac{-\sum h}{2\sum h/Q} = \frac{-(-33.91)}{2 \times 1191.82} = 0.01423 = 14.23 \text{ L/s}.$$

	Pipe	Q (L/s)	Re ($\times 10^5$)	λ	K	h (m)	$h/Q\left(\frac{\text{m}}{\text{m}^3/\text{s}}\right)$
Loop 2	BC	50.0	3.76	0.0174	11 359.7	28.40	567.98
	CD	10.0	1.13	0.0205	33 877.0	3.39	338.77
	DE	−20.0	1.50	0.0189	12 338.9	−4.94	246.78
	EB	−24.23	2.73	0.0189	31 232.9	−18.34	756.77
					$\sum$	−8.51	1910.30

$\Rightarrow \Delta Q = -2.23$ L/s.

(Note that the previously corrected value of flow in the 'common' pipe EB has been used in Loop 2.)

	Pipe	Q (L/s)	Re ($\times 10^5$)	λ	K	h (m)	$h/Q\left(\frac{\text{m}}{\text{m}^3/\text{s}}\right)$
Loop 1	AB	134.23	6.05	0.0156	791.9	14.27	106.30
	BE	26.46	2.98	0.0188	31 067.7	21.75	822.05
	EF	−45.77	3.44	0.0175	11 424.9	−23.93	522.92
	FA	−85.77	4.83	0.0164	846.9	−6.23	72.64
					$\sum$	5.86	1523.91

$\Rightarrow \Delta Q = -1.92$ L/s.

Proceed to loop 2 again, and continuing in this way the solution is obtained within the required specified limit on $\sum h$ in any loop after several further iterations. The solution given is obtained for $\sum h < 0.01$ m but an acceptable result may be achieved with a larger tolerance.

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Final values

Pipe	Q (L/s)	h (m)
AB	131.55	13.70
BE	25.02	19.55
FE	48.45	26.67
AF	88.45	6.59
BC	46.53	24.74
CD	6.55	1.52
ED	23.47	6.69

Note: Flows in direction of pipe identifier, e.g. A → B.

Pressure heads

Node	Pressure head (m)
A	40.00
B	31.29
C	11.57
D	10.05
E	14.74
F	38.41

Example 5.2

In the network shown in Figure 5.4 a valve in BC is partially closed to produce a local head loss of $10V_{BC}^2/2g$. Analyse the flows in the network.

Pipe	AB	BC	CD	DE	BE	EF	AF
Length (m)	500	400	200	400	200	600	300
Diameter (mm)	250	150	100	150	150	200	250

Note: Roughness of all pipes is 0.06 mm.

Solution:

The procedure is identical with that of the previous problem. K_{BC} is now composed of the valve loss coefficient and the friction loss coefficient.

With the initial assumed flows shown in the table below, $Q_{BC} = 50$ L/s; $Re = 3.7 \times 10^5$; $k/D = 0.0004$; $\lambda = 0.0174$ (from the Moody chart). Hence, $K_f = 7573$, $K_m = 1632$ and $K_{BC} = 9205$.

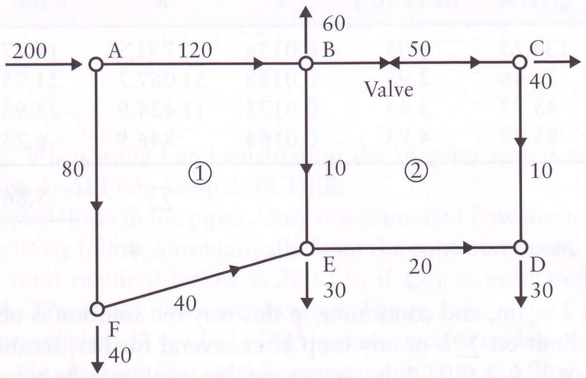

Figure 5.4 Pipe network with valve losses.

	Pipe	k/D	Q (L/s)	$Re\ (\times 10^5)$	λ	K	h (m)	$h/Q\left(\frac{\mathrm{m}}{\mathrm{m}^3/\mathrm{s}}\right)$
Loop 1	AB	0.00024	120.00	5.41	0.0157	664.2	9.56	79.70
	BE	0.00040	10.00	0.75	0.0208	4 526.5	0.45	45.26
	EF	0.00030	−40.00	2.25	0.0175	2 711.2	−4.34	108.45
	FA	0.00024	−80.00	3.61	0.0163	413.7	−2.65	33.10
						$\sum$	3.03	266.51

$\Rightarrow \Delta Q = -5.69$ L/s.

	Pipe	k/D	Q (L/s)	$Re\ (\times 10^5)$	λ	K	h (m)	$h/Q\left(\frac{\mathrm{m}}{\mathrm{m}^3/\mathrm{s}}\right)$
Loop 2	BC	0.0004	50.00	3.75	0.0174	9 205.2	23.01	460.26
	CD	0.0006	10.00	1.13	0.0205	33 877.0	3.39	338.77
	DE	0.0004	−20.00	1.50	0.0190	8 226.0	−3.29	164.52
	EB	0.0004	−4.31	0.32	0.0242	5 266.4	−0.10	22.70
						$\sum$	23.01	986.25

$\Rightarrow \Delta Q = -11.67$ L/s.

Proceeding in this way the solution is obtained within a small limit on $\sum h$ in any loop:

Final values

Pipe	AB	BE	FE	FA	BC	CD	ED
Q (L/s)	111.52	16.48	48.48	88.48	35.05	4.95	34.95
h_L (m)	8.31	1.15	6.26	3.20	11.57	0.91	9.52

Example 5.3

If in the network shown in Example 5.2 a pump is installed in line BC boosting the flow towards C and the valve removed, analyse the network. Assume that the pump delivers a head of 10 m. (*Note:* In reality, it would not be possible to predict the head generated by the pump since this will depend upon the discharge. The head–discharge relationship for the pump, e.g. $H = AQ^2 + BQ + C$, must therefore be solved for the discharge in the pipe at each iteration. However, for the purpose of illustration of the basic effect of a pump the head in this case is assumed to be known.) An example of a network analysis in which the pump head–discharge curve is used is given in Chapter 6 (Example 6.8). Consider length BC (see Figure 5.5).

The net loss of head along $BC(Z_B - Z_C)$ is $(h_f - H_p)$, where H_p is the total head delivered by pump. The value of K for BC is now due to friction only; the head loss for BC in the table now becomes $h_{L,BC} = (KQ_{BC}^2 - 10)$ m. Otherwise the iterative procedure is as before.

