Chapter

1

Water Systems Piping

Introduction

Water systems piping consists of pipes, valves, fittings, pumps, and associated appurtenances that make up water transportation systems. These systems may be used to transport fresh water or nonpotable water at room temperatures or at elevated temperatures. In this chapter we will discuss the physical properties of water and how pressure drop due to friction is calculated using the various formulas. In addition, total pressure required and an estimate of the power required to transport water in pipelines will be covered. Some cost comparisons for economic transportation of various pipeline systems will also be discussed.

1.1 Properties of Water

1.1.1 Mass and weight

Mass is defined as the quantity of matter. It is measured in slugs (slug) in U.S. Customary System (USCS) units and kilograms (kg) in Système International (SI) units. A given mass of water will occupy a certain volume at a particular temperature and pressure. For example, a mass of water may be contained in a volume of 500 cubic feet (ft^3) at a temperature of 60°F and a pressure of 14.7 pounds per square inch (lb/in^2 or psi). Water, like most liquids, is considered incompressible. Therefore, pressure and temperature have a negligible effect on its volume. However, if the properties of water are known at standard conditions such as 60°F and 14.7 psi pressure, these properties will be slightly different at other temperatures and pressures. By the principle of conservation of mass, the mass of a given quantity of water will remain the same at all temperatures and pressures.

Weight is defined as the gravitational force exerted on a given mass at a particular location. Hence the weight varies slightly with the geographic location. By Newton's second law the weight is simply the product of the mass and the acceleration due to gravity at that location. Thus

$$W = mg \tag{1.1}$$

where W = weight, lb
m = mass, slug
g = acceleration due to gravity, ft/s^2

In USCS units g is approximately 32.2 ft/s^2, and in SI units it is 9.81 m/s^2. In SI units, weight is measured in newtons (N) and mass is measured in kilograms. Sometimes mass is referred to as pound-mass (lbm) and force as pound-force (lbf) in USCS units. Numerically we say that 1 lbm has a weight of 1 lbf.

1.1.2 Density and specific weight

Density is defined as mass per unit volume. It is expressed as slug/ft^3 in USCS units. Thus, if 100 ft^3 of water has a mass of 200 slug, the density is 200/100 or 2 slug/ft^3. In SI units, density is expressed in kg/m^3. Therefore water is said to have an approximate density of 1000 kg/m^3 at room temperature.

Specific weight, also referred to as weight density, is defined as the weight per unit volume. By the relationship between weight and mass discussed earlier, we can state that the specific weight is as follows:

$$\gamma = \rho g \tag{1.2}$$

where γ = specific weight, lb/ft^3
ρ = density, slug/ft^3
g = acceleration due to gravity

The volume of water is usually measured in gallons (gal) or cubic ft (ft^3) in USCS units. In SI units, cubic meters (m^3) and liters (L) are used. Correspondingly, the flow rate in water pipelines is measured in gallons per minute (gal/min), million gallons per day (Mgal/day), and cubic feet per second (ft^3/s) in USCS units. In SI units, flow rate is measured in cubic meters per hour (m^3/h) or liters per second (L/s). One ft^3 equals 7.48 gal. One m^3 equals 1000 L, and 1 gal equals 3.785 L. A table of conversion factors for various units is provided in App. A.

Example 1.1 Water at 60°F fills a tank of volume 1000 ft^3 at atmospheric pressure. If the weight of water in the tank is 31.2 tons, calculate its density and specific weight.

Solution

$$\text{Specific weight} = \frac{\text{weight}}{\text{volume}} = \frac{31.2 \times 2000}{1000} = 62.40 \text{ lb/ft}^3$$

From Eq. (1.2) the density is

$$\text{Density} = \frac{\text{specific weight}}{g} = \frac{62.4}{32.2} = 1.9379 \text{ slug/ft}^3$$

Example 1.2 A tank has a volume of 5 m^3 and contains water at 20°C. Assuming a density of 990 kg/m^3, calculate the weight of the water in the tank. What is the specific weight in N/m^3 using a value of 9.81 m/s^2 for gravitational acceleration?

Solution

$$\text{Mass of water} = \text{volume} \times \text{density} = 5 \times 990 = 4950 \text{ kg}$$

$$\text{Weight of water} = \text{mass} \times g = 4950 \times 9.81 = 48{,}559.5 \text{ N} = 48.56 \text{ kN}$$

$$\text{Specific weight} = \frac{\text{weight}}{\text{volume}} = \frac{48.56}{5} = 9.712 \text{ N/m}^3$$

1.1.3 Specific gravity

Specific gravity is a measure of how heavy a liquid is compared to water. It is a ratio of the density of a liquid to the density of water at the same temperature. Since we are dealing with water only in this chapter, the specific gravity of water by definition is always equal to 1.00.

1.1.4 Viscosity

Viscosity is a measure of a liquid's resistance to flow. Each layer of water flowing through a pipe exerts a certain amount of frictional resistance to the adjacent layer. This is illustrated in the shear stress versus velocity gradient curve shown in Fig. 1.1*a*. Newton proposed an equation that relates the frictional shear stress between adjacent layers of flowing liquid with the velocity variation across a section of the pipe as shown in the following:

$$\text{Shear stress} = \mu \times \text{velocity gradient}$$

or

$$\tau = \mu \frac{dv}{dy} \tag{1.3}$$

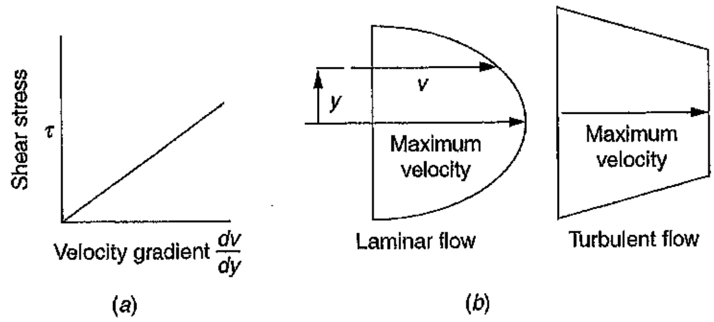


Figure 1.1 Shear stress versus velocity gradient curve.

where τ = shear stress
μ = absolute viscosity, (lb · s)/ft^2 or slug/(ft · s)
$\frac{dv}{dy}$ = velocity gradient

The proportionality constant μ in Eq. (1.3) is referred to as the *absolute viscosity* or *dynamic viscosity*. In SI units, μ is expressed in poise or centipoise (cP).

The viscosity of water, like that of most liquids, decreases with an increase in temperature, and vice versa. Under room temperature conditions water has an absolute viscosity of 1 cP.

Kinematic viscosity is defined as the absolute viscosity divided by the density. Thus

$$\nu = \frac{\mu}{\rho} \tag{1.4}$$

where ν = kinematic viscosity, ft^2/s
μ = absolute viscosity, (lb · s)/ft^2 or slug/(ft · s)
ρ = density, slug/ft^3

In SI units, kinematic viscosity is expressed as stokes or centistokes (cSt). Under room temperature conditions water has a kinematic viscosity of 1.0 cSt. Properties of water are listed in Table 1.1.

Example 1.3 Water has a dynamic viscosity of 1 cP at 20°C. Calculate the kinematic viscosity in SI units.

Solution

$$\text{Kinematic viscosity} = \frac{\text{absolute viscosity } \mu}{\text{density } \rho}$$

$$= \frac{1.0 \times 10^{-2} \times 0.1\ (\text{N} \cdot \text{s})/\text{m}^2}{1.0 \times 1000\ \text{kg/m}^3} = 10^{-6}\ \text{m}^2/\text{s}$$

since 1.0 N = 1.0 (kg · m)/s^2.

TABLE 1.1 Properties of Water at Atmospheric Pressure

Temperature °F	Density slug/ft^3	Specific weight lb/ft^3	Dynamic viscosity (lb · s)/ft^2	Vapor pressure psia
		USCS units		
32	1.94	62.4	3.75×10^{-5}	0.08
40	1.94	62.4	3.24×10^{-5}	0.12
50	1.94	62.4	2.74×10^{-5}	0.17
60	1.94	62.4	2.36×10^{-5}	0.26
70	1.94	62.3	2.04×10^{-5}	0.36
80	1.93	62.2	1.80×10^{-5}	0.51
90	1.93	62.1	1.59×10^{-5}	0.70
100	1.93	62.0	1.42×10^{-5}	0.96
Temperature °C	**Density kg/m^3**	**Specific weight kN/m^3**	**Dynamic viscosity (N · s)/m^2**	**Vapor pressure kPa**
		SI units		
0	1000	9.81	1.75×10^{-3}	0.611
10	1000	9.81	1.30×10^{-3}	1.230
20	998	9.79	1.02×10^{-3}	2.340
30	996	9.77	8.00×10^{-4}	4.240
40	992	9.73	6.51×10^{-4}	7.380
50	988	9.69	5.41×10^{-4}	12.300
60	984	9.65	4.60×10^{-4}	19.900
70	978	9.59	4.02×10^{-4}	31.200
80	971	9.53	3.50×10^{-4}	47.400
90	965	9.47	3.11×10^{-4}	70.100
100	958	9.40	2.82×10^{-4}	101.300

1.2 Pressure

Pressure is defined as the force per unit area. The pressure at a location in a body of water is by Pascal's law constant in all directions. In USCS units pressure is measured in lb/in^2 (psi), and in SI units it is expressed as N/m^2 or pascals (Pa). Other units for pressure include lb/ft^2, kilopascals (kPa), megapascals (MPa), kg/cm^2, and bar. Conversion factors are listed in App. A.

Therefore, at a depth of 100 ft below the free surface of a water tank the intensity of pressure, or simply the pressure, is the force per unit area. Mathematically, the column of water of height 100 ft exerts a force equal to the weight of the water column over an area of 1 in^2. We can calculate the pressures as follows:

$$\text{Pressure} = \frac{\text{weight of 100-ft column of area 1.0 in}^2}{1.0\ \text{in}^2}$$

$$= \frac{100 \times (1/144) \times 62.4}{1.0}$$

In this equation, we have assumed the specific weight of water to be 62.4 lb/ft^3. Therefore, simplifying the equation, we obtain

$$\text{Pressure at a depth of 100 ft} = 43.33 \text{ lb/in}^2 \text{ (psi)}$$

A general equation for the pressure in a liquid at a depth h is

$$P = \gamma h \tag{1.5}$$

where P = pressure, psi
γ = specific weight of liquid
h = liquid depth

Variable γ may also be replaced with ρg where ρ is the density and g is gravitational acceleration.

Generally, pressure in a body of water or a water pipeline is referred to in psi above that of the atmospheric pressure. This is also known as the *gauge pressure* as measured by a pressure gauge. The *absolute pressure* is the sum of the gauge pressure and the atmospheric pressure at the specified location. Mathematically,

$$P_{\text{abs}} = P_{\text{gauge}} + P_{\text{atm}} \tag{1.6}$$

To distinguish between the two pressures, psig is used for gauge pressure and psia is used for the absolute pressure. In most calculations involving water pipelines the gauge pressure is used. Unless otherwise specified, psi means the gauge pressure.

Liquid pressure may also be referred to as *head pressure*, in which case it is expressed in feet of liquid head (or meters in SI units). Therefore, a pressure of 1000 psi in a liquid such as water is said to be equivalent to a pressure head of

$$h = \frac{1000 \times 144}{62.4} = 2308 \text{ ft}$$

In a more general form, the pressure P in psi and liquid head h in feet for a specific gravity of Sg are related by

$$P = \frac{h \times \text{Sg}}{2.31} \tag{1.7}$$

where P = pressure, psi
h = liquid head, ft
Sg = specific gravity of water

In SI units, pressure P in kilopascals and head h in meters are related by the following equation:

$$P = \frac{h \times \text{Sg}}{0.102} \tag{1.8}$$

Example 1.4 Calculate the pressure in psi at a water depth of 100 ft assuming the specific weight of water is 62.4 lb/ft^3. What is the equivalent pressure in kilopascals? If the atmospheric pressure is 14.7 psi, calculate the absolute pressure at that location.

Solution Using Eq. (1.5), we calculate the pressure:

$$P = \gamma h = 62.4 \text{ lb/ft}^3 \times 100 \text{ ft} = 6240 \text{ lb/ft}^2$$

$$= \frac{6240}{144} \text{ lb/in}^2 = 43.33 \text{ psig}$$

$$\text{Absolute pressure} = 43.33 + 14.7 = 58.03 \text{ psia}$$

In SI units we can calculate the pressures as follows:

$$\text{Pressure} = 62.4 \times \frac{1}{2.2025}(3.281)^3 \text{ kg/m}^3 \times \left(\frac{100}{3.281}\text{m}\right)(9.81 \text{ m/s}^2)$$

$$= 2.992 \times 10^5 (\text{kg} \cdot \text{m})/(\text{s}^2 \cdot \text{m}^2)$$

$$= 2.992 \times 10^5 \text{ N/m}^2 = 299.2 \text{ kPa}$$

Alternatively,

$$\text{Pressure in kPa} = \frac{\text{pressure in psi}}{0.145}$$

$$= \frac{43.33}{0.145} = 298.83 \text{ kPa}$$

The 0.1 percent discrepancy between the values is due to conversion factor round-off.

