## Hardy Cross Method

The Hardy Cross method is an iterative method for determining the flow in pipe network systems where the inputs and outputs are known, but the flow inside the network is unknown. The method was first published in November 1936 by its namesake, Hardy Cross, a structural engineering professor at the University of Illinois at Urbana–Champaign. The Hardy Cross method is an adaptation of the Moment distribution method, which was also developed by Hardy Cross as a way to determine the moments in indeterminate structures.

The introduction of the Hardy Cross method for analyzing pipe flow networks revolutionized municipal water supply design. Before the method was introduced, solving complex pipe systems for distribution was extremely difficult due to the nonlinear relationship between head loss and flow. The method was later made obsolete by computer solving algorithms employing the Newton-Raphson method or other solving methods that prevent the need to solve nonlinear systems of equations by hand.

The Hardy Cross method provides a system for calculating the value of the correction to be made, each loop or junction being considered in turn and corrected assuming that conditions in the reminder of the network remain unaltered.
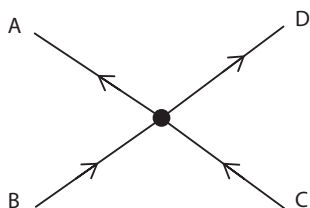
Two main assuptions used in Hardy Cross method:
1. The losses of head between any two junctions must be the same for all routes between these junctions.
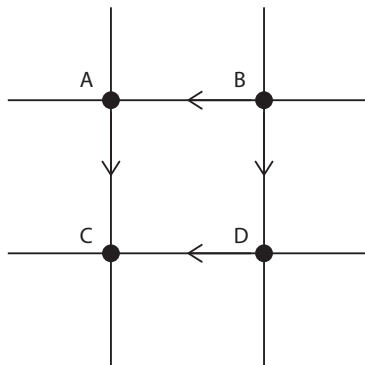
Head loss in pipe BA + Head loss in pipe DC
= Head loss in pipe AC + Head loss in pipe BD

2. The inflow to each junction must equal the outflow from that junction.

$Q_B + Q_C = Q_A + Q_D$

The method of balancing heads uses an initial guess that satisfies continuity of flow at each junction and then balances the flows until continuity of potential is also achieved over each loop in the system.

This method can be used if total flow rate is known. Head or pressure at junctions is not compulsory item.

From previous lesson, major loss can be determined as:

$$h_f = f \cdot \frac{L}{D} \cdot \frac{V^2}{2g} = f \cdot \frac{L}{D^5} \cdot \frac{16}{2g\pi^2} \cdot Q^2 = K \cdot Q^2$$

Head loss in pipe in which flow is clockwise;

$$\sum h_c = \sum K_C \cdot Q_C^2$$

Head loss in pipe in which flow is counter-clockwise;

$$\sum h_{cc} = \sum K_{CC} \cdot Q_{CC}^2$$

Total head loss is constant, whether it is measured clockwise or counter-clockwise.

$$\sum K_C \cdot Q_C^2 = \sum K_{CC} \cdot Q_{CC}^2$$

In first assumption, we assumed that the losses value is not balanced.

$$\sum K_C \cdot (Q_C - \Delta Q)^2 = \sum K_{CC} \cdot (Q_{CC} + \Delta Q)^2$$

$$\sum K_C \cdot (Q_C^2 - 2Q_C \cdot \Delta Q + \Delta Q^2) = \sum K_{CC} \cdot (Q_{CC}^2 - 2Q_{CC} \cdot \Delta Q + \Delta Q^2)$$

$$\Delta Q^2 \text{ can be neglected.}$$

Then,

$$\Delta Q = \frac{\sum K_C \cdot Q_C^2 - \sum K_{CC} \cdot Q_{CC}^2}{2(\sum K_C \cdot Q_C - K_{CC} \cdot Q_{CC})}$$

It is known that;

$$h_f = K \cdot Q^2$$
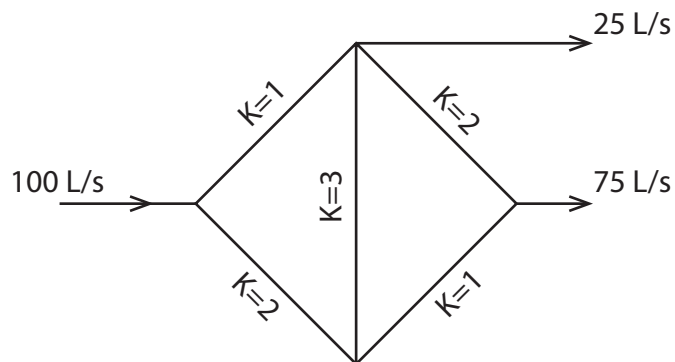$$K \cdot Q = \frac{h_f}{Q}$$