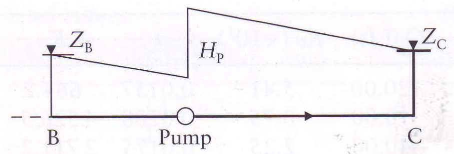

Figure 5.5 Network of Example 5.2 with pump.

Solution:

	Pipe	k/D	Q (L/s)	Re ($\times 10^5$)	λ	K	h (m)	$h/Q\left(\frac{\text{m}}{\text{m}^3/\text{s}}\right)$
Loop 1	AB	0.00024	120.00	5.41	0.0157	664.2	9.56	79.70
	BE	0.00040	10.00	0.75	0.0208	4 526.5	0.45	45.26
	EF	0.00030	−40.00	2.25	0.0175	2 711.2	−4.34	108.45
	FA	0.00024	−80.00	3.61	0.0163	413.7	−2.65	33.10
						$\sum$	3.03	266.51

$\Rightarrow \Delta Q = -5.69$ L/s.

	Pipe	k/D	Q (L/s)	Re ($\times 10^5$)	λ	K	h (m)	$h/Q\left(\frac{\text{m}}{\text{m}^3/\text{s}}\right)$
Loop 2	BC	0.00040	50.00	3.76	0.0174	7 573.0	8.93	178.66
	CD	0.00060	10.00	1.13	0.0205	33 877.0	3.39	333.77
	DE	0.00040	−20.00	1.50	0.0189	8 225.96	−3.29	164.52
	EB	0.00040	−4.31	0.32	0.0242	5 266.4	−0.10	22.70
						$\sum$	8.93	704.65

$\Rightarrow \Delta Q = -6.34$ L/s.

	Pipe	Q (L/s)	Re ($\times 10^5$)	λ	K	h (m)	$h/Q\left(\frac{\text{m}}{\text{m}^3/\text{s}}\right)$
Loop 1	AB	114.31	5.15	0.0158	668.4	8.73	76.41
	BE	10.65	0.80	0.0206	4 482.9	0.51	47.74
	EF	−45.69	2.57	0.0173	2 680.2	−5.59	122.46
	FA	−85.69	3.66	0.0162	411.2	−3.02	35.24
					$\sum$	0.63	281.85

$\Rightarrow \Delta Q = -1.11$ L/s.

After similar further iterations:

Final values

Pipe	AB	BE	FE	FA	BC	CD	ED
Q (L/s)	113.21	8.90	46.79	86.79	44.30	4.30	25.70
h_L (m)	8.57	0.37	5.83	3.10	4.95	0.71	5.29

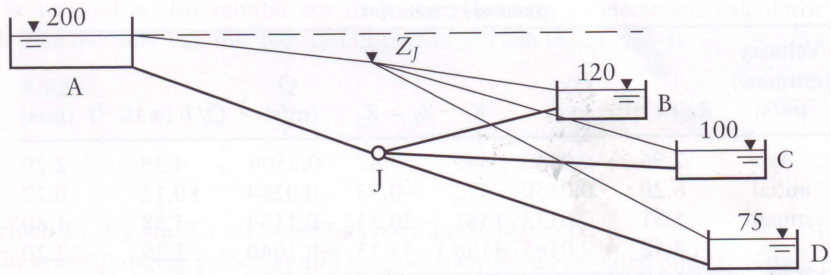

Figure 5.6 Network connecting multi-reservoirs.

Example 5.4

Determine the discharges in the pipes of the network shown in Figure 5.6 neglecting minor losses.

Pipe	Length (m)	Diameter (mm)
AJ	10 000	450
BJ	2 000	350
CJ	3 000	300
DJ	3 000	250

Note: Roughness size of all pipes is 0.06 mm.

The friction factor λ may be obtained from the Moody diagram, or using Barr's equation, using an initially estimated velocity in each pipe. Subsequently, λ can be based on the previously calculated discharges. However, unless there is a serious error in the initial velocity estimates, much effort is saved by retaining the initial λ values until perhaps the penultimate or final correction.

Solution:

Estimate Z_J (pressure head elevation at J) = 150.0 m a.o.d. (*Note*: the elevation of the pipe junction itself does not affect the solution.) See tables below and on p. 128.

First correction

Pipe	Velocity (estimate) (m/s)	$Re\,(\times 10^5)$	λ	K	$Z_I - Z_J$	Q (m/s)	$Q/h\,(\times 10^{-3})$	Q/A (m/s)
AJ	2.0	7.96	0.0145	649	+50	0.2775	5.55	1.75
BJ	2.0	6.20	0.0150	472	−30	−0.2521	8.40	2.62
CJ	2.0	5.31	0.0155	1581	−50	−0.1778	3.56	2.50
DJ	2.0	4.42	0.0165	4188	−75	−0.1338	1.78	2.73
					Σ	−0.2862	0.0193	

$$\Rightarrow \text{Correction to } Z_J = \frac{2(-0.2862)}{0.0193} = -29.67;\ Z_J = 120.33\,\text{m}.$$

Second correction

Pipe	Velocity (estimate) (m/s)	Re ($\times 10^5$)	λ	K	$Z_I - Z_J$	Q (m/s)	Q/h ($\times 10^{-3}$)	Q/A (m/s)
AJ	As	7.96	0.0145	649	79.67	0.3504	4.39	2.20
BJ	initial	6.20	0.0150	472	−0.33	−0.0264	80.12	0.27
CJ	estimate	5.31	0.0155	1581	−20.33	−0.1134	5.58	1.60
DJ		4.42	0.0165	4188	−45.33	−0.1040	2.29	2.20
					$\sum$	+0.1066	+0.092	

$\Rightarrow \Delta Z_J = +2.30$ m; $Z_J = 122.63$ m.

Comment: The velocity in BJ has changed significantly but it may oscillate; it is therefore estimated at 1.0 m/s for next correction. Note that λ (BJ) altered accordingly.

Third correction

Pipe	Velocity (estimate) (m/s)	λ	K	$Z_I - Z_J$	Q (m³/s)	Q/h ($\times 10^{-3}$)	Q/A (m/s)
AJ	2.0	0.0145	649	77.37	0.3452	4.46	2.17
BJ	1.0	0.016	503	−2.63	−0.0723	27.50	0.75
CJ	1.8	0.0155	1581	−22.63	−0.1196	5.29	1.69
DJ	2.3	0.016	4061	47.63	−0.1083	2.27	2.21
				$\sum$	+0.0450	0.0395	

$\Rightarrow \Delta Z_J = 2.27$ m; $Z_J = 124.90$ m.