1.3 Velocity

The velocity of flow in a water pipeline depends on the pipe size and flow rate. If the flow rate is uniform throughout the pipeline (steady flow), the velocity at every cross section along the pipe will be a constant value. However, there is a variation in velocity along the pipe cross section. The velocity at the pipe wall will be zero, increasing to a maximum at the centerline of the pipe. This is illustrated in Fig. 1.1*b*.

We can define a bulk velocity or an average velocity of flow as follows:

$$\text{Velocity} = \frac{\text{flow rate}}{\text{area of flow}}$$

Considering a circular pipe with an inside diameter D and a flow rate of Q, we can calculate the average velocity as

$$V = \frac{Q}{\pi D^2/4} \tag{1.9}$$

Employing consistent units of flow rate Q in ft^3/s and pipe diameter in inches, the velocity in ft/s is as follows:

$$V = \frac{144Q}{\pi D^2/4}$$

or

$$V = 183.3461\frac{Q}{D^2} \tag{1.10}$$

where V = velocity, ft/s
Q = flow rate, ft^3/s
D = inside diameter, in

Additional formulas for velocity in different units are as follows:

$$V = 0.4085\frac{Q}{D^2} \tag{1.11}$$

where V = velocity, ft/s
Q = flow rate, gal/min
D = inside diameter, in

In SI units, the velocity equation is as follows:

$$V = 353.6777\frac{Q}{D^2} \tag{1.12}$$

where V = velocity, m/s
Q = flow rate, m^3/h
D = inside diameter, mm

Example 1.5 Water flows through an NPS 16 pipeline (0.250-in wall thickness) at the rate of 3000 gal/min. Calculate the average velocity for steady flow. (*Note*: The designation NPS 16 means nominal pipe size of 16 in.)

Solution From Eq. (1.11), the average flow velocity is

$$V = 0.4085\frac{3000}{15.5^2} = 5.10 \text{ ft/s}$$

Example 1.6 Water flows through a DN 200 pipeline (10-mm wall thickness) at the rate of 75 L/s. Calculate the average velocity for steady flow.

Solution The designation DN 200 means metric pipe size of 200-mm outside diameter. It corresponds to NPS 8 in USCS units. From Eq. (1.12) the average flow velocity is

$$V = 353.6777\left(\frac{75 \times 60 \times 60 \times 10^{-3}}{180^2}\right) = 2.95 \text{ m/s}$$

The variation of flow velocity in a pipe depends on the type of flow. In laminar flow, the velocity variation is parabolic. As the flow rate becomes turbulent the velocity profile approximates a trapezoidal shape. Both types of flow are depicted in Fig. 1.1*b*. Laminar and turbulent flows are discussed in Sec. 1.5 after we introduce the concept of the Reynolds number.

1.4 Reynolds Number

The Reynolds number is a dimensionless parameter of flow. It depends on the pipe size, flow rate, liquid viscosity, and density. It is calculated from the following equation:

$$R = \frac{VD\rho}{\mu} \tag{1.13}$$

or

$$R = \frac{VD}{\nu} \tag{1.14}$$

where R = Reynolds number, dimensionless
V = average flow velocity, ft/s
D = inside diameter of pipe, ft
ρ = mass density of liquid, slug/ft^3
μ = dynamic viscosity, slug/(ft · s)
ν = kinematic viscosity, ft^2/s

Since R must be dimensionless, a consistent set of units must be used for all items in Eq. (1.13) to ensure that all units cancel out and R has no dimensions.

Other variations of the Reynolds number for different units are as follows:

$$R = 3162.5\frac{Q}{D\nu} \tag{1.15}$$

where R = Reynolds number, dimensionless
Q = flow rate, gal/min
D = inside diameter of pipe, in
ν = kinematic viscosity, cSt

In SI units, the Reynolds number is expressed as follows:

$$R = 353{,}678 \frac{Q}{\nu D} \tag{1.16}$$

where R = Reynolds number, dimensionless
Q = flow rate, m^3/h
D = inside diameter of pipe, mm
ν = kinematic viscosity, cSt

Example 1.7 Water flows through a 20-in pipeline (0.375-in wall thickness) at 6000 gal/min. Calculate the average velocity and Reynolds number of flow. Assume water has a viscosity of 1.0 cSt.

Solution Using Eq. (1.11), the average velocity is calculated as follows:

$$V = 0.4085 \frac{6000}{19.25^2} = 6.61 \text{ ft/s}$$

From Eq. (1.15), the Reynolds number is

$$R = 3162.5 \frac{6000}{19.25 \times 1.0} = 985{,}714$$

Example 1.8 Water flows through a 400-mm pipeline (10-mm wall thickness) at 640 m^3/h. Calculate the average velocity and Reynolds number of flow. Assume water has a viscosity of 1.0 cSt.

Solution From Eq. (1.12) the average velocity is

$$V = 353.6777 \frac{640}{380^2} = 1.57 \text{ m/s}$$

From Eq. (1.16) the Reynolds number is

$$R = 353{,}678 \frac{640}{380 \times 1.0} = 595{,}668$$

1.5 Types of Flow

Flow through pipe can be classified as laminar flow, turbulent flow, or critical flow depending on the Reynolds number of flow. If the flow is such that the Reynolds number is less than 2000 to 2100, the flow is said to be *laminar*. When the Reynolds number is greater than 4000, the flow is said to be *turbulent*. *Critical flow* occurs when the Reynolds number is in the range of 2100 to 4000. Laminar flow is characterized by smooth flow in which no eddies or turbulence are visible. The flow is said to occur in laminations. If dye was injected into a transparent pipeline, laminar flow would be manifested in the form of smooth streamlines of dye. Turbulent flow occurs at higher velocities and is accompanied by eddies and other disturbances in the liquid. Mathematically, if R represents the Reynolds number of flow, the flow types are defined as follows:

Laminar flow: $R \leq 2100$

Critical flow: $2100 < R \leq 4000$

Turbulent flow: $R > 4000$

In the critical flow regime, where the Reynolds number is between 2100 and 4000, the flow is undefined as far as pressure drop calculations are concerned.

1.6 Pressure Drop Due to Friction

As water flows through a pipe there is friction between the adjacent layers of water and between the water molecules and the pipe wall. This friction causes energy to be lost, being converted from pressure energy and kinetic energy to heat. The pressure continuously decreases as water flows down the pipe from the upstream end to the downstream end. The amount of pressure loss due to friction, also known as head loss due to friction, depends on the flow rate, properties of water (specific gravity and viscosity), pipe diameter, pipe length, and internal roughness of the pipe. Before we discuss the frictional pressure loss in a pipeline we must introduce Bernoulli's equation, which is a form of the energy equation for liquid flow in a pipeline.

1.6.1 Bernoulli's equation

Bernoulli's equation is another way of stating the principle of conservation of energy applied to liquid flow through a pipeline. At each point along the pipeline the total energy of the liquid is computed by taking into consideration the liquid energy due to pressure, velocity, and elevation combined with any energy input, energy output, and energy losses. The total energy of the liquid contained in the pipeline at any point is a constant. This is also known as the principle of conservation of energy.

Consider a liquid flow through a pipeline from point A to point B as shown in Fig. 1.2. The elevation of point A is Z_A and the elevation at B is Z_B above some common datum, such as mean sea level. The pressure at point A is P_A and that at B is P_B. It is assumed that the pipe diameter at A and B are different, and hence the flow velocity at A and B will be represented by V_A and V_B, respectively. A particle of the liquid of

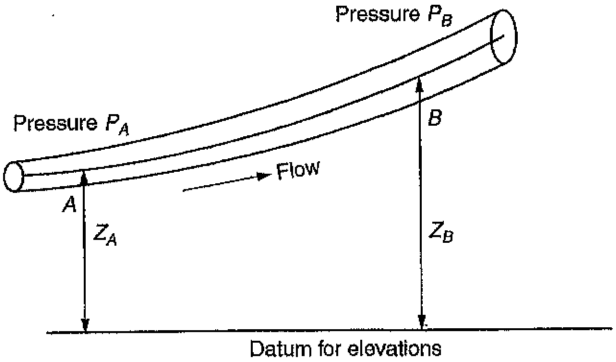


Figure 1.2 Total energy of water in pipe flow.

unit weight at point A in the pipeline possesses a total energy E which consists of three components:

$$\text{Potential energy} = Z_A$$

$$\text{Pressure energy} = \frac{P_A}{\gamma}$$

$$\text{Kinetic energy} = \left(\frac{V_A}{2g}\right)^2$$

where γ is the specific weight of liquid.

Therefore the total energy E is

$$E = Z_A + \frac{P_A}{\gamma} + \frac{V_A{}^2}{2g} \tag{1.17}$$

Since each term in Eq. (1.17) has dimensions of length, we refer to the total energy at point A as H_A in feet of liquid head. Therefore, rewriting the total energy in feet of liquid head at point A, we obtain

$$H_A = Z_A + \frac{P_A}{\gamma} + \frac{V_A{}^2}{2g} \tag{1.18}$$

Similarly, the same unit weight of liquid at point B has a total energy per unit weight equal to H_B given by

$$H_B = Z_B + \frac{P_B}{\gamma} + \frac{V_B{}^2}{2g} \tag{1.19}$$

By the principle of conservation of energy

$$H_A = H_B \tag{1.20}$$

Therefore,

$$Z_A + \frac{P_A}{\gamma} + \frac{{V_A}^2}{2g} = Z_B + \frac{P_B}{\gamma} + \frac{{V_B}^2}{2g} \tag{1.21}$$

In Eq. (1.21), referred to as Bernoulli's equation, we have not considered any energy added to the liquid, energy taken out of the liquid, or energy losses due to friction. Therefore, modifying Eq. (1.21) to take into account the addition of energy (such as from a pump at A) and accounting for frictional head losses h_f, we get the more common form of Bernoulli's equation as follows:

$$Z_A + \frac{P_A}{\gamma} + \frac{{V_A}^2}{2g} + H_p = Z_B + \frac{P_B}{\gamma} + \frac{{V_B}^2}{2g} + h_f \tag{1.22}$$

where H_P is the equivalent head added to the liquid by the pump at A and h_f represents the total frictional head losses between points A and B.

We will next discuss how the head loss due to friction h_f in Bernoulli's equation is calculated for various conditions of water flow in pipelines. We begin with the classical pressure drop equation known as the Darcy-Weisbach equation, or simply the Darcy equation.

1.6.2 Darcy equation

The Darcy equation, also called Darcy-Weisbach equation, is one of the oldest formulas used in classical fluid mechanics. It can be used to calculate the pressure drop in pipes transporting any type of fluid, such as a liquid or gas.

As water flows through a pipe from point A to point B the pressure decreases due to friction between the water and the pipe wall. The Darcy equation may be used to calculate the pressure drop in water pipes as follows:

$$h = f\frac{L}{D}\frac{V^2}{2g} \tag{1.23}$$

where h = frictional pressure loss, ft of head
f = Darcy friction factor, dimensionless
L = pipe length, ft
D = inside pipe diameter, ft
V = average flow velocity, ft/s
g = acceleration due to gravity, ft/s^2

In USCS units, $g = 32.2$ ft/s^2, and in SI units, $g = 9.81$ m/s^2.

Note that the Darcy equation gives the frictional pressure loss in feet of head of water. It can be converted to pressure loss in psi using Eq. (1.7). The term $V^2/2g$ in the Darcy equation is called the velocity head, and it represents the kinetic energy of the water. The term *velocity head* will be used in subsequent sections of this chapter when discussing frictional head loss through pipe fittings and valves.

Another form of the Darcy equation with frictional pressure drop expressed in psi/mi and using a flow rate instead of velocity is as follows:

$$P_m = 71.16\frac{fQ^2}{D^5} \tag{1.24}$$

where P_m = frictional pressure loss, psi/mi
f = Darcy friction factor, dimensionless
Q = flow rate, gal/min
D = pipe inside diameter, in

In SI units, the Darcy equation may be written as

$$h = 50.94\frac{fLV^2}{D} \tag{1.25}$$

where h = frictional pressure loss, meters of liquid head
f = Darcy friction factor, dimensionless
L = pipe length, m
D = pipe inside diameter, mm
V = average flow velocity, m/s

Another version of the Darcy equation in SI units is as follows:

$$P_{km} = (6.2475 \times 10^{10})\frac{fQ^2}{D^5} \tag{1.26}$$

where P_{km} = pressure drop due to friction, kPa/km
Q = liquid flow rate, m³/h
f = Darcy friction factor, dimensionless
D = pipe inside diameter, mm

In order to calculate the friction loss in a water pipeline using the Darcy equation, we must know the friction factor f. The friction factor f in the Darcy equation is the only unknown on the right-hand side of Eq. (1.23). This friction factor is a nondimensional number between 0.0 and 0.1 (usually around 0.02 for turbulent flow) that depends on the internal roughness of the pipe, the pipe diameter, and the Reynolds number, and therefore the type of flow (laminar or turbulent).