Finally,

$$\Delta Q = \frac{\sum h_{fc} - \sum h_{fcc}}{2(\sum \frac{h_{fc}}{Q_c} + \sum \frac{h_{fcc}}{Q_{cc}})}$$

Calculation will be done many times until the value of ΔQ is approaching zero.

Example

Determine the distribution of flowrate?

Basic step:
1. Determine loop or cycle.
2. Predict the initial flow rate and its direction. No need to convert unit into SI unit.
3. Create table to fulfill the Hardy Cross method equation.
4. Redraw the figure, and propose new flow rate. ΔQ value is not the answers.
5. Repeat steps 1-4 until ΔQ is approaching zero. Propose the new flow rate.

Answer

First trial
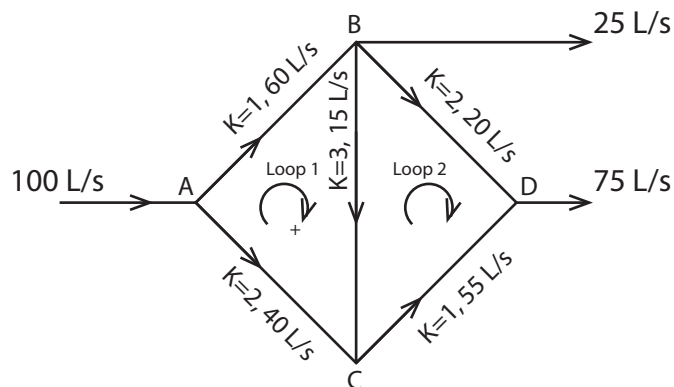
Assumption:
clockwise --> positive
counter-clockwise --> negative

Branch	K	Q (L/s)	$h_f = K \cdot Q^2$
AB	1	60	3600
AC	2	40	3200
BC	3	15	675
CD	1	55	3025
BD	2	20	800

Basic equation: $\Delta Q = \dfrac{\sum h_{fC} - \sum h_{fCC}}{2(\sum \frac{h_{fC}}{Q_C} + \sum \frac{h_{fCC}}{Q_{CC}})}$		
	Loop 1	Loop 2
$\sum h_{fC} - \sum h_{fCC}$	$3600 + 675 - 3200 = 1075$	$800 - 3025 - 675 = -2900$
$2\left(\sum \dfrac{h_{fC}}{Q_C} + \sum \dfrac{h_{fCC}}{Q_{CC}}\right)$	$2\left(\dfrac{3600}{60} + \dfrac{675}{15} + \dfrac{3200}{40}\right) = 370$	$2\left(\dfrac{800}{20} + \dfrac{3025}{55} + \dfrac{675}{15}\right) = 280$
ΔQ (L/s)	$\dfrac{1075}{370} = 2.91$	$\dfrac{-2900}{280} = -10.36$

Modification of flow rate value need to be done base on the value ΔQ calculated above.

Modification (add or substrate) rule of thumb:
Clockwise → substrate with ΔQ
Counter-clockwise → add with ΔQ

| Branch	Flow direction	Loop	ΔQ	New Q (L/s)
AB	C (-)	1	2.91	$60 - 2.91 = 57.09$
AC	CC (+)	1	2.91	$40 + 2.91 = 42.91$
BC	C (-) & CC (+)	1 & 2	2.91 & -10.36	$15 - 2.91 + (-10.36) = 1.73$
CD	CC (+)	2	-10.36	$55 + (-10.36) = 44.64$
BD	C (-)	2	-10.36	$20 - (-10.36) = 30.36$

New propose flow rate and its direction is:
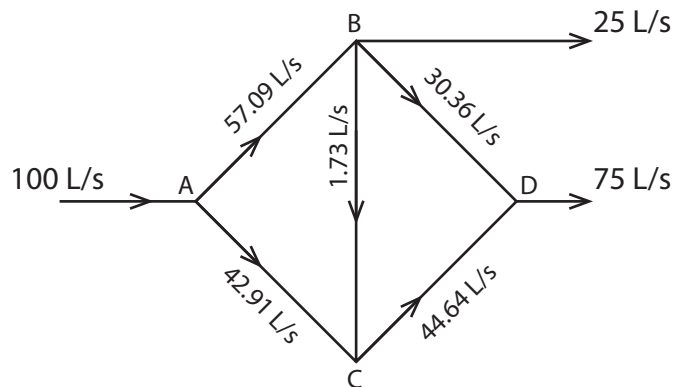
However, this is not the final flow rate, because the ΔQ is still shows high value. Second trial calculation need to be done. Repeat the procedure until the value of ΔQ is almost zero.