Final values:

$Q_{AJ} = 0.344$ m³/s; $Q_{JB} = 0.105$ m³/s; $Q_{JC} = 0.127$ m³/s; $Q_{JD} = 0.112$ m³/s

Example 5.5

If in the network of Example 5.4 the flow to C is regulated by a valve to 100 L/s, calculate the effect on the flows to the other reservoirs; determine the head loss to be provided by the valve.

The principle of the solution is identical with that of the previous example except that the flow in JC is prescribed and simply treated as an external outflow at J. In this example the flow rates in the pipes have been evaluated directly from Equation 5.6.

$$Q = -2A\sqrt{2gD\frac{h}{L}}\log\left(\frac{k}{3.7D} + \frac{2.51\nu}{D\sqrt{2gDh_f/L}}\right)$$

in which $h = Z_I - Z_J$, since there are no minor losses. This approach is ideal for computer analysis; if minor losses are present use the iterative procedure described in Example 4.2.

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The method is also suitable for desk analysis using an electronic calculator since for each pipe the only variable is h and Equation 5.6 can be written as

$$Q = -C_1\sqrt{h}\,\log\left(C_2 + \frac{C_3}{\sqrt{h}}\right)$$

in which C_1, C_2 and C_3 are constants for a particular pipe.

The corresponding velocities and λ values have been evaluated and tabulated; these data may be useful for those who wish to work through the example using the Moody diagram as shown in Example 5.4.

Note that Q is expressed in litres per second; in evaluating $\sum Q/h$ the flow is also expressed in litres per second so that the units in the correction term $\Delta Z = 2(\sum Q - F)/(\sum Q/h)$ are consistent.

Example 5.5 calculation

Pipe	AJ	BJ	DJ
k/D	0.000133	0.000171	0.000240

Note: Estimate $Z_J = 150.00$ a.o.a.

First correction

	Pipe	$Z_I - Z_J$ ($=h$) (m)	Q (L/s)	Q/h	V (m/s)	λ
	AJ	50.00	279.32	5.59	1.76	0.0143
Junction J	BJ	−30.00	−255.95	8.53	2.66	0.0146
	DJ	−75.00	−137.90	1.84	2.81	0.0155
		$\sum$	−114.53	15.96		

$$\text{Correction to } Z_J = \frac{2(\sum Q - F)}{\sum Q/h} = \frac{2(-144.53 - 100)}{15.96} = -26.89\text{ m}$$

$$Z_J = 123.11\text{ m}$$

Second correction

	Pipe	$Z_I - Z_J$	Q (L/s)	Q/h	V (m/s)	λ
	AJ	76.89	349.70	4.55	2.20	0.0140
Junction J	BJ	−3.11	−77.61	24.96	0.81	0.0164
	DJ	−48.11	−109.50	2.28	2.23	0.0158

$\Rightarrow \Delta Z_J = 3.94$ m; $Z_J = 127.05$ m.

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Third correction

	Pipe	$Z_I - Z_J$	Q (L/s)	Q/h	V (m/s)	λ
	AJ	72.95	340.2	4.66	2.14	0.0141
Junction J	BJ	−7.05	−119.94	17.01	1.25	0.0156
	DJ	−52.05	−114.08	2.19	2.32	0.0158

$$\Rightarrow \Delta Z_J = 0.52 \text{ m}; \quad Z_J = 127.57 \text{ m}.$$

Final values

Pipe	AJ	JB	JD
Q (L/s)	338.98	124.36	114.65

Head loss due to friction along JC:

$$\text{Diameter} = 300 \text{ mm}; \quad A = 0.0707 \text{ m}^2; \quad Q = 0.100 \text{ m}^3/\text{s}; \quad V = 1.415 \text{ m/s}$$

$$Re = \frac{1.415 \times 0.3}{1.13 \times 10^{-6}} = 3.76 \times 10^5; \quad \frac{k}{D} = 0.0002$$

$$\text{whence } \lambda = 0.016; \quad h_f = \frac{0.016 \times 3000 \times 1.415^2}{19.62 \times 0.3} = 16.33 \text{ m}$$

(see Figure 5.7).

$$\begin{aligned}\Rightarrow \text{Head loss at valve} &= Z_J - Z_C - h_f \\ &= 127.55 - 100.00 - 16.33 \\ &= 11.22 \text{ m}\end{aligned}$$

Example 5.6

In the network as before, a pump P is installed on JB to boost the flow to B. With the flows to C and D uncontrolled and the pump delivering 10 metres head, determine the flows in the pipes (see Figure 5.8).

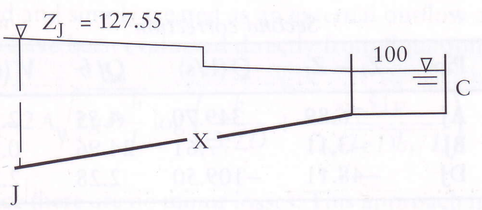

Figure 5.7 Network of Example 5.4 with valve losses.

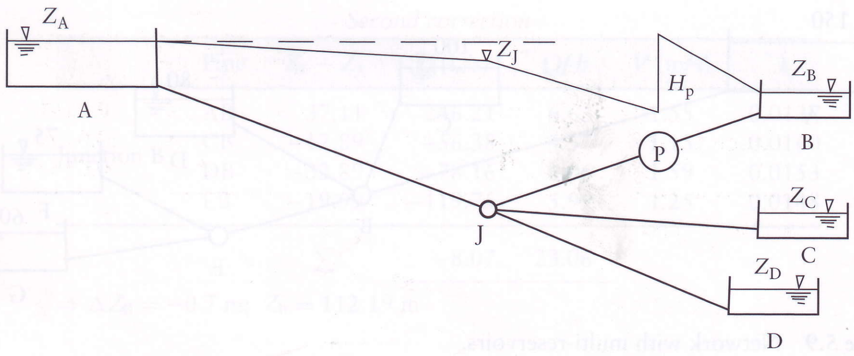

Figure 5.8 Network of with pump.

Note: In the case of rotodynamic pumps the manometric head delivered varies with the discharge (see Chapter 6). Thus it is not strictly possible to specify the head and it is necessary to solve the pump equation $H_p = AQ^2 + BQ + C$ together with the resistance equation for JB. However to illustrate the effect of a pump in this example let us assume that the head does not vary with flow.