For laminar flow, the friction factor f depends only on the Reynolds number and is calculated as follows:

$$f = \frac{64}{R} \tag{1.27}$$

where f is the friction factor for laminar flow and R is the Reynolds number for laminar flow ($R < 2100$) (dimensionless).

Therefore, if the Reynolds number for a particular flow is 1200, the friction factor for this laminar flow is $64/1200 = 0.0533$. If this pipeline has a 400-mm inside diameter and water flows through it at 500 m^3/h, the pressure loss per kilometer would be, from Eq. (1.26),

$$P_{km} = 6.2475 \times 10^{10} \times 0.0533 \times \frac{(500)^2}{(400)^5} = 81.3 \text{ kPa/km}$$

If the flow is turbulent ($R > 4000$), calculation of the friction factor is not as straightforward as that for laminar flow. We will discuss this next.

1.6.3 Colebrook-White equation

In turbulent flow the calculation of friction factor f is more complex. The friction factor depends on the pipe inside diameter, the pipe roughness, and the Reynolds number. Based on work by Moody, Colebrook-White, and others, the following empirical equation, known as the Colebrook-White equation, has been proposed for calculating the friction factor in turbulent flow:

$$\frac{1}{\sqrt{f}} = -2 \log_{10}\left(\frac{e}{3.7D} + \frac{2.51}{R\sqrt{f}}\right) \tag{1.28}$$

where f = Darcy friction factor, dimensionless
D = pipe inside diameter, in
e = absolute pipe roughness, in
R = Reynolds number, dimensionless

The absolute pipe roughness depends on the internal condition of the pipe. Generally a value of 0.002 in or 0.05 mm is used in most calculations, unless better data are available. Table 1.2 lists the pipe roughness for various types of pipe. The ratio e/D is known as the relative pipe roughness and is dimensionless since both pipe absolute roughness e and pipe inside diameter D are expressed in the same units (inches in USCS units and millimeters in SI units). Therefore, Eq. (1.28) remains the same for SI units, except that, as stated, the absolute pipe roughness e and the pipe diameter D are both expressed in millimeters. All other terms in the equation are dimensionless.

TABLE 1.2 Pipe Internal Roughness

Pipe material	Roughness in	Roughness mm
Riveted steel	0.035–0.35	0.9–9.0
Commercial steel/welded steel	0.0018	0.045
Cast iron	0.010	0.26
Galvanized iron	0.006	0.15
Asphalted cast iron	0.0047	0.12
Wrought iron	0.0018	0.045
PVC, drawn tubing, glass	0.000059	0.0015
Concrete	0.0118–0.118	0.3–3.0

It can be seen from Eq. (1.28) that the calculation of the friction factor f is not straightforward since it appears on both sides of the equation. Successive iteration or a trial-and-error approach is used to solve for the friction factor.

1.6.4 Moody diagram

The Moody diagram is a graphical plot of the friction factor f for all flow regimes (laminar, critical, and turbulent) against the Reynolds number at various values of the relative roughness of pipe. The graphical method of determining the friction factor for turbulent flow using the Moody diagram (see Fig. 1.3) is discussed next.

For a given Reynolds number on the horizontal axis, a vertical line is drawn up to the curve representing the relative roughness e/D. The friction factor is then read by going horizontally to the vertical axis on the left. It can be seen from the Moody diagram that the turbulent region is further divided into two regions: the "transition zone" and the "complete turbulence in rough pipes" zone. The lower boundary is designated as "smooth pipes," and the transition zone extends up to the dashed line. Beyond the dashed line is the complete turbulence in rough pipes zone. In this zone the friction factor depends very little on the Reynolds number and more on the relative roughness. This is evident from the Colebrook-White equation, where at large Reynolds numbers, the second term within the parentheses approaches zero. The friction factor thus depends only on the first term, which is proportional to the relative roughness e/D. In contrast, in the transition zone both R and e/D influence the value of friction factor f.

Example 1.9 Water flows through a 16-in pipeline (0.375-in wall thickness) at 3000 gal/min. Assuming a pipe roughness of 0.002 in, calculate the friction factor and head loss due to friction in 1000 ft of pipe length.

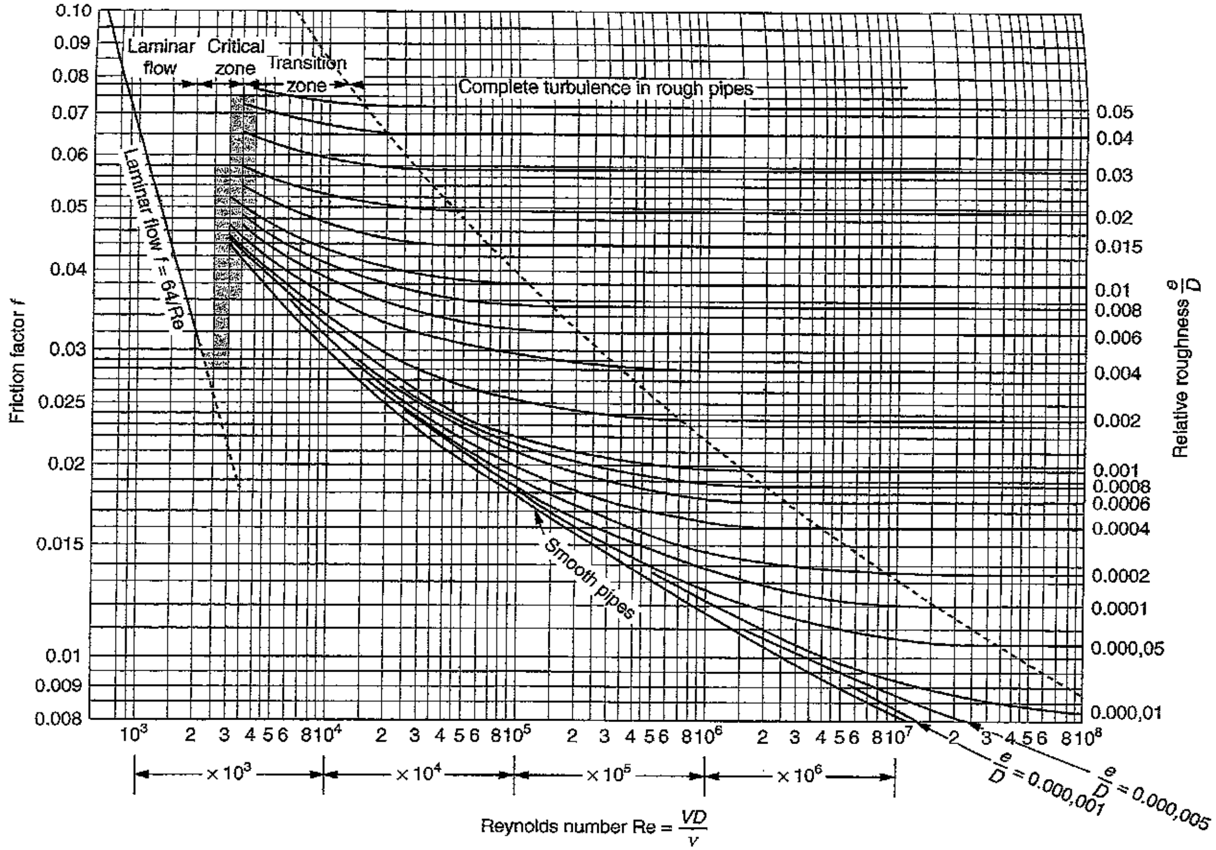


Figure 1.3 Moody diagram.

Solution Using Eq. (1.11) we calculate the average flow velocity:

$$V = 0.4085\frac{3000}{(15.25)^2} = 5.27 \text{ ft/s}$$

Using Eq. (1.15) we calculate the Reynolds number as follows:

$$R = 3162.5\frac{3000}{15.25 \times 1.0} = 622{,}131$$

Thus the flow is turbulent, and we can use the Colebrook-White equation (1.28) to calculate the friction factor.

$$\frac{1}{\sqrt{f}} = -2\log_{10}\left(\frac{0.002}{3.7 \times 15.25} + \frac{2.51}{622{,}131\sqrt{f}}\right)$$

This equation must be solved for f by trial and error. First assume that $f = 0.02$. Substituting in the preceding equation, we get a better approximation for f as follows:

$$\frac{1}{\sqrt{f}} = -2\log_{10}\left(\frac{0.002}{3.7 \times 15.25} + \frac{2.51}{622{,}131\sqrt{0.02}}\right) \quad \text{or} \quad f = 0.0142$$

Recalculating using this value

$$\frac{1}{\sqrt{f}} = -2\log_{10}\left(\frac{0.002}{3.7 \times 15.25} + \frac{2.51}{(622{,}131\sqrt{0.0142}}\right) \quad \text{or} \quad f = 0.0145$$

and finally

$$\frac{1}{\sqrt{f}} = -2\log_{10}\left(\frac{0.002}{3.7 \times 15.25} + \frac{2.51}{622{,}131\sqrt{0.0145}}\right) \quad \text{or} \quad f = 0.0144$$

Thus the friction factor is 0.0144. (We could also have used the Moody diagram to find the friction factor graphically, for Reynolds number $R = 622{,}131$ and $e/D = 0.002/15.25 = 0.0001$. From the graph, we get $f = 0.0145$, which is close enough.)

The head loss due to friction can now be calculated using the Darcy equation (1.23).

$$h = 0.0144\frac{1000 \times 12}{15.25}\frac{5.27^2}{64.4} = 4.89 \text{ ft of head of water}$$

Converting to psi using Eq. (1.7), we get

$$\text{Pressure drop due to friction} = \frac{4.89 \times 1.0}{2.31} = 2.12 \text{ psi}$$

Example 1.10 A concrete pipe (2-m inside diameter) is used to transport water from a pumping facility to a storage tank 5 km away. Neglecting any difference in elevations, calculate the friction factor and pressure loss in kPa/km due to friction at a flow rate of 34,000 m^3/h. Assume a pipe roughness of 0.05 mm. If a delivery pressure of 4 kPa must be maintained at the delivery point and the storage tank is at an elevation of 200 m above that of the pumping facility, calculate the pressure required at the pumping facility at the given flow rate, using the Moody diagram.

Solution The average flow velocity is calculated using Eq. (1.12).

$$V = 353.6777 \frac{34{,}000}{(2000)^2} = 3.01 \text{ m/s}$$

Next using Eq. (1.16), we get the Reynolds number as follows:

$$R = 353{,}678 \frac{34{,}000}{1.0 \times 2000} = 6{,}012{,}526$$

Therefore, the flow is turbulent. We can use the Colebrook-White equation or the Moody diagram to determine the friction factor. The relative roughness is

$$\frac{e}{D} = \frac{0.05}{2000} = 0.00003$$

Using the obtained values for relative roughness and the Reynolds number, from the Moody diagram we get friction factor $f = 0.01$.

The pressure drop due to friction can now be calculated using the Darcy equation (1.23) for the entire 5-km length of pipe as

$$h = 0.01 \frac{5000}{2.0} \frac{3.01^2}{2 \times 9.81} = 11.54 \text{ m of head of water}$$

Using Eq. (1.8) we calculate the pressure drop in kilopascals as

$$\text{Total pressure drop in 5 km} = \frac{11.54 \times 1.0}{0.102} = 113.14 \text{ kPa}$$

Therefore,

$$\text{Pressure drop in kPa/km} = \frac{113.14}{5} = 22.63 \text{ kPa/km}$$

The pressure required at the pumping facility is calculated by adding the following three items:

1. Pressure drop due to friction for 5-km length.
2. The static elevation difference between the pumping facility and storage tank.
3. The delivery pressure required at the storage tank.

We can also state the calculation mathematically.

$$P_t = P_f + P_{elev} + P_{del} \tag{1.29}$$

where P_t = total pressure required at pump
P_f = frictional pressure head
P_{elev} = pressure head due to elevation difference
P_{del} = delivery pressure at storage tank

All pressures must be in the same units: either meters of head or kilopascals.

$$P_t = 113.14 \text{ kPa} + 200 \text{ m} + 4 \text{ kPa}$$

Changing all units to kilopascals we get

$$P_t = 113.14 + \frac{200 \times 1.0}{0.102} + 4 = 2077.92 \text{ kPa}$$

Therefore, the pressure required at the pumping facility is 2078 kPa.

1.6.5 Hazen-Williams equation

A more popular approach to the calculation of head loss in water piping systems is the use of the Hazen-Williams equation. In this method a coefficient C known as the Hazen-Williams C factor is used to account for the internal pipe roughness or efficiency. Unlike the Moody diagram or the Colebrook-White equation, the Hazen-Williams equation does not require use of the Reynolds number or viscosity of water to calculate the head loss due to friction.