Second trial

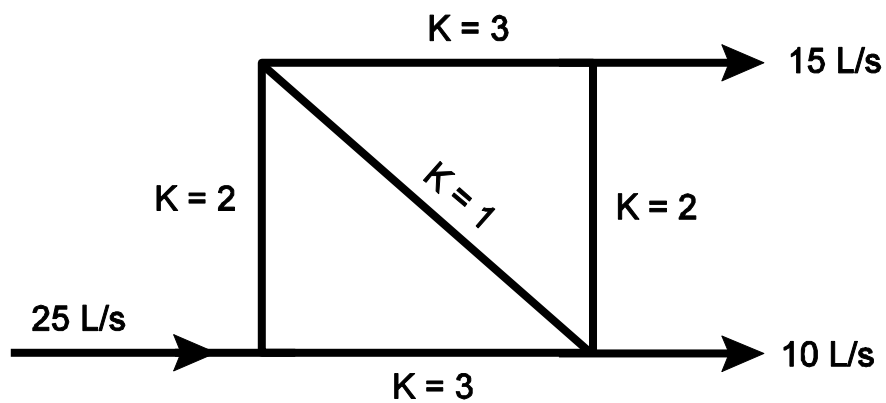Calculation should starts with this flow rate value.

Tutorial #1

K = 2

25 L/s

K = 3

K = 2

K = 2

50 L/s

K = 3

25 L/s
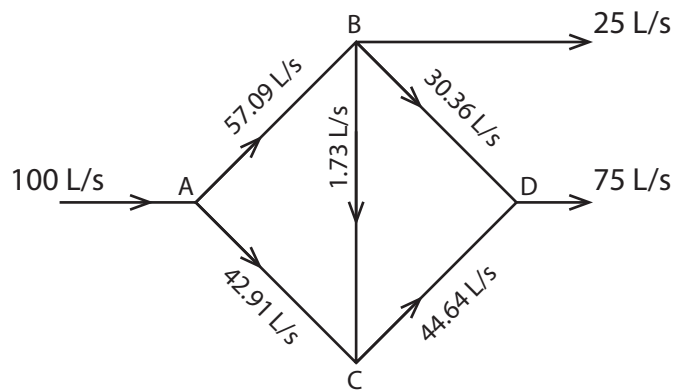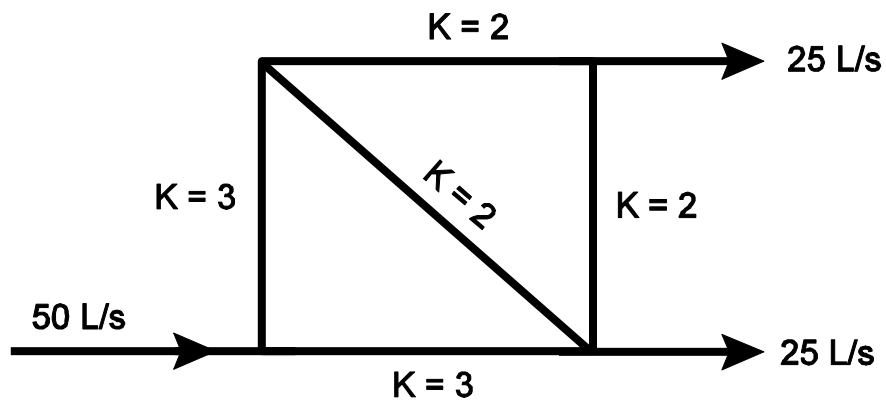
Answer: 24.35, 9.90, 25.65, 14.45, 10.55

Tutorial #2

K = 3

15 L/s

K = 2

K = 1

K = 2

25 L/s

K = 3

10 L/s

Answer: 13.45, 6.19, 11.55, 7.26, 7.74