Solution:

The analysis is straightforward, and follows the procedure of Example 5.5.

The head giving flow along JB is

$$h_{L,JB} = Z_J - Z_B - H_p$$

The final solution is as follows:

Pipe	AJ	JB	JC	JD
Q (L/s)	357.7	141.6	110.8	105.3

Note: $Z_J = 119.66$ m o.d.

Example 5.7

Determine the flows in the network shown in Figure 5.9 neglecting minor losses.

Pipe	AB	BC	BD	BE	EF	EG
Length (m)	10 000	3 000	4 000	6 000	3 000	3 000
Diameter (mm)	450	250	250	350	250	200

Note: Roughness of all pipes is 0.03 mm (=k).

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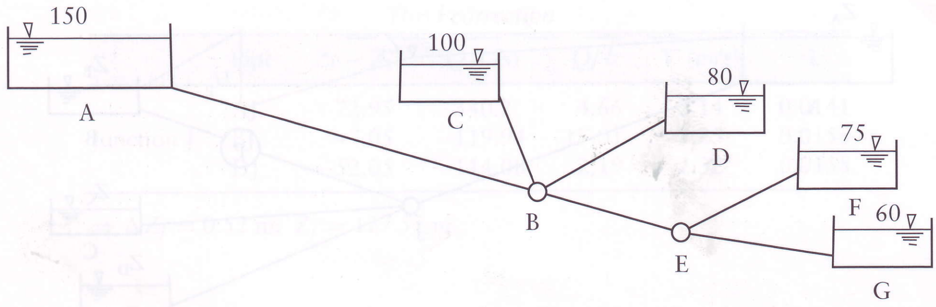

Figure 5.9 Network with multi-reservoirs.

Solution:

In this case there are two unknown pressure head elevations which must therefore be both initially estimated and corrected alternately.

$$\text{Estimate } Z_B = 120.0 \text{ m o.d.}; \qquad Z_E = 95.0 \text{ m o.d.}$$

First correction

	Pipe	$Z_I - Z_J$ (=h)	Q (L/s)	Q/ h	V (m/s)	λ
Junction B	AB	30.00	219.77	7.33	1.38	0.0139
	CB	−20.00	−71.38	3.57	1.45	0.0155
	DB	−40.00	−86.75	4.34	1.77	0.0151
	EB	−25.00	−135.00	5.40	1.40	0.0145
		Σ	−73.35	20.63		

$\Rightarrow \Delta Z_B = +7.11$ m; $Z_B = 112.89$ m.

Proceed to Junction E noting that the amended value of Z_B is now used:

	Pipe	$Z_I - Z_J$	Q (L/s)	Q/ h	V (m/s)	λ
Junction E	BE	17.89	112.81	6.31	1.17	0.0149
	FE	−20.00	−71.38	3.57	1.45	0.0155
	GE	−35.00	−53.38	1.53	1.70	0.0159
		Σ	11.95	11.40		

$\Rightarrow \Delta Z_E = -2.1$ m; $Z_E = 92.9$ m.

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Second correction

	Pipe	$Z_I - Z_J$	Q (L/s)	Q/h	V (m/s)	λ
Junction B	AB	37.11	246.21	6.63	1.55	0.0137
	CB	−12.89	−56.38	4.37	1.15	0.0160
	DB	−32.89	−78.16	6.06	1.59	0.0153
	EB	−19.99	−119.75	5.99	1.25	0.0148
		Σ	−8.07	23.06		

$\Rightarrow \Delta Z_B = -0.7$ m; $Z_B = 112.19$ m.

	Pipe	$Z_I - Z_J$	Q (L/s)	Q/h	V (m/s)	λ
Junction E	BE	92.9	117.48	6.09	1.22	0.0148
	FE	−17.9	−67.26	3.76	1.37	0.0156
	GE	−32.9	−51.64	1.57	1.64	0.0159
		Σ	−1.43	11.42		

$\Rightarrow \Delta Z_E = -0.25$ m; $Z_E = 92.65$ m.

Example 5.8

In the network shown in Figure 5.10, a valve in pipe 2-3 is partially closed, producing a local head loss of $10V_{2\text{-}3}^2/2g$. The head at node 1 is 100 m of water. The roughness of all pipes is 0.06 mm. The pipe lengths are in metres and the demand discharges are in litres per second.

The pipe diameters are pipes 1-2 and 1-6, 250 mm; pipe 6-5, 200 mm; pipes 2-3 and 4-5, 150 mm; pipes 2-5 and 3-4, 100 mm. Analyse the network using the gradient method.

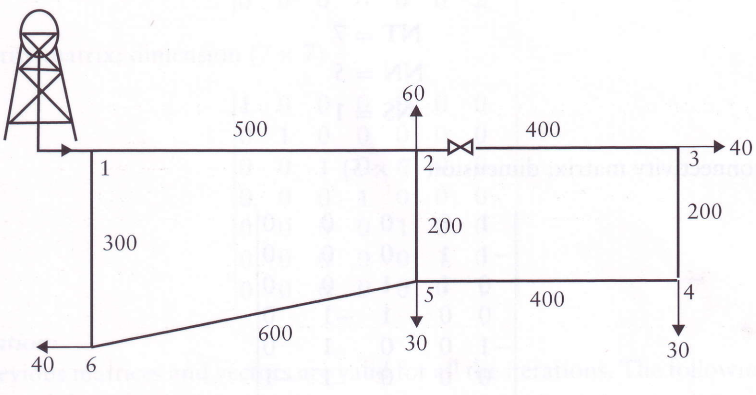

Figure 5.10 Pipe network with valve loss.

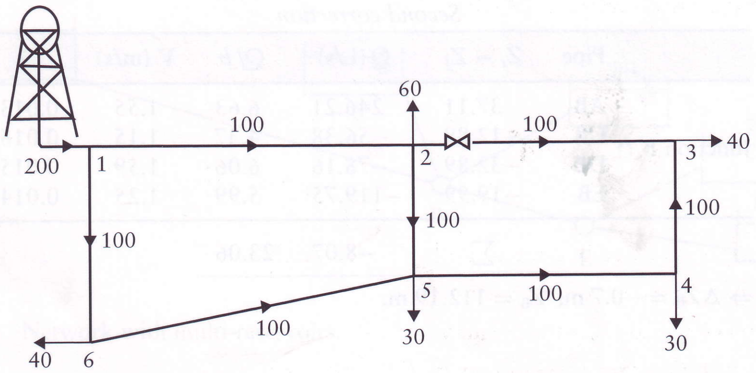

Figure 5.11 Network solution.