The Hazen-Williams equation for head loss is expressed as follows:

$$h = \frac{4.73\ L(Q/C)^{1.852}}{D^{4.87}} \tag{1.30}$$

where h = frictional head loss, ft
L = length of pipe, ft
D = inside diameter of pipe, ft
Q = flow rate, ft^3/s
C = Hazen-Williams C factor or roughness coefficient, dimensionless

Commonly used values of the Hazen-Williams C factor for various applications are listed in Table 1.3.

TABLE 1.3 Hazen-Williams *C* Factor

Pipe material	*C* factor
Smooth pipes (all metals)	130–140
Cast iron (old)	100
Iron (worn/pitted)	60–80
Polyvinyl chloride (PVC)	150
Brick	100
Smooth wood	120
Smooth masonry	120
Vitrified clay	110

On examining the Hazen-Williams equation, we see that the head loss due to friction is calculated in feet of head, similar to the Darcy equation. The value of h can be converted to psi using the head-to-psi conversion [Eq. (1.7)]. Although the Hazen-Williams equation appears to be simpler to use than the Colebrook-White and Darcy equations to calculate the pressure drop, the unknown term C can cause uncertainties in the pressure drop calculation.

Usually, the C factor, or Hazen-Williams roughness coefficient, is based on experience with the water pipeline system, such as the pipe material or internal condition of the pipeline system. When designing a new pipeline, proper judgment must be exercised in choosing a C factor since considerable variation in pressure drop can occur by selecting a particular value of C compared to another. Because of the inverse proportionality effect of C on the head loss h, using $C = 140$ instead of $C = 100$ will result in a $[1 - (\frac{100}{140})^{1.852}]$ or 46 percent less pressure drop. Therefore, it is important that the C value be chosen judiciously.

Other forms of the Hazen-Williams equation using different units are discussed next. In the following formulas the presented equations calculate the flow rate from a given head loss, or vice versa.

In USCS units, the following forms of the Hazen-Williams equation are used.

$$Q = (6.755 \times 10^{-3}) C D^{2.63} h^{0.54} \tag{1.31}$$

$$h = 10{,}460 \left(\frac{Q}{C}\right)^{1.852} \frac{1}{D^{4.87}} \tag{1.32}$$

$$P_m = 23{,}909 \left(\frac{Q}{C}\right)^{1.852} \frac{1}{D^{4.87}} \tag{1.33}$$

where Q = flow rate, gal/min
h = friction loss, ft of water per 1000 ft of pipe
P_m = friction loss, psi per mile of pipe
D = inside diameter of pipe, in
C = Hazen-Williams C factor, dimensionless (see Table 1.3)

In SI units, the Hazen-Williams equation is expressed as follows:

$$Q = (9.0379 \times 10^{-8}) C D^{2.63} \left(\frac{P_{\text{km}}}{\text{Sg}}\right)^{0.54} \tag{1.34}$$

$$P_{\text{km}} = 1.1101 \times 10^{13} \left(\frac{Q}{C}\right)^{1.852} \frac{\text{Sg}}{D^{4.87}} \tag{1.35}$$

where Q = flow rate, m^3/h
D = pipe inside diameter, mm
P_{km} = frictional pressure drop, kPa/km
Sg = liquid specific gravity (water = 1.00)
C = Hazen-Williams C factor, dimensionless (see Table 1.3)

1.6.6 Manning equation

The Manning equation was originally developed for use in open-channel flow of water. It is also sometimes used in pipe flow. The Manning equation uses the Manning index n, or roughness coefficient, which like the Hazen-Williams C factor depends on the type and internal condition of the pipe. The values used for the Manning index for common pipe materials are listed in Table 1.4.

The following is a form of the Manning equation for pressure drop due to friction in water piping systems:

$$Q = \frac{1.486}{n} AR^{2/3} \left(\frac{h}{L}\right)^{1/2} \tag{1.36}$$

where Q = flow rate, ft^3/s
A = cross-sectional area of pipe, ft^2
R = hydraulic radius = $D/4$ for circular pipes flowing full
n = Manning index, or roughness coefficient, dimensionless
D = inside diameter of pipe, ft
h = friction loss, ft of water
L = pipe length, ft

TABLE 1.4 Manning Index

Pipe material	Resistance factor
PVC	0.009
Very smooth	0.010
Cement-lined ductile iron	0.012
New cast iron, welded steel	0.014
Old cast iron, brick	0.020
Badly corroded cast iron	0.035
Wood, concrete	0.016
Clay, new riveted steel	0.017
Canals cut through rock	0.040
Earth canals average condition	0.023
Rivers in good conditions	0.030

In SI units, the Manning equation is expressed as follows:

$$Q = \frac{1}{n} A R^{2/3} \left(\frac{h}{L}\right)^{1/2} \tag{1.37}$$

where Q = flow rate, m^3/s
A = cross-sectional area of pipe, m^2
R = hydraulic radius = $D/4$ for circular pipes flowing full
n = Manning index, or roughness coefficient, dimensionless
D = inside diameter of pipe, m
h = friction loss, ft of water
L = pipe length, m

Example 1.11 Water flows through a 16-in pipeline (0.375-in wall thickness) at 3000 gal/min. Using the Hazen-Williams equation with a C factor of 120, calculate the pressure loss due to friction in 1000 ft of pipe length.

Solution First we calculate the flow rate using Eq. (1.31):

$$Q = 6.755 \times 10^{-3} \times 120 \times (15.25)^{2.63} h^{0.54}$$

where h is in feet of head per 1000 ft of pipe.

Rearranging the preceding equation, using $Q = 3000$ and solving for h, we get

$$h^{0.54} = \frac{3000}{6.755 \times 10^{-3} \times 120 \times (15.25)^{2.63}}$$

Therefore,

$$h = 7.0 \text{ ft per 1000 ft of pipe}$$

$$\text{Pressure drop} = \frac{7.0 \times 1.0}{2.31} = 3.03 \text{ psi}$$

Compare this with the same problem described in Example 1.9. Using the Colebrook-White and Darcy equations we calculated the pressure drop to be 4.89 ft per 1000 ft of pipe. Therefore, we can conclude that the C value used in the Hazen-Williams equation in this example is too low and hence gives us a comparatively higher pressure drop. Therefore, we will recalculate the pressure drop using a C factor = 140 instead.

$$h^{0.54} = \frac{3000}{6.755 \times 10^{-3} \times 140 \times (15.25)^{2.63}}$$

Therefore,

$$h = 5.26 \text{ ft per 1000 ft of pipe}$$

$$\text{Pressure drop} = \frac{5.26 \times 1.0}{2.31} = 2.28 \text{ psi}$$

It can be seen that we are closer now to the results using the Colebrook-White and Darcy equations. The result is still 7.6 percent higher than that obtained using the Colebrook-White and Darcy equations. The conclusion is that the

C factor in the preceding Hazen-Williams calculation should probably be slightly higher than 140. In fact, using a C factor of 146 will get the result closer to the 4.89 ft per 1000 ft we got using the Colebrook-White equation.

Example 1.12 A concrete pipe with a 2-m inside diameter is used to transport water from a pumping facility to a storage tank 5 km away. Neglecting differences in elevation, calculate the pressure loss in kPa/km due to friction at a flow rate of 34,000 m^3/h. Use the Hazen-Williams equation with a C factor of 140. If a delivery pressure of 400 kPa must be maintained at the delivery point and the storage tank is at an elevation of 200 m above that of the pumping facility, calculate the pressure required at the pumping facility at the given flow rate.

Solution The flow rate Q in m^3/h is calculated using the Hazen-Williams equation (1.35) as follows:

$$P_{\mathrm{km}} = (1.1101 \times 10^{13}) \left(\frac{34{,}000}{140}\right)^{1.852} \times \frac{1}{(2000)^{4.87}}$$
$$= 24.38 \text{ kPa/km}$$

The pressure required at the pumping facility is calculated by adding the pressure drop due to friction to the delivery pressure required and the static elevation head between the pumping facility and storage tank using Eq. (1.29).

$$P_t = P_f + P_{\mathrm{elev}} + P_{\mathrm{del}}$$
$$= (24.38 \times 5) \text{ kPa } + 200 \text{ m} + 400 \text{ kPa}$$

Changing all units to kPa we get

$$P_t = 121.9 + \frac{200 \times 1.0}{0.102} + 400 = 2482.68 \text{ kPa}$$

Thus the pressure required at the pumping facility is 2483 kPa.

1.7 Minor Losses

So far, we have calculated the pressure drop per unit length in straight pipe. We also calculated the total pressure drop considering several miles of pipe from a pump station to a storage tank. Minor losses in a water pipeline are classified as those pressure drops that are associated with piping components such as valves and fittings. Fittings include elbows and tees. In addition there are pressure losses associated with pipe diameter enlargement and reduction. A pipe nozzle exiting from a storage tank will have entrance and exit losses. All these pressure drops are called *minor losses*, as they are relatively small compared to friction loss in a straight length of pipe.

Generally, minor losses are included in calculations by using the equivalent length of the valve or fitting or using a resistance factor or

TABLE 1.5 Equivalent Lengths of Valves and Fittings

Description	L/D
Gate valve	8
Globe valve	340
Angle valve	55
Ball valve	3
Plug valve straightway	18
Plug valve 3-way through-flow	30
Plug valve branch flow	90
Swing check valve	100
Lift check valve	600
Standard elbow	
90°	30
45°	16
Long radius 90°	16
Standard tee	
Through-flow	20
Through-branch	60
Miter bends	
$\alpha = 0$	2
$\alpha = 30$	8
$\alpha = 60$	25
$\alpha = 90$	60

K factor multiplied by the velocity head $V^2/2g$. The term minor losses can be applied only where the pipeline lengths and hence the friction losses are relatively large compared to the pressure drops in the fittings and valves. In a situation such as plant piping and tank farm piping the pressure drop in the straight length of pipe may be of the same order of magnitude as that due to valves and fittings. In such cases the term minor losses is really a misnomer. In any case, the pressure losses through valves, fittings, etc., can be accounted for approximately using the equivalent length or K times the velocity head method. It must be noted that this way of calculating the minor losses is valid only in turbulent flow. No data are available for laminar flow.

1.7.1 Valves and fittings

Table 1.5 shows the equivalent lengths of commonly used valves and fittings in a typical water pipeline. It can be seen from this table that a gate valve has an L/D ratio of 8 compared to straight pipe. Therefore, a 20-in-diameter gate valve may be replaced with a $20 \times 8 = 160$-in-long piece of pipe that will match the frictional pressure drop through the valve.

Example 1.13 A piping system is 2000 ft of NPS 20 pipe that has two 20-in gate valves, three 20-in ball valves, one swing check valve, and four

90° standard elbows. Using the equivalent length concept, calculate the total pipe length that will include all straight pipe and valves and fittings.

Solution Using Table 1.5, we can convert all valves and fittings in terms of 20-in pipe as follows:

$$\text{Two 20-in gate valves} = 2 \times 20 \times 8 = 320 \text{ in of 20-in pipe}$$

$$\text{Three 20-in ball valves} = 3 \times 20 \times 3 = 180 \text{ in of 20-in pipe}$$

$$\text{One 20-in swing check valve} = 1 \times 20 \times 50 = 1000 \text{ in of 20-in pipe}$$

$$\text{Four 90° elbows} = 4 \times 20 \times 30 = 2400 \text{ in of 20-in pipe}$$

$$\text{Total for all valves and fittings} = 4220 \text{ in of 20-in pipe}$$

$$= 351.67 \text{ ft of 20-in pipe}$$

Adding the 2000 ft of straight pipe, the total equivalent length of straight pipe and all fittings is

$$L_e = 2000 + 351.67 = 2351.67 \text{ ft}$$

The pressure drop due to friction in the preceding piping system can now be calculated based on 2351.67 ft of pipe. It can be seen in this example that the valves and fittings represent roughly 15 percent of the total pipeline length. In plant piping this percentage may be higher than that in a long-distance water pipeline. Hence, the reason for the term *minor losses*.