The iterative process can be summarised in the following steps:

1. Assume initial discharges in each of the network pipes. (They can be unbalanced at each node.)
2. Solve the system represented by Equation 5.17 using a standard method for the solution of simultaneous linear equations.
3. With the calculated $[H_{i+1}]$ (Step 2), $[Q_{i+1}]$ is solved by Equation 5.18.
4. With the new $[Q_{i+1}]$, Equation 5.17 is solved (Step 2) to find a new $[H_{i+1}]$.
5. Process continues until

$$[H_{i+1}] \approx [H_i]$$

For all pipes initial discharges of 100 L/s have been assumed with the directions as shown in Figure 5.11.

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Solution:

All the matrices and vectors needed for the gradient method are as follows:

$$NT = 7$$
$$NN = 5$$
$$NS = 1$$

[A12] = connectivity matrix; dimension (7 × 5)

$$\begin{vmatrix} 1 & 0 & 0 & 0 & 0 \\ -1 & 1 & 0 & 0 & 0 \\ 0 & 1 & -1 & 0 & 0 \\ 0 & 0 & 1 & -1 & 0 \\ -1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 0 & 1 \end{vmatrix}$$

[A21] = transposed matrix of [A12]

$$\begin{vmatrix} 1 & -1 & 0 & 0 & -1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 1 \end{vmatrix}$$

[A10] = topologic matrix node to node; dimension (7 × 1)
[Q] = discharges vector; dimension (7 × 1)
[H] = unknown piezometric head vector; dimension (5 × 1)
[H_0] = fixed piezometric head vector; dimension (1 × 1)
[q] = water demand vector; dimension (5 × 1)

$$\begin{array}{ccccc} [A10] & \begin{array}{c}[Q] \\ (m^3/s)\end{array} & [H] & \begin{array}{c}[H_0] \\ (m)\end{array} & \begin{array}{c}[q] \\ (m^3/s)\end{array} \\ \begin{vmatrix} -1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ -1 \end{vmatrix} & \begin{vmatrix} 0.10 \\ 0.10 \\ 0.10 \\ 0.10 \\ 0.10 \\ 0.10 \\ 0.10 \end{vmatrix} & \begin{vmatrix} H_2 \\ H_3 \\ H_4 \\ H_5 \\ H_6 \end{vmatrix} & \begin{vmatrix} 100 \end{vmatrix} & \begin{vmatrix} 0.06 \\ 0.04 \\ 0.03 \\ 0.03 \\ 0.04 \end{vmatrix} \end{array}$$

[N] = diagonal matrix; dimension (7 × 7); having 2 in the diagonal (from the Darcy–Weisbach head loss equation)

$$\begin{vmatrix} 2 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 2 \end{vmatrix}$$

[I] = identity matrix; dimension (7 × 7)

$$\begin{vmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{vmatrix}$$

First iteration:

The previous matrices and vectors are valid for all the iterations. The following matrices change in each iteration:

[A21] = transposed matrix of [A12]

$$\begin{vmatrix} 1 & -1 & 0 & 0 & -1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 1 \end{vmatrix}$$

[A10] = topologic matrix node to node; dimension (7 × 1)
[Q] = discharges vector; dimension (7 × 1)
[H] = unknown piezometric head vector; dimension (5 × 1)
[H_0] = fixed piezometric head vector; dimension (1 × 1)
[q] = water demand vector; dimension (5 × 1)

$$\underset{[A10]}{\begin{vmatrix} -1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ -1 \end{vmatrix}} \quad \underset{\substack{[Q] \\ (m^3/s)}}{\begin{vmatrix} 0.10 \\ 0.10 \\ 0.10 \\ 0.10 \\ 0.10 \\ 0.10 \\ 0.10 \end{vmatrix}} \quad \underset{[H]}{\begin{vmatrix} H_2 \\ H_3 \\ H_4 \\ H_5 \\ H_6 \end{vmatrix}} \quad \underset{\substack{[H_0] \\ (m)}}{\begin{vmatrix} 100 \end{vmatrix}} \quad \underset{\substack{[q] \\ (m^3/s)}}{\begin{vmatrix} 0.06 \\ 0.04 \\ 0.03 \\ 0.03 \\ 0.04 \end{vmatrix}}$$

[N] = diagonal matrix; dimension (7 × 7); having 2 in the diagonal (from the Darcy–Weisbach head loss equation)

$$\begin{vmatrix} 2 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 2 \end{vmatrix}$$

[I] = identity matrix; dimension (7 × 7)

$$\begin{vmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{vmatrix}$$

First iteration:

The previous matrices and vectors are valid for all the iterations. The following matrices change in each iteration:

[A11] = diagonal matrix; dimension (7 × 7); having the value $\alpha_i Q_i^{(n_i-1)}$ on the diagonal, with coefficients β and γ zero as no pumps exist in the network

The following table shows the calculated values for α:

Pipe	Q (m^3/s)	λ	V (m/s)	h_f (m)	$h_f + h_m$ (m)	α
1-2	0.10	0.0159	1.974	6.22	6.22	622.28
2-3	0.10	0.0166	5.482	66.89	82.21	8 220.77
3-4	0.10	0.0178	12.335	271.02	271.02	27 101.65
5-4	0.10	0.0166	5.482	66.89	66.89	6 688.98
2-5	0.10	0.0178	12.335	270.99	270.09	27 098.90
6-5	0.10	0.0161	3.084	23.09	23.09	2 308.78
6-1	0.10	0.0159	1.974	3.73	3.73	373.42

Matrix [A11]:

$$\begin{vmatrix} 62.23 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 822.08 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 2710.16 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 668.90 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 2709.89 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 230.88 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 37.34 \end{vmatrix}$$

[A11]′ = diagonal matrix; dimension (7 × 7); having the value $\alpha_i Q_i^{(n_i-1)}$ on the diagonal

For this network, [A11′] = [A11].

$$\begin{vmatrix} 62.23 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 822.08 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 2710.16 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 668.90 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 2709.89 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 230.88 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 37.34 \end{vmatrix}$$

To find H_{i+1} by Equation 5.17 following a step-by-step analysis, the following matrices can be found:

[N][A11]′

$$\begin{vmatrix} 124.46 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1644.15 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 5420.33 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1337.80 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 5419.78 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 461.76 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 74.68 \end{vmatrix}$$

$$([N][A11]')^{-1}$$

$$\begin{vmatrix} 0.00804 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0.00061 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.00018 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.00075 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.00018 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0.00219 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0.01339 \end{vmatrix}$$

$$[A21]([N][A11]')^{-1}$$

$$\begin{vmatrix} 0.00804 & -0.00061 & 0 & 0 & -0.00018 & 0 & 0 \\ 0 & 0.00061 & 0.00018 & 0 & 0 & 0 & 0 \\ 0 & 0 & -0.00018 & 0.00075 & 0 & 0 & 0 \\ 0 & 0 & 0 & -0.00075 & 0.00018 & 0.00219 & 0 \\ 0 & 0 & 0 & 0 & 0 & -0.00219 & 0.01339 \end{vmatrix}$$

$$[A21]([N][A11]')^{-1}[A12]$$

$$\begin{vmatrix} 0.00804 & -0.00061 & 0 & -0.00018 & 0 \\ -0.00061 & 0.00079 & -0.00018 & 0 & 0 \\ 0 & -0.00018 & 0.00093 & -0.00075 & 0 \\ -0.00018 & 0 & -0.00075 & 0.00310 & -0.00217 \\ 0 & 0 & 0 & -0.00217 & 0.01556 \end{vmatrix}$$

$$-([A21]([N][A11]')^{-1}[A12])^{-1}$$

$$\begin{vmatrix} -120.541 & -100.256 & -33.383 & -16.879 & -2.350 \\ -100.256 & -1423.476 & -365.444 & -104.310 & -14.522 \\ -33.383 & -365.444 & -1460.157 & -392.548 & -54.651 \\ -16.879 & -104.310 & -392.548 & -463.689 & -64.555 \\ -2.350 & -14.522 & -54.651 & -64.555 & -73.273 \end{vmatrix}$$

$$[A11][Q] \quad [A10][H_0] \quad ([A11][Q]) + ([A10][H_0])$$

$$\begin{vmatrix} 6.223 \\ 82.208 \\ 271.016 \\ 66.890 \\ 270.989 \\ 23.088 \\ 3.734 \end{vmatrix} \quad \begin{vmatrix} -100 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ -100 \end{vmatrix} \quad \begin{vmatrix} -93.777 \\ 82.208 \\ 271.016 \\ 66.890 \\ 270.989 \\ 23.088 \\ -96.266 \end{vmatrix}$$

$$([A21]([N][A11]')^{-1}([A11][Q][A10][H_0])) \quad [A21][Q] \quad ([A21]([N][A11]')^{-1}([A11][Q] + [A10][H_0]) - ([A21][Q] - [q]))$$

$$\begin{vmatrix} -0.853 \\ 0.1 \\ 0 \\ 0.05 \\ -1.339 \end{vmatrix} \quad \begin{vmatrix} -0.1 \\ 0.2 \\ 0 \\ 0.1 \\ 0 \end{vmatrix} \quad \begin{vmatrix} -0.6935 \\ -0.06 \\ 0.03 \\ -0.02 \\ -1.299 \end{vmatrix}$$

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Thus

$$H_{i+1} = -([A21]([N][A11'])^{-1}([A12])^{-1}([A21]([N][A11'])^{-1}([A11][Q]$$
$$+[A10][H_0] - ([A21][Q] - [q]))$$

$$\begin{array}{c} \text{Node} \\ \left|\begin{array}{c} 2 \\ 3 \\ 4 \\ 5 \\ 6 \end{array}\right| \end{array} = \begin{array}{c} \text{(m)} \\ \left|\begin{array}{c} 92.000 \\ 164.922 \\ 80.115 \\ 99.317 \\ 97.335 \end{array}\right| \end{array}$$

To find Q_{i+1} by Equation 5.18 following a step-by-step analysis, the following matrices can be found:

$$\begin{array}{ccc} [A12][H_{i+1}] & [A12][H_{i+1}]+[A10][H_0] & ([N][A11']^{-1})(([A12][H_{i+1}])+([A10][H_0])) \\ \left|\begin{array}{r} 92.00 \\ 72.92 \\ 84.81 \\ -19.20 \\ 7.32 \\ 1.98 \\ 97.33 \end{array}\right| & \left|\begin{array}{r} -8.00 \\ 72.92 \\ 84.81 \\ -19.20 \\ 7.32 \\ 1.98 \\ -2.67 \end{array}\right| & \left|\begin{array}{r} -0.0643 \\ 0.0444 \\ 0.0157 \\ -0.0144 \\ 0.0014 \\ 0.0043 \\ -0.0357 \end{array}\right| \end{array}$$

$$([N][A11'])^{-1}[A11]$$

$$\left|\begin{array}{ccccccc} 0.5 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0.5 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.5 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.5 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.5 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0.5 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0.5 \end{array}\right|$$

$$\begin{array}{cc} ([I] - ([N])[A11'])^{-1}[A11]) & ([I] - ([N][A11'])^{-1} \times [A11])[Q] \\ \left|\begin{array}{ccccccc} 0.5 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0.5 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.5 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.5 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.5 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0.5 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0.5 \end{array}\right| & \left|\begin{array}{c} 0.05 \\ 0.05 \\ 0.05 \\ 0.05 \\ 0.05 \\ 0.05 \\ 0.05 \end{array}\right| \end{array}$$

Thus

$$Q_{i+1} = ([I] - [N][A11'])^{-1}[A11])[Q] - (([N][A11'])^{-1}([A12][H_{i+1}] + [A10][H_0]))$$

Pipe		(m^3/s)
1-2		0.114
2-3		0.006
3-4		0.034
5-4	=	0.064
2-5		0.049
6-5		0.046
6-1		0.086

After only five iterations the following are the results.

Head at each node:

Node		(m)
2		92.960
3		81.358
4	=	81.780
5		89.812
6		96.727

Pipe discharges:

Pipe		(m^3/s)
1-2		0.10667
2-3		0.03658
3-4		0.00342
5-4	=	0.03342
2-5		0.01009
6-5		0.05333
6-1		0.09333

References and recommended reading

Featherstone, R. E. and El Jumailly, K. K. (1983) Optimal diameter selection for pipe networks. *Journal of the Hydraulics Division, American Society of Civil Engineers*, 109, 221–234.

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Twort, A. C., Ratnayaka, D. D. and Brandt, M. J. (2000) *Water Supply*, 5th edn, Arnold, London; reprinted by Butterworth-Heinemann, 2001.