Another approach to accounting for minor losses is using the resistance coefficient or K factor. The K factor and the velocity head approach to calculating pressure drop through valves and fittings can be analyzed as follows using the Darcy equation. From the Darcy equation (1.23), the pressure drop in a straight length of pipe is given by

$$h = f \frac{L}{D} \frac{V^2}{2g} \tag{1.38}$$

The term $f(L/D)$ may be substituted with a head loss coefficient K (also known as the resistance coefficient) and Eq. (1.38) then becomes

$$h = K \frac{V^2}{2g} \tag{1.39}$$

In Eq. (1.39), the head loss in a straight piece of pipe is represented as a multiple of the velocity head $V^2/2g$. Following a similar analysis, we can state that the pressure drop through a valve or fitting can also be represented by $K(V^2/2g)$, where the coefficient K is specific to the valve or fitting. Note that this method is only applicable to turbulent flow through pipe fittings and valves. No data are available for laminar flow in fittings and valves. Typical K factors for valves and fittings are listed in Table 1.6. It can be seen that the K factor depends on the

TABLE 1.6 Friction Loss in Valves—Resistance Coefficient K

		Nominal pipe size, in											
Description	L/D	$\frac{1}{2}$	$\frac{3}{4}$	1	$1\frac{1}{4}$	$1\frac{1}{2}$	2	$2\frac{1}{2}$–3	4	6	8–10	12–16	18–24
Gate valve	8	0.22	0.20	0.18	0.18	0.15	0.15	0.14	0.14	0.12	0.11	0.10	0.10
Globe valve	340	9.20	8.50	7.80	7.50	7.10	6.50	6.10	5.80	5.10	4.80	4.40	4.10
Angle valve	55	1.48	1.38	1.27	1.21	1.16	1.05	0.99	0.94	0.83	0.77	0.72	0.66
Ball valve	3	0.08	0.08	0.07	0.07	0.06	0.06	0.05	0.05	0.05	0.04	0.04	0.04
Plug valve straightway	18	0.49	0.45	0.41	0.40	0.38	0.34	0.32	0.31	0.27	0.25	0.23	0.22
Plug valve 3-way through-flow	30	0.81	0.75	0.69	0.66	0.63	0.57	0.54	0.51	0.45	0.42	0.39	0.36
Plug valve branch flow	90	2.43	2.25	2.07	1.98	1.89	1.71	1.62	1.53	1.35	1.26	1.17	1.08
Swing check valve	50	1.40	1.30	1.20	1.10	1.10	1.00	0.90	0.90	0.75	0.70	0.65	0.60
Lift check valve	600	16.20	15.00	13.80	13.20	12.60	11.40	10.80	10.20	9.00	8.40	7.80	7.22
Standard elbow													
90°	30	0.81	0.75	0.69	0.66	0.63	0.57	0.54	0.51	0.45	0.42	0.39	0.36
45°	16	0.43	0.40	0.37	0.35	0.34	0.30	0.29	0.27	0.24	0.22	0.21	0.19
Long radius 90°	16	0.43	0.40	0.37	0.35	0.34	0.30	0.29	0.27	0.24	0.22	0.21	0.19
Standard tee													
Through-flow	20	0.54	0.50	0.46	0.44	0.42	0.38	0.36	0.34	0.30	0.28	0.26	0.24
Through-branch	60	1.62	1.50	1.38	1.32	1.26	1.14	1.08	1.02	0.90	0.84	0.78	0.72
Mitre bends													
$\alpha = 0$	2	0.05	0.05	0.05	0.04	0.04	0.04	0.04	0.03	0.03	0.03	0.03	0.02
$\alpha = 30$	8	0.22	0.20	0.18	0.18	0.17	0.15	0.14	0.14	0.12	0.11	0.10	0.10
$\alpha = 60$	25	0.68	0.63	0.58	0.55	0.53	0.48	0.45	0.43	0.38	0.35	0.33	0.30
$\alpha = 90$	60	1.62	1.50	1.38	1.32	1.26	1.14	1.08	1.02	0.90	0.84	0.78	0.72

nominal pipe size of the valve or fitting. The equivalent length, on the other hand, is given as a ratio of L/D for a particular fitting or valve.

From Table 1.6, it can be seen that a 6-in gate valve has a K factor of 0.12, while a 20-in gate valve has a K factor of 0.10. However, both sizes of gate valves have the same equivalent length–to–diameter ratio of 8. The head loss through the 6-in valve can be estimated to be $0.12\,(V^2/2g)$ and that in the 20-in valve is $0.10\,(V^2/2g)$. The velocities in both cases will be different due to the difference in diameters.

If the flow rate was 1000 gal/min, the velocity in the 6-in valve will be approximately

$$V_6 = 0.4085 \frac{1000}{6.125^2} = 10.89 \text{ ft/s}$$

Similarly, at 1000 gal/min, the velocity in the 20-in valve will be approximately

$$V_6 = 0.4085 \frac{1000}{19.5^2} = 1.07 \text{ ft/s}$$

Therefore,

$$\text{Head loss in 6-in gate valve} = \frac{0.12\,(10.89)^2}{64.4} = 0.22 \text{ ft}$$

and

$$\text{Head loss in 20-in gate valve} = \frac{0.10\,(1.07)^2}{64.4} = 0.002 \text{ ft}$$

These head losses appear small since we have used a relatively low flow rate in the 20-in valve. In reality the flow rate in the 20-in valve may be as high as 6000 gal/min and the corresponding head loss will be 0.072 ft.

1.7.2 Pipe enlargement and reduction

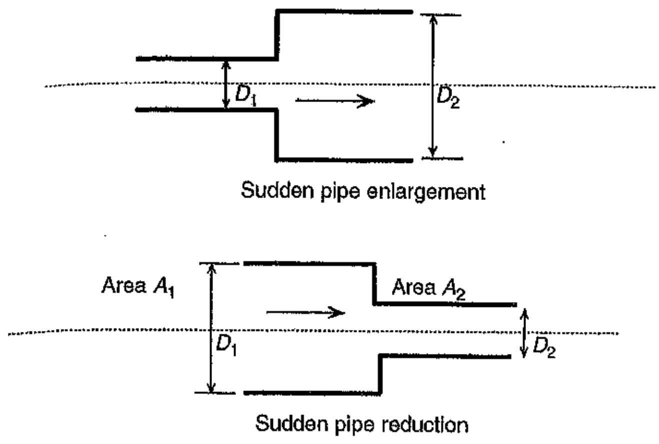
Pipe enlargements and reductions contribute to head loss that can be included in minor losses. For sudden enlargement of pipes, the following head loss equation may be used:

$$h_f = \frac{(v_1 - v_2)^2}{2g} \tag{1.40}$$

where v_1 and v_2 are the velocities of the liquid in the two pipe sizes D_1 and D_2 respectively. Writing Eq. (1.40) in terms of pipe cross-sectional areas A_1 and A_2,

$$h_f = \left(1 - \frac{A_1}{A_2}\right)^2 \left(\frac{{v_1}^2}{2g}\right) \tag{1.41}$$

for sudden enlargement. This is illustrated in Fig. 1.4.



A_1/A_2	0.00	0.10	0.20	0.30	0.40	0.50	0.60	0.70	0.80	0.90	1.00
C_c	0.585	0.624	0.632	0.643	0.659	0.681	0.712	0.755	0.813	0.892	1.000

Figure 1.4 Sudden pipe enlargement and reduction.

For sudden contraction or reduction in pipe size as shown in Fig. 1.4, the head loss is calculated from

$$h_f = \left(\frac{1}{C_c} - 1\right)\frac{{v_2}^2}{2g} \tag{1.42}$$

where the coefficient C_c depends on the ratio of the two pipe cross-sectional areas A_1 and A_2 as shown in Fig. 1.4.

Gradual enlargement and reduction of pipe size, as shown in Fig. 1.5, cause less head loss than sudden enlargement and sudden reduction. For gradual expansions, the following equation may be used:

$$h_f = \frac{C_c(v_1 - v_2)^2}{2g} \tag{1.43}$$

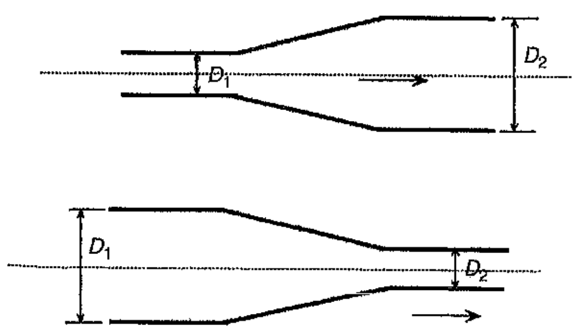


Figure 1.5 Gradual pipe enlargement and reduction.

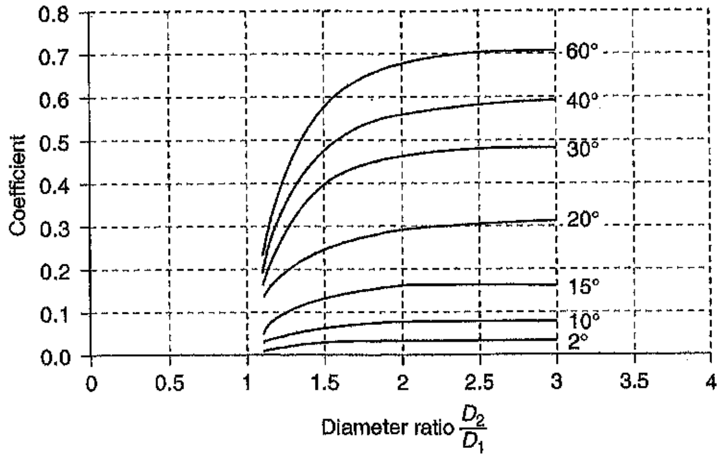


Figure 1.6 Gradual pipe expansion head loss coefficient.

where C_c depends on the diameter ratio D_2/D_1 and the cone angle β in the gradual expansion. A graph showing the variation of C_c with β and the diameter ratio is shown in Fig. 1.6.

1.7.3 Pipe entrance and exit losses

The K factors for computing the head loss associated with pipe entrance and exit are as follows:

$$K = \begin{cases} 0.5 & \text{for pipe entrance, sharp edged} \\ 1.0 & \text{for pipe exit, sharp edged} \\ 0.78 & \text{for pipe entrance, inward projecting} \end{cases}$$

1.8 Complex Piping Systems

So far we have discussed straight length of pipe with valves and fittings. Complex piping systems include pipes of different diameters in series and parallel configuration.

1.8.1 Series piping

Series piping in its simplest form consists of two or more different pipe sizes connected end to end as illustrated in Fig. 1.7. Pressure drop calculations in series piping may be handled in one of two ways. The first approach would be to calculate the pressure drop in each pipe size and add them together to obtain the total pressure drop. Another approach is to consider one of the pipe diameters as the base size and convert other pipe sizes into equivalent lengths of the base pipe size. The resultant equivalent lengths are added together to form one long piece

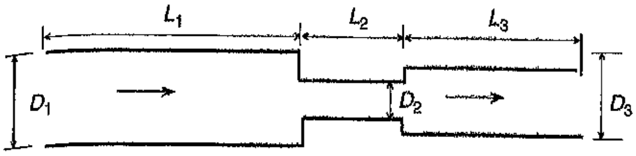


Figure 1.7 Series piping.

of pipe of constant diameter equal to the base diameter selected. The pressure drop can now be calculated for this single-diameter pipeline. Of course, all valves and fittings will also be converted to their respective equivalent pipe lengths using the L/D ratios from Table 1.5.

Consider three sections of pipe joined together in series. Using subscripts 1, 2, and 3 and denoting the pipe length as L, inside diameter as D, flow rate as Q, and velocity as V, we can calculate the equivalent length of each pipe section in terms of a base diameter. This base diameter will be selected as the diameter of the first pipe section D_1. Since equivalent length is based on the same pressure drop in the equivalent pipe as the original pipe diameter, we will calculate the equivalent length of section 2 by finding that length of diameter D_1 that will match the pressure drop in a length L_2 of pipe diameter D_2. Using the Darcy equation and converting velocities in terms of flow rate from Eq. (1.11), we can write

$$\text{Head loss} = \frac{f(L/D)(0.4085Q/D_2)^2}{2g} \tag{1.44}$$

For simplicity, assuming the same friction factor,

$$\frac{L_e}{D_1{}^5} = \frac{L_2}{D_2{}^5} \tag{1.45}$$

Therefore, the equivalent length of section 2 based on diameter D_1 is

$$L_e = L_2\left(\frac{D_1}{D_2}\right)^5 \tag{1.46}$$

Similarly, the equivalent length of section 3 based on diameter D_1 is

$$L_e = L_3\left(\frac{D_1}{D_3}\right)^5 \tag{1.47}$$

The total equivalent length of all three pipe sections based on diameter D_1 is therefore

$$L_t = L_1 + L_2\left(\frac{D_1}{D_2}\right)^5 + L_3\left(\frac{D_1}{D_3}\right)^5 \tag{1.48}$$

The total pressure drop in the three sections of pipe can now be calculated based on a single pipe of diameter D_1 and length L_t.

Example 1.14 Three pipes with 14-, 16-, and 18-in diameters, respectively, are connected in series with pipe reducers, fittings, and valves as follows:

14-in pipeline, 0.250-in wall thickness, 2000 ft long

16-in pipeline, 0.375-in wall thickness, 3000 ft long

18-in pipeline, 0.375-in wall thickness, 5000 ft long

One 16 × 14 in reducer

One 18 × 16 in reducer

Two 14-in 90° elbows

Four 16-in 90° elbows

Six 18-in 90° elbows

One 14-in gate valve

One 16-in ball valve

One 18-in gate valve

(a) Use the Hazen-Williams equation with a C factor of 140 to calculate the total pressure drop in the series water piping system at a flow rate of 3500 gal/min. Flow starts in the 14-in piping and ends in the 18-in piping.
(b) If the flow rate is increased to 6000 gal/min, estimate the new total pressure drop in the piping system, keeping everything else the same.

Solution

(a) Since we are going to use the Hazen-Williams equation, the pipes in series analysis will be based on the pressure loss being inversely proportional to $D^{4.87}$, where D is the inside diameter of pipe, per Eq. (1.30).

We will first calculate the total equivalent lengths of all 14-in pipe, fittings, and valves in terms of the 14-in-diameter pipe.

$$\text{Straight pipe: 14 in., 2000 ft} = \text{2000 ft of 14-in pipe}$$

$$\text{Two 14-in 90° elbows} = \frac{2 \times 30 \times 14}{12} = \text{70 ft of 14-in pipe}$$

$$\text{One 14-in gate valve} = \frac{1 \times 8 \times 14}{12} = \text{9.33 ft of 14-in pipe}$$

Therefore, the total equivalent length of 14-in pipe, fittings, and valves = 2079.33 ft of 14-in pipe.

Similarly we get the total equivalent length of 16-in pipe, fittings, and valve as follows:

$$\text{Straight pipe: 16-in, 3000 ft} = \text{3000 ft of 16-in pipe}$$

$$\text{Four 16-in 90° elbows} = \frac{4 \times 30 \times 16}{12} = \text{160 ft of 16-in pipe}$$

$$\text{One 16-in ball valve} = \frac{1 \times 3 \times 16}{12} = \text{4 ft of 16-in pipe}$$

Therefore, the total equivalent length of 16-in pipe, fittings, and valve = 3164 ft of 16-in pipe.

Finally, we calculate the total equivalent length of 18-in pipe, fittings, and valve as follows:

$$\text{Straight pipe: 18-in, 5000 ft} = 5000 \text{ ft of 18-in pipe}$$

$$\text{Six 18-in } 90^\circ \text{ elbows} = \frac{6 \times 30 \times 18}{12} = 270 \text{ ft of 18-in pipe}$$

$$\text{One 18-in gate valve} = \frac{1 \times 8 \times 18}{12} = 12 \text{ ft of 18-in pipe}$$

Therefore, the total equivalent length of 18-in pipe, fittings, and valve = 5282 ft of 18-in pipe.

Next we convert all the preceding pipe lengths to the equivalent 14-in pipe based on the fact that the pressure loss is inversely proportional to $D^{4.87}$, where D is the inside diameter of pipe.

$$2079.33 \text{ ft of 14-in pipe} = 2079.33 \text{ ft of 14-in pipe}$$

$$3164 \text{ ft of 16-in pipe} = 3164 \times \left(\frac{13.5}{15.25}\right)^{4.87} = 1748 \text{ ft of 14-in pipe}$$

$$5282 \text{ ft of 18-in pipe} = 5282 \times \left(\frac{13.5}{17.25}\right)^{4.87} = 1601 \text{ ft of 14-in pipe}$$

Therefore adding all the preceding lengths we get

$$\text{Total equivalent length in terms of 14-in pipe} = 5429 \text{ ft of 14-in pipe}$$

We still have to account for the 16 × 14 in and 18 × 16 in reducers. The reducers can be considered as sudden enlargements for the approximate calculation of the head loss, using the K factor and velocity head method. For sudden enlargements, the resistance coefficient K is found from

$$K = \left[1 - \left(\frac{d_1}{d_2}\right)^2\right]^2 \tag{1.49}$$

where d_1 is the smaller diameter and d_2 is the larger diameter.

For the 16 × 14 in reducer,

$$K = \left[1 - \left(\frac{13.5}{15.25}\right)^2\right]^2 = 0.0468$$

and for the 18 × 16 in reducer,

$$K = \left[1 - \left(\frac{15.25}{17.25}\right)^2\right]^2 = 0.0477$$

The head loss through the reducers will then be calculated based on $K(V^2/2g)$.

Flow velocities in the three different pipe sizes at 3500 gal/min will be calculated using Eq. (1.11):

$$\text{Velocity in 14-in pipe: } V_{14} = \frac{0.4085 \times 3500}{(13.5)^2} = 7.85 \text{ ft/s}$$

$$\text{Velocity in 16-in pipe: } V_{16} = \frac{0.4085 \times 3500}{(15.25)^2} = 6.15 \text{ ft/s}$$

$$\text{Velocity in 18-in pipe: } V_{18} = \frac{0.4085 \times 3500}{(17.25)^2} = 4.81 \text{ ft/s}$$

The head loss through the 16 × 14 in reducer is

$$h_1 = 0.0468\frac{7.85^2}{64.4} = 0.0448 \text{ ft}$$

and the head loss through the 18 × 16 in reducer is

$$h_1 = 0.0477\frac{6.15^2}{64.4} = 0.028 \text{ ft}$$

These head losses are insignificant and hence can be neglected in comparison with the head loss in straight length of pipe. Therefore, the total head loss in the entire piping system will be based on a total equivalent length of 5429 ft of 14-in pipe.

Using the Hazen-Williams equation (1.32) the pressure drop at 3500 gal/min is

$$h = 10{,}460\left(\frac{3500}{140}\right)^{1.852}\frac{1.0}{(13.5)^{4.87}} = 12.70 \text{ ft per 1000 ft of pipe}$$

Therefore, for the 5429 ft of equivalent 14-in pipe, the total pressure drop is

$$h = \frac{12.7 \times 5429}{1000} = 68.95 \text{ ft} = \frac{68.95}{2.31} = 29.85 \text{ psi}$$

(b) When the flow rate is increased to 6000 gal/min, we can use proportions to estimate the new total pressure drop in the piping as follows:

$$h = \left(\frac{6000}{3500}\right)^{1.852} \times 12.7 = 34.46 \text{ ft per 1000 ft of pipe}$$

Therefore, the total pressure drop in 5429 ft of 14-in. pipe is

$$h = 34.46 \times \frac{5429}{1000} = 187.09 \text{ ft} = \frac{187.09}{2.31} = 81.0 \text{ psi}$$

Example 1.15 Two pipes with 400- and 600-mm diameters, respectively, are connected in series with pipe reducers, fittings, and valves as follows:

400-mm pipeline, 6-mm wall thickness, 600 m long

600-mm pipeline, 10-mm wall thickness, 1500 m long

One 600 × 400 mm reducer

Two 400-mm 90° elbows

Four 600-mm 90° elbows

One 400-mm gate valve

One 600-mm gate valve

Use the Hazen-Williams equation with a C factor of 120 to calculate the total pressure drop in the series water piping system at a flow rate of 250 L/s. What will the pressure drop be if the flow rate were increased to 350 L/s?

Solution The total equivalent length on 400-mm-diameter pipe is the sum of the following:

$$\text{Straight pipe length} = 600 \text{ m}$$

$$\text{Two } 90^\circ \text{ elbows} = \frac{2 \times 30 \times 400}{1000} = 24 \text{ m}$$

$$\text{One gate valve} = \frac{1 \times 8 \times 400}{1000} = 3.2 \text{ m}$$

Thus,

$$\text{Total equivalent length on 400-mm-diameter pipe} = 627.2 \text{ m}$$

The total equivalent length on 600-mm-diameter pipe is the sum of the following:

$$\text{Straight pipe length} = 1500 \text{ m}$$

$$\text{Four } 90^\circ \text{ elbows} = \frac{4 \times 30 \times 600}{1000} = 72 \text{ m}$$

$$\text{One gate valve} = \frac{1 \times 8 \times 600}{1000} = 4.8 \text{ m}$$

Thus,

$$\text{Total equivalent length on 600-mm-diameter pipe} = 1576.8 \text{ m}$$

Reducers will be neglected since they have insignificant head loss. Convert all pipe to 400-mm equivalent diameter.

$$1576.8 \text{ m of 600-mm pipe} = 1576.8 \left(\frac{388}{580}\right)^{4.87}$$

$$= 222.6 \text{ m of 400-mm pipe}$$

$$\text{Total equivalent length on 400-mm-diameter pipe} = 627.2 + 222.6 = 849.8 \text{ m}$$

$$Q = 250 \times 10^{-3} \times 3600 = 900 \text{ m}^3/\text{h}$$

The pressure drop from Eq. (1.35) is

$$P_m = 1.1101 \times 10^{13} \left(\frac{900}{120}\right)^{1.852} \frac{1}{(388)^{4.87}}$$

$$= 114.38 \text{ kPa/km}$$

$$\text{Total pressure drop} = \frac{114.38 \times 849.8}{1000} = 97.2 \text{ kPa}$$

When the flow rate is increased to 350 L/s, we can calculate the pressure drop using proportions as follows:

$$\text{Revised head loss at 350 L/s} = \left(\frac{350}{250}\right)^{1.852} \times 114.38 = 213.3 \text{ kPa/km}$$

Therefore,

$$\text{Total pressure drop} = 213.3 \times 0.8498 = 181.3 \text{ kPa}$$

1.8.2 Parallel piping

Water pipes in parallel are set up such that the multiple pipes are connected so that water flow splits into the multiple pipes at the beginning and the separate flow streams subsequently rejoin downstream into another single pipe as depicted in Fig. 1.8.

Figure 1.8 shows a parallel piping system in the horizontal plane with no change in pipe elevations. Water flows through a single pipe *AB*, and at the junction *B* the flow splits into two pipe branches *BCE* and *BDE*. At the downstream end at junction *E*, the flows rejoin to the initial flow rate and subsequently flow through the single pipe *EF*.

To calculate the flow rates and pressure drop due to friction in the parallel piping system, shown in Fig. 1.8, two main principles of parallel piping must be followed. These are flow conservation at any junction point and common pressure drop across each parallel branch pipe.

Based on flow conservation, at each junction point of the pipeline, the incoming flow must exactly equal the total outflow. Therefore, at junction *B*, the flow *Q* entering the junction must exactly equal the sum of the flow rates in branches *BCE* and *BDE*.

Thus,

$$Q = Q_{BCE} + Q_{BDE} \tag{1.50}$$

where Q_{BCE} = flow through branch *BCE*
Q_{BDE} = flow through branch *BDE*
Q = incoming flow at junction *B*

The other requirement in parallel pipes concerns the pressure drop in each branch piping. Based on this the pressure drop due to friction

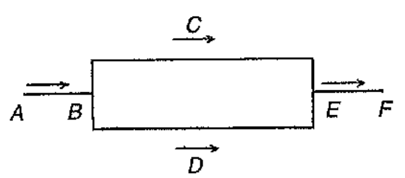


Figure 1.8 Parallel piping.

in branch BCE must exactly equal that in branch BDE. This is because both branches have a common starting point (B) and a common ending point (E). Since the pressure at each of these two points is a unique value, we can conclude that the pressure drop in branch pipe BCE and that in branch pipe BDE are both equal to $P_B - P_E$, where P_B and P_E represent the pressure at the junction points B and E, respectively.

Another approach to calculating the pressure drop in parallel piping is the use of an equivalent diameter for the parallel pipes. For example in Fig. 1.8, if pipe AB has a diameter of 14 in and branches BCE and BDE have diameters of 10 and 12 in, respectively, we can find some equivalent diameter pipe of the same length as one of the branches that will have the same pressure drop between points B and C as the two branches. An approximate equivalent diameter can be calculated using the Darcy equation.

The pressure loss in branch BCE (10-in diameter) can be calculated as

$$h_1 = \frac{f(L_1/D_1)V_1^2}{2g} \tag{1.51}$$

where the subscript 1 is used for branch BCE and subscript 2 for branch BDE.

Similarly, for branch BDE

$$h_2 = \frac{f(L_2/D_2)V_2^2}{2g} \tag{1.52}$$

For simplicity we have assumed the same friction factors for both branches. Since h_1 and h_2 are equal for parallel pipes, and representing the velocities V_1 and V_2 in terms of the respective flow rates Q_1 and Q_2, using Eq. (1.23) we have the following equations:

$$\frac{f(L_1/D_1)V_1^2}{2g} = \frac{f(L_2/D_2)V_2^2}{2g} \tag{1.53}$$

$$V_1 = 0.4085\frac{Q_1}{D_1^2} \tag{1.54}$$

$$V_2 = 0.4085\frac{Q_2}{D_2^2} \tag{1.55}$$

In these equations we are assuming flow rates in gal/min and diameters in inches.

Simplifying Eqs. (1.53) to (1.55), we get

$$\frac{L_1}{D_1}\left(\frac{Q_1}{D_1^2}\right)^2 = \frac{L_2}{D_2}\left(\frac{Q_2}{D_2^2}\right)^2$$

or

$$\frac{Q_1}{Q_2} = \left(\frac{L_2}{L_1}\right)^{0.5} \left(\frac{D_1}{D_2}\right)^{2.5} \tag{1.56}$$

Also by conservation of flow

$$Q_1 + Q_2 = Q \tag{1.57}$$

Using Eqs. (1.56) and (1.57), we can calculate the flow through each branch in terms of the inlet flow Q. The equivalent pipe will be designated as D_e in diameter and L_e in length. Since the equivalent pipe will have the same pressure drop as each of the two branches, we can write

$$\frac{L_e}{D_e} \left(\frac{Q_e}{D_e^2}\right)^2 = \frac{L_1}{D_1} \left(\frac{Q_1}{D_1^2}\right)^2 \tag{1.58}$$

where Q_e is the same as the inlet flow Q since both branches have been replaced with a single pipe. In Eq. (1.58), there are two unknowns L_e and D_e. Another equation is needed to solve for both variables. For simplicity, we can set L_e to be equal to one of the lengths L_1 or L_2. With this assumption, we can solve for the equivalent diameter D_e as follows:

$$D_e = D_1 \left(\frac{Q}{Q_1}\right)^{0.4} \tag{1.59}$$

Example 1.16 A 10-in water pipeline consists of a 2000-ft section of NPS 12 pipe (0.250-in wall thickness) starting at point A and terminating at point B. At point B, two pieces of pipe (4000 ft long each and NPS 10 pipe with 0.250-in wall thickness) are connected in parallel and rejoin at a point D. From D, 3000 ft of NPS 14 pipe (0.250-in wall thickness) extends to point E. Using the equivalent diameter method calculate the pressures and flow rate throughout the system when transporting water at 2500 gal/min. Compare the results by calculating the pressures and flow rates in each branch. Use the Colebrook-White equation for the friction factor.