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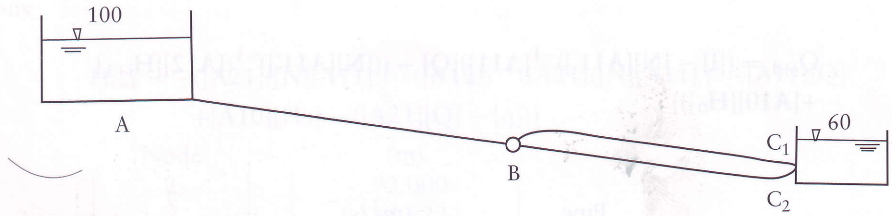

Figure 5.12 Pipes in parallel.

Problems

1. Calculate the flows in the pipes of the pipe system illustrated in Figure 5.12. Minor losses are given by $C_m V^2/2g$.

Pipe	Length (m)	Diameter (mm)	Roughness (mm)	Minor loss coefficients (C_m)
AB	5000	400	0.15	10.0
BC_1	7000	250	0.15	15.0
BC_2	7000	250	0.06	10.0

(*Note:* While this problem could be solved by the method of Example 4.1, the method of quantity balance facilitates a convenient method of solution. Note that the pressure head elevations at the ends of C_1 and C_2 are identical.)

2. In the system shown in Problem 1, an axial flow pump producing a total head of 5.0 m is installed in pipe BC_1 to boost the flow in this branch. Determine the flows in the pipes. (*Note:* Although it is not strictly possible to predict the head generated by a rotodynamic pump since this varies with the discharge (see Chapter 6), axial flow pumps often produce a fairly flat head–discharge curve in the mid-discharge range.)

3. Determine the flows in the network illustrated in Figure 5.13. Minor losses are given by $C_m V^2/2g$.

Pipe	Length (m)	Diameter (mm)	k (mm)	C_m
AB	20 000	500	0.3	20
BC	5 000	350	0.3	10
BD_1	6 000	300	0.3	10
BD_2	6 000	250	0.06	10

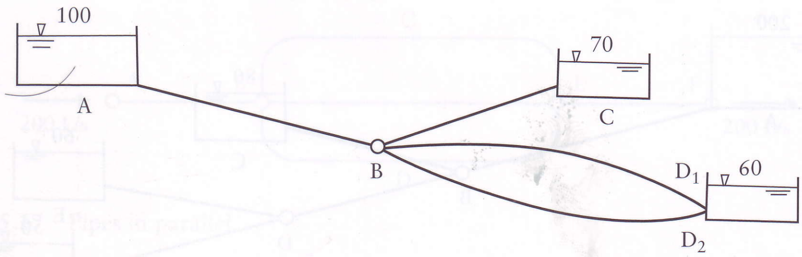

Figure 5.13 Network with reservoirs.

4. In the system illustrated in Figure. 5.14, a pump is installed in pipe BC to provide a flow of 40 L/s to Reservoir C. Neglecting minor losses calculate the total head to be generated by the pump and the power consumption assuming an overall efficiency of 60%. Determine also the flow rates in the other pipes.

Pipe	Length (m)	Diameter (mm)	Roughness (mm)
AB	10 000	400	0.06
BC	4 000	250	0.06
BD	5 000	250	0.06

5. Determine the pressure head elevations at B and D and the discharges in the branches in the system illustrated in Figure 5.15. Neglect minor losses.

Pipe	Length (m)	Diameter (mm)	Roughness (mm)
AB	20 000	600	0.06
BC	2 000	250	0.06
BD	2 000	450	0.06
DE	2 000	300	0.06
DF	2 000	250	0.06

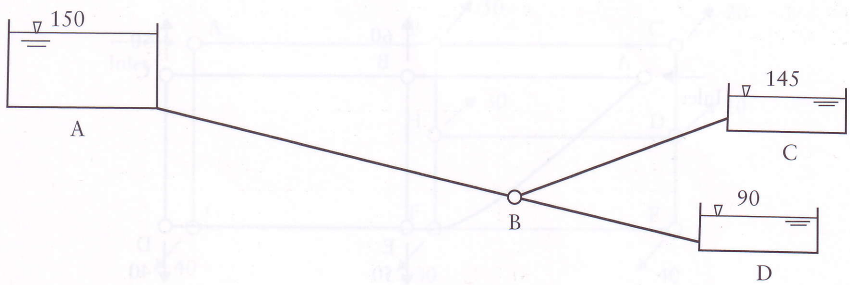

Figure 5.14 Network with reservoirs.

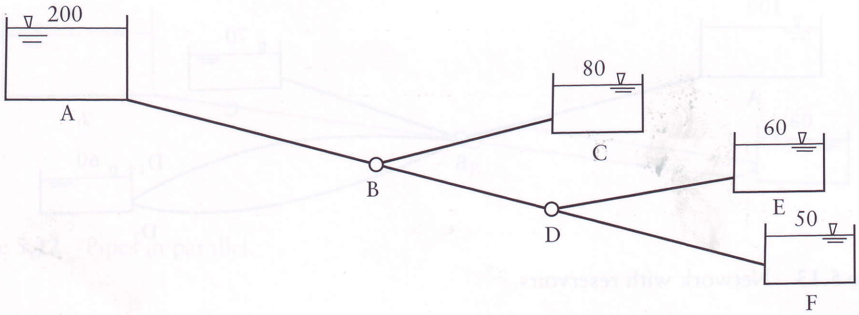

Figure 5.15 Network with reservoirs.

6. Determine the flows in a pipe system similar in configuration to that in Problem 5. A valve is installed in BC producing a minor loss of $20V^2/2g$; otherwise consider only friction losses.

Pipe	Length (m)	Diameter (mm)	Roughness (mm)
AB	20 000	450	0.06
BC	2 000	300	0.06
BD	10 000	400	0.06
DE	3 000	250	0.06
DF	4 000	300	0.06

7. Determine the flow in the pipes and the pressure head elevations at the junctions of the closed-loop pipe network illustrated, neglecting minor losses. All pipes have the same roughness size of 0.03 mm. The outflows at the junctions are shown in litres per second (see Figure 5.16).

Pipe	AB	BC	CD	DE	EA	BE
Length (m)	500	600	200	600	600	200
Diameter (mm)	200	150	100	150	200	100

Pressure head elevation at A = 60 m a.o.d.