Solution Since the pipe loops between B and D are each NPS 10 and 4000 ft long, the flow will be equally split between the two branches. Each branch pipe will carry 1250 gal/min.

The equivalent diameter for section BD is found from Eq. (1.59):

$$D_e = D_1 \left(\frac{Q}{Q_1}\right)^{0.4} = 10.25 \times (2)^{0.4} = 13.525 \text{ in}$$

Therefore we can replace the two 4000-ft NPS 10 pipes between B and D with a single pipe that is 4000 ft long and has a 13.525-in inside diameter.

The Reynolds number for this pipe at 2500 gal/min is found from Eq. (1.15):

$$R = \frac{3162.5 \times 2500}{13.525 \times 1.0} = 584{,}566$$

Considering that the pipe roughness is 0.002 in for all pipes:

$$\text{Relative roughness } \frac{e}{D} = \frac{0.002}{13.525} = 0.0001$$

From the Moody diagram, the friction factor $f = 0.0147$. The pressure drop in section BD is [using Eq. (1.24)]

$$P_m = 71.16 \frac{fQ^2}{D^5}$$

$$= 71.16 \frac{0.0147 \times (2500)^2 \times 1}{(13.525)^5} = 14.45 \text{ psi/mi}$$

Therefore,

$$\text{Total pressure drop in } BD = \frac{14.45 \times 4000}{5280} = 10.95 \text{ psi}$$

For section AB we have,

$$R = \frac{3162.5 \times 2500}{12.25 \times 1.0} = 645{,}408$$

$$\text{Relative roughness } \frac{e}{D} = \frac{0.002}{12.25} = 0.0002$$

From the Moody diagram, the friction factor $f = 0.0147$. The pressure drop in section AB is [using Eq. (1.24)]

$$P_m = 71.16 \frac{0.0147 \times (2500)^2 \times 1}{(12.25)^5} = 22.66 \text{ psi/mi}$$

Therefore,

$$\text{Total pressure drop in } AB = \frac{22.66 \times 2000}{5280} = 8.58 \text{ psi}$$

Finally, for section DE we have,

$$R = \frac{3162.5 \times 2500}{13.5 \times 1.0} = 585{,}648$$

$$\text{Relative roughness } \frac{e}{D} = \frac{0.002}{13.5} = 0.0001$$

From the Moody diagram, the friction factor $f = 0.0147$. The pressure drop in section DE is

$$P_m = 71.16\ \frac{0.0147 \times (2500)^2 \times 1}{(13.5)^5} = 14.58 \text{ psi/mi}$$

Therefore,

$$\text{Total pressure drop in } DE = \frac{14.58 \times 3000}{5280} = 8.28 \text{ psi}$$

Finally,

$$\text{Total pressure drop in entire piping system} = 8.58 + 10.95 + 8.28$$
$$= 27.81 \text{ psi}$$

Next for comparison we will analyze the branch pressure drops considering each branch separately flowing at 1250 gal/min.

$$R = \frac{3162.5 \times 1250}{10.25 \times 1.0} = 385{,}671$$

$$\text{Relative roughness } \frac{e}{D} = \frac{0.002}{10.25} = 0.0002$$

From the Moody diagram, the friction factor $f = 0.0158$. The pressure drop in section BD is [using Eq. (1.24)]

$$P_m = 71.16 \frac{0.0158 \times (1250)^2 \times 1}{(10.25)^5} = 15.53 \text{ psi/mi}$$

This compares with the pressure drop of 14.45 psi/mi we calculated using an equivalent diameter of 13.525. It can be seen that the difference between the two pressure drops is approximately 7.5 percent.

Example 1.17 A waterline 5000 m long is composed of three sections A, B, and C. Section A has a 200-m inside diameter and is 1500 m long. Section C has a 400-mm inside diameter and is 2000 m long. The middle section B consists of two parallel pipes each 3000 m long. One of the parallel pipes has a 150-mm inside diameter and the other has a 200-mm inside diameter. Assume no elevation change throughout. Calculate the pressures and flow rates in this piping system at a flow rate of 500 m^3/h, using the Hazen-Williams formula with a C factor of 1.20.

Solution We will replace the two 3000-m pipe branches in section B with a single equivalent diameter pipe to be determined. Since the pressure drop according to the Hazen-Williams equation is inversely proportional to the 4.87 power of the pipe diameter, we calculate the equivalent diameter for section B as follows:

$$\frac{Q_e{}^{1.852}}{D_e{}^{4.87}} = \frac{Q_1{}^{1.852}}{D_1{}^{4.87}} = \frac{Q_2{}^{1.852}}{D_2{}^{4.87}}$$

Therefore,

$$\frac{D_e}{D_1} = \left(\frac{Q_e}{Q_1}\right)^{0.3803}$$

Also $Q_e = Q_1 + Q_2$ and

$$\frac{Q_1}{Q_2} = \left(\frac{D_1}{D_2}\right)^{2.63} = \left(\frac{150}{200}\right)^{2.63} = 0.4693$$

Solving for Q_1 and Q_2, with $Q_e = 500$, we get

$$Q_1 = 159.7\text{m}^3/\text{hr} \qquad \text{and} \qquad Q_2 = 340.3\text{m}^3/\text{h}$$

Therefore, the equivalent diameter is

$$D_e = D_1\left(\frac{Q_e}{Q_1}\right)^{0.3803} = 150 \times \left(\frac{500}{159.7}\right)^{0.3803} = 231.52 \text{ mm}$$

The pressure drop in section A, using Hazen-Williams equation (1.35), is

$$P_m = 1.1101 \times 10^{13} \times \left(\frac{500}{120}\right)^{1.852} \times \frac{1}{(200)^{4.87}} = 970.95 \text{ kPa/km}$$

$$\Delta P_a = 970.95 \times 1.5 = 1456.43 \text{ kPa}$$

The pressure drop in section B, using Hazen-Williams equation, is

$$P_m = 1.1101 \times 10^{13} \times \left(\frac{500}{120}\right)^{1.852} \times \frac{1}{(231.52)^{4.87}} = 476.07 \text{ kPa/km}$$

$$\Delta P_b = 476.07 \times 3.0 = 1428.2 \text{ kPa}$$

The pressure drop in section C, using Hazen-Williams equation, is

$$P_m = 1.1101 \times 10^{13} \times \left(\frac{500}{120}\right)^{1.852} \times \frac{1}{(400)^{4.87}} = 33.20 \text{ kPa/km}$$

$$\Delta P_c = 33.2 \times 2.0 = 66.41 \text{ kPa}$$

Therefore,

$$\text{Total pressure drop of sections A, B, and C} = 1456.43 + 1428.20 + 66.41 = 2951.04 \text{ kPa}$$

1.9 Total Pressure Required

So far we have examined the frictional pressure drop in water systems piping consisting of pipe, fittings, valves, etc. We also calculated the total pressure required to pump water through a pipeline up to a delivery station at an elevated point. The total pressure required at the beginning of a pipeline, for a specified flow rate, consists of three distinct components:

1. Frictional pressure drop
2. Elevation head
3. Delivery pressure

$$P_t = P_f + P_{\text{elev}} + P_{\text{del}} \qquad \text{from Eq. (1.29)}$$

The first item is simply the total frictional head loss in all straight pipe, fittings, valves, etc. The second item accounts for the pipeline elevation difference between the origin of the pipeline and the delivery terminus. If the origin of the pipeline is at a lower elevation than that of the pipeline terminus or delivery point, a certain amount of positive pressure is required to compensate for the elevation difference. On the other hand, if the delivery point were at a lower elevation than the beginning of the pipeline, gravity will assist the flow and the pressure required at the beginning of the pipeline will be reduced by this elevation difference. The third component, delivery pressure at the terminus, simply ensures that a certain minimum pressure is maintained at the delivery point, such as a storage tank.

For example, if a water pipeline requires 800 psi to take care of frictional losses and the minimum delivery pressure required is 25 psi, the total pressure required at the beginning of the pipeline is calculated as follows. If there were no elevation difference between the beginning of the pipeline and the delivery point, the elevation head (component 2) is zero. Therefore, the total pressure P_t required is

$$P_t = 800 + 0 + 25 = 825 \text{ psi}$$

Next consider elevation changes. If the elevation at the beginning is 100 ft and the elevation at the delivery point is 500 ft, then

$$P_t = 800 + \frac{(500 - 100) \times 1.0}{2.31} + 25 = 998.16 \text{ psi}$$

The middle term in this equation represents the static elevation head difference converted to psi. Finally, if the elevation at the beginning is 500 ft and the elevation at the delivery point is 100 ft, then

$$P_t = 800 + \frac{(100 - 500) \times 1.0}{2.31} + 25 = 651.84 \text{ psi}$$

It can be seen from the preceding that the 400-ft advantage in elevation in the final case reduces the total pressure required by approximately 173 psi compared to the situation where there was no elevation difference between the beginning of the pipeline and delivery point.

1.9.1 Effect of elevation

The preceding discussion illustrated a water pipeline that had a flat elevation profile compared to an uphill pipeline and a downhill pipeline. There are situations, where the ground elevation may have drastic peaks and valleys, that require careful consideration of the pipeline topography. In some instances, the total pressure required to transport a given volume of water through a long pipeline may depend more on the ground elevation profile than the actual frictional pressure drop. In the preceding we calculated the total pressure required for a flat pipeline as 825 psi and an uphill pipeline to be 998 psi. In the uphill case the static elevation difference contributed to 17 percent of the total pressure required. Thus the frictional component was much higher than the elevation component. We will examine a case where the elevation differences in a long pipeline dictate the total pressure required more than the frictional head loss.

Example 1.18 A 20-in (0.375-in wall thickness) water pipeline 500 mi long has a ground elevation profile as shown in Fig. 1.9. The elevation at Corona is 600 ft and at Red Mesa is 2350 ft. Calculate the total pressure required at the Corona pump station to transport 11.5 Mgal/day of water to Red Mesa storage tanks, assuming a minimum delivery pressure of 50 psi at Red Mesa. Use the Hazen-Williams equation with a C factor of 140. If the pipeline operating pressure cannot exceed 1400 psi, how many pumping stations, besides Corona, will be required to transport the given flow rate?

Solution The flow rate Q in gal/min is

$$Q = \frac{11.5 \times 10^6}{24 \times 60} = 7986.11 \text{ gal/min}$$

If P_m is the head loss in psi/mi of pipe, using the Hazen-Williams equation (1.33),

$$P_m = 23{,}909 \left(\frac{7986.11}{140}\right)^{1.852} \frac{1}{19.25^{4.87}} = 23.76 \text{ psi/mi}$$

Therefore,

$$\text{Frictional pressure drop} = 23.76 \text{ psi/mi}$$

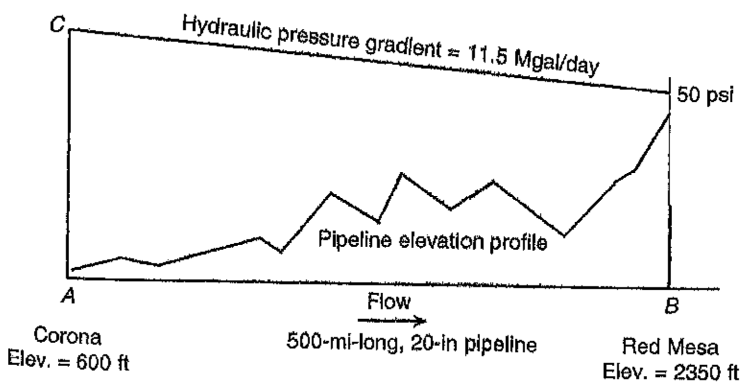


Figure 1.9 Corona to Red Mesa pipeline.