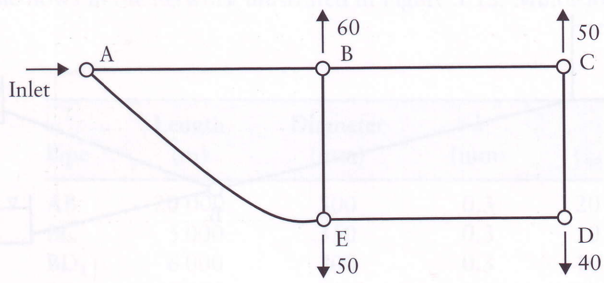

Figure 5.16 Two-loop network.

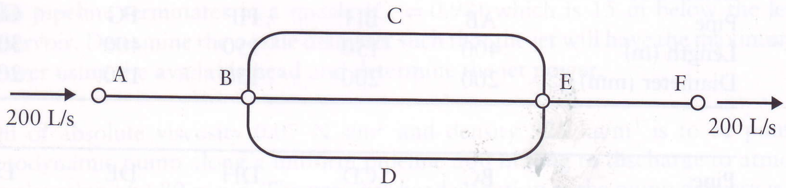

Figure 5.17 Pipes in parallel.

(*Note:* A more rapid solution is obtained by using the head balance method. However the network can be analysed by the quantity balance method but in this case four unknown pressure heads, at B, C, D and E, are to be corrected. If the quantity balance method is used, set a fixed arbitrary pressure head elevation to A, say 100 m.)

8. Determine the flow distribution in the pipe system illustrated in Figure 5.17 and the total head loss between A and F. Neglect minor losses. A total discharge of 200 L/s passes through the system.

Pipe	AB	BCE	BE	BDE	EF
Length (m)	1000	3000	2000	3000	1000
Diameter (mm)	450	300	250	350	450
Roughness (mm)	0.15	0.06	0.15	0.06	0.15

9. In the system shown in Problem 7 (Figure 5.16) a pump is installed in BC to boost the flow to C. Neglecting minor losses determine the flow distribution and head elevations at the junctions if the pump delivers a head of 15.0 m.

10. Determine the flows in the pipes and the pressure head elevations at the junctions in the network shown in Figure 5.18. Neglect minor losses and take the pressure head elevation at A to be 100 m. The outflows are in litres per second. All pipes have a roughness of 0.06 mm.

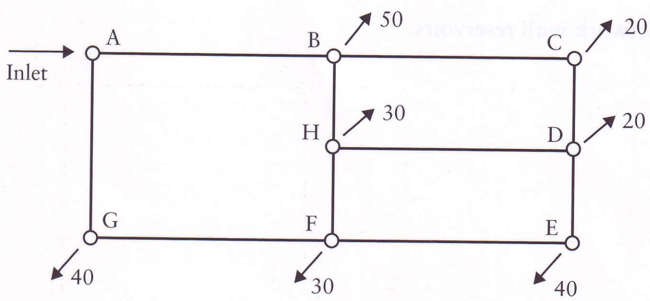

Figure 5.18 Three-loop network.

Pipe	AB	BH	HF	FG	GA
Length (m)	400	150	150	400	300
Diameter (mm)	200	200	150	150	200

Pipe	BC	CD	DH	DE	EF
Length (m)	300	150	300	150	300
Diameter (mm)	150	150	150	150	150

11. Analyse the flows and pressure heads in the pipe system shown in Figure 5.19. Neglect minor losses.

Pipe	AB	BC	CD	DE	EF	EF
Length (m)	1000	400	300	400	800	300
Diameter (mm)	250	200	150	150	250	200
Roughness (mm)	0.06	0.15	0.15	0.15	0.06	0.15

12. Solve the network in Problem 10 using the gradient method.

13. Analyse the network of Example 5.1 by the gradient method.

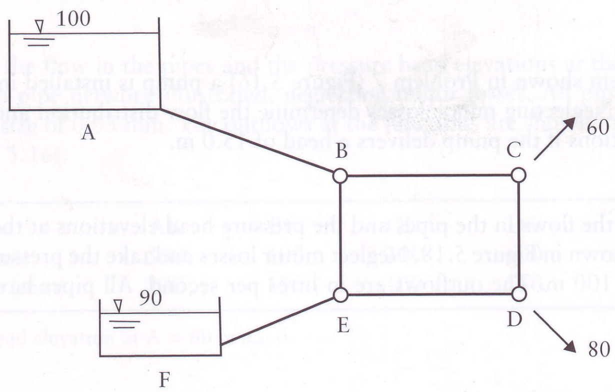

Figure 5.19 Network with reservoirs.

The pipeline terminates in a nozzle ($C_v = 0.98$) which is 15 m below the level in the reservoir. Determine the nozzle diameter such that the jet will have the maximum possible power using the available head and determine the jet power.

8. Oil of absolute viscosity 0.07 N s/m^2 and density 925 kg/m^3 is to be pumped by a rotodynamic pump along a uniform pipeline 500 m long to discharge to atmosphere at an elevation of +80 m o.d. The pressure head elevation at the pump delivery is 95 m o.d. Neglecting minor losses, compare the discharges attained when the pipe of roughness 0.06 mm is (a) 100 mm and (b) 150 mm diameter, and state in each whether the flow is laminar or turbulent.

9. A pipeline 10 km long is to be designed to deliver water from a river through a pumping station to the inlet tank of a treatment works. Elevation of delivery pressure head at pumping station is 50 m o.d.; elevation of water in tank is 30 m o.d. Neglecting minor losses, compare the discharges obtainable using
(a) a 300 mm diameter plastic pipeline which may be considered to be smooth
(i) using the Colebrook–White equation
(ii) using the Blasius equation
(b) a 300 mm diameter pipeline with an effective roughness of 0.6 mm
(i) using the Kármán–Prandtl rough law
(ii) using the Colebrook–White equation.

10. Determine the hydraulic gradient in a rectangular concrete culvert 1 m wide and 0.6 m high of roughness size 0.06 mm when running full and conveying water at a rate of 2.5 m^3/s.

The pipeline terminates in a nozzle ($C_v = 0.98$) which is 15 m below the level in the reservoir. Determine the nozzle diameter such that the jet will have the maximum possible power using the available head and determine the jet power.

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10. Determine the hydraulic gradient in a rectangular concrete culvert 1 m wide and 0.6 m high of roughness size 0.06 mm when running full and conveying water at a rate of 2.5 m^3/s.