The total pressure required at Corona is calculated by adding the pressure drop due to friction to the delivery pressure required at Red Mesa and the static elevation head between Corona and Red Mesa.

$$P_t = P_f + P_{\text{elev}} + P_{\text{del}} \qquad \text{from Eq. (1.29)}$$

$$= (23.76 \times 500) + \frac{2350 - 600}{2.31} + 50$$

$$= 11{,}880 + 757.58 + 50 = 12{,}688 \text{ psi} \qquad \text{rounded off to the nearest psi}$$

Since a total pressure of 12,688 psi at Corona far exceeds the maximum operating pressure of 1400 psi, it is clear that we need additional intermediate booster pump stations besides Corona. The approximate number of pump stations required without exceeding the pipeline pressure of 1400 psi is

$$\text{Number of pump stations} = \frac{12{,}688}{1400} = 9.06 \text{ or } 10 \text{ pump stations}$$

With 10 pump stations the average pressure per pump station will be

$$\text{Average pump station pressure} = \frac{12{,}688}{10} = 1269 \text{ psi}$$

1.9.2 Tight line operation

When there are drastic elevation differences in a long pipeline, sometimes the last section of the pipeline toward the delivery terminus may operate in an open-channel flow. This means that the pipeline section will not be full of water and there will be a vapor space above the water. Such situations are acceptable in water pipelines compared to high vapor pressure liquids such as liquefied petroleum gas (LPG). To prevent such open-channel flow or slack line conditions, we pack the line by providing adequate back pressure at the delivery terminus as illustrated in Fig. 1.10.

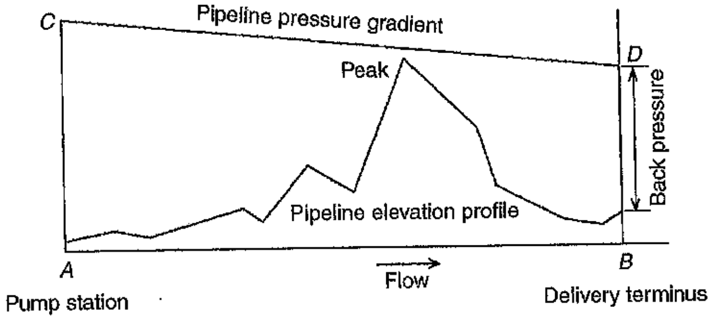


Figure 1.10 Tight line operation.

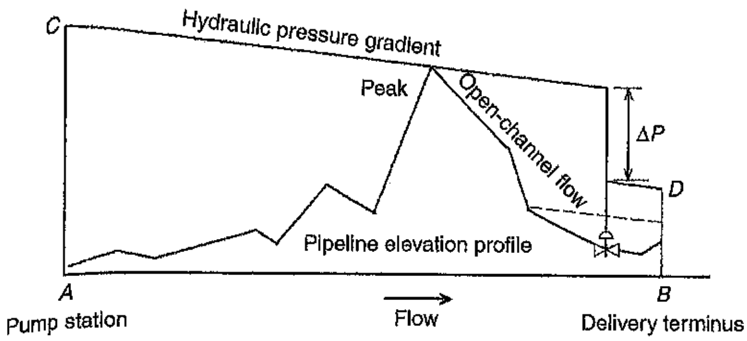


Figure 1.11 Slack line flow.

1.9.3 Slack line flow

Slack line or open-channel flow occurs in the last segment of a long-distance water pipeline where a large elevation difference exists between the delivery terminus and intermediate point in the pipeline as indicated in Fig. 1.11.

If the pipeline were packed to avoid slack line flow, the hydraulic gradient is as shown by the solid line in Fig. 1.11. However, the piping system at the delivery terminal may not be able to handle the higher pressure due to line pack. Therefore, we may have to reduce the pressure at some point within the delivery terminal using a pressure control valve. This is illustrated in Fig. 1.11.

1.10 Hydraulic Gradient

The graphical representation of the pressures along the pipeline, as shown in Fig. 1.12, is called the hydraulic pressure gradient. Since elevation is measured in feet, the pipeline pressures are converted to feet of head and plotted against the distance along the pipeline superimposed

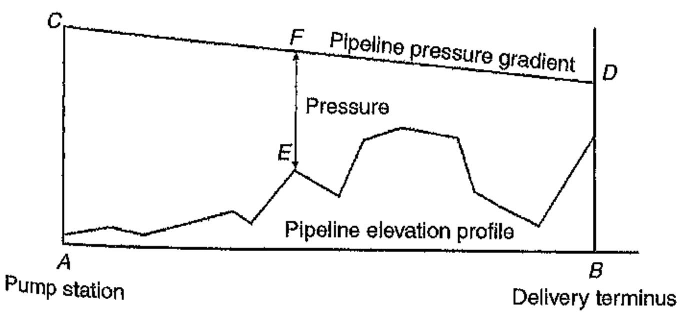


Figure 1.12 Hydraulic pressure gradient.

on the elevation profile. If we assume a beginning elevation of 100 ft, a delivery terminus elevation of 500 ft, a total pressure of 1000 psi required at the beginning, and a delivery pressure of 25 psi at the terminus, we can plot the hydraulic pressure gradient graphically by the following method.

At the beginning of the pipeline the point C representing the total pressure will be plotted at a height of

$$100 \text{ ft} + (1000 \times 2.31) = 2410 \text{ ft}$$

Similarly, at the delivery terminus the point D representing the total head at delivery will be plotted at a height of

$$500 + (25 \times 2.31) = 558 \text{ ft} \qquad \text{rounded off to the nearest foot}$$

The line connecting the points C and D represents the variation of the total head in the pipeline and is termed the *hydraulic gradient*. At any intermediate point such as E along the pipeline the pipeline pressure will be the difference between the total head represented by point F on the hydraulic gradient and the actual elevation of the pipeline at E.

If the total head at F is 1850 ft and the pipeline elevation at E is 250 ft, the actual pipeline pressure at E is

$$(1850 - 250)\text{ft} = \frac{1600}{2.31} = 693 \text{ psi}$$

It can be seen that the hydraulic gradient clears all peaks along the pipeline. If the elevation at E were 2000 ft, we would have a negative pressure in the pipeline at E equivalent to

$$(1850 - 2000)\text{ft} = -150 \text{ ft} = -\frac{150}{2.31} = -65 \text{ psi}$$

Since a negative pressure is not acceptable, the total pressure at the beginning of the pipeline will have to be higher by the preceding amount.

$$\text{Revised total head at } A = 2410 + 150 = 2560 \text{ ft}$$

This will result in zero gauge pressure in the pipeline at peak E. The actual pressure in the pipeline will therefore be equal to the atmospheric pressure at that location. Since we would like to always maintain some positive pressure above the atmospheric pressure, in this case the total head at A must be slightly higher than 2560 ft. Assuming a 10-psi positive pressure is desired at the highest peak such as E (2000-ft elevation), the revised total pressure at A would be

$$\text{Total pressure at } A = 1000 + 65 + 10 = 1075 \text{ psi}$$

Therefore,

$$\text{Total head at } C = 100 + (1075 \times 2.31) = 2483 \text{ ft}$$

This will ensure a positive pressure of 10 psi at the peak E.

1.11 Gravity Flow

Gravity flow in a water pipeline occurs when water flows from a source at point A at a higher elevation than the delivery point B, without any pumping pressure at A and purely under gravity. This is illustrated in Fig. 1.13.

The volume flow rate under gravity flow for the reservoir pipe system shown in Fig. 1.13 can be calculated as follows. If the head loss in the pipeline is h ft/ft of pipe length, the total head loss in length L is $(h \times L)$. Since the available driving force is the difference in tank levels at A and B, we can write

$$H_1 - (h \times L) = H_2 \tag{1.60}$$

Therefore,

$$hL = H_1 - H_2 \tag{1.61}$$

and

$$h = \frac{H_1 - H_2}{L} \tag{1.62}$$

where h = head loss in pipe, ft/ft
L = length of pipe
H_1 = head in tank A
H_2 = head in tank B

In the preceding analysis, we have neglected the entrance and exit losses at A and B. Using the Hazen-Williams equation we can then calculate flow rate based on a C value.

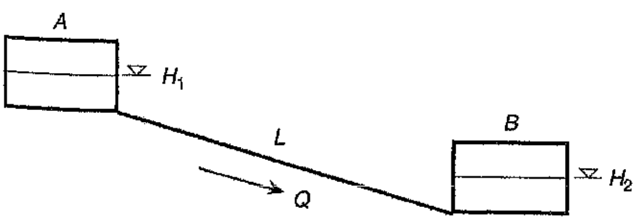


Figure 1.13 Gravity flow from reservoir.

Example 1.19 The gravity feed system shown in Fig. 1.13 consists of a 16-inch (0.250-in wall thickness) 3000-ft-long pipeline, with a tank elevation at $A = 500$ ft and elevation at $B = 150$ ft. Calculate the flow rate through this gravity flow system. Use a Hazen-Williams C factor of 130.

Solution

$$h = \frac{500 - 150}{3000} = 0.1167 \text{ ft/ft}$$

Substituting in Hazen-Williams equation (1.32), we get

$$0.1167 \times 1000 = 10{,}460 \times \left(\frac{Q}{130}\right)^{1.852} \left(\frac{1}{15.5}\right)^{4.87}$$

Solving for flow rate Q,

$$Q = 15{,}484 \text{ gal/min}$$

Compare the results using the Colebrook-White equation assuming $e = 0.002$.

$$\frac{e}{D} = \frac{0.002}{15.5} = 0.0001$$

We will assume a friction factor $f = 0.02$ initially. Head loss due to friction per Eq. (1.24) is

$$P_m = 71.16 \times \frac{0.02(Q^2)}{(15.5)^5} \text{ psi/mi}$$

or

$$P_m = 1.5908 \times 10^{-6} Q^2 \quad \text{psi/mi}$$

$$= \left(1.5908 \times 10^{-6} \frac{2.31}{5280}\right) Q^2 \quad \text{ft/ft}$$

$$= (6.9596 \times 10^{-10}) Q^2 \quad \text{ft/ft}$$

$$0.1167 = (6.9596 \times 10^{-10}) Q^2$$

Solving for flow rate Q, we get

$$Q = 12{,}949 \text{ gal/min}$$

Solving for the Reynolds number, we get

$$\text{Re} = 3162.5 \times \frac{12{,}949}{15.5} \times 1 = 2{,}642{,}053$$

From the Moody diagram, $f = 0.0128$. Now we recalculate P_m,

$$P_m = 71.16 \times 0.0128 \times \frac{Q^2}{(15.5)^5} \text{ psi/mi}$$

$$= 4.4541 \times 10^{-10} Q^2 \quad \text{ft/ft}$$

Solving for Q again,

$$Q = 16{,}186 \text{ gal/min}$$

By successive iteration we arrive at the final flow rate of 16,379 gal/min using the Colebrook-White equation. Comparing this with 15,484 gal/min obtained using the Hazen-Williams equation, we see that the flow rate is underestimated probably because the assumed Hazen-Williams C factor ($C = 130$) was too low.

Example 1.20 The two-reservoir system described in Fig. 1.13 is modified to include a second source of water from a tank located at C between the two tanks located at A and B and away from the pipeline AB. The tank at C is at an elevation of 300 ft and connects to the piping from A to B via a new 16-inch, 1000-ft-long pipe CD. The common junction D is located along the pipe AB at a distance of 1500 ft from the tank at B. Determine the flow rates Q_1 from A to D, Q_2 from C to D, and Q_3 from D to B. Use the Hazen-Williams equation with $C = 130$.

Solution At the common junction D we can apply the conservation of flow principle as follows:

$$Q_1 + Q_2 = Q_3$$

Also since D is a common junction, the head H_D at point D is common to the three legs AD, CD, and DB. Designating the head loss due to friction in the respective pipe segments AD, CD, and DB as h_{fAD}, h_{fCD}, and h_{fDB}, we can write the following pressure balance equations for the three pipe legs.

$$H_D = H_A - h_{fAD}$$

$$H_D = H_C - h_{fCD}$$

$$H_D = H_B + h_{fDB}$$

Since the pipe sizes are all 16 in and the C factor is 130, using the Hazen-Williams equation (1.32) we can write

$$h_{fAD} = 10{,}460 \times \frac{L_{AD}}{1000}\left(\frac{Q_1}{130}\right)^{1.852}\left(\frac{1}{15.5}\right)^{4.87} = KL_{AD} \times Q_1^{1.852}$$

where K is a constant for all pipes and is equal to

$$K = 10{,}460 \times \frac{1}{1000}\left(\frac{1}{130}\right)^{1.852}\left(\frac{1}{15.5}\right)^{4.87} = 2.0305 \times 10^{-9}$$

and

$$L_{AD} = \text{length of pipe from } A \text{ to } D = 1500 \text{ ft}$$

Similarly, we can write

$$h_{fCD} = KL_{CD} \times Q_2^{1.852}$$

and for leg DB

$$h_{fDB} = KL_{DB} \times Q_3^{1.852}$$

Substituting the values in the preceding H_D equations, we get

$$H_D = 500 - K \times 1500 \times Q_1^{1.852}$$

$$H_D = 300 - K \times 1000 \times Q_2^{1.852}$$

$$H_D = 150 + K \times 1000 \times Q_3^{1.852}$$

Simplifying these equations by eliminating H_D, we get the following two equations:

$$1.5Q_1^{1.852} - Q_2^{1.852} = \frac{0.2}{K} \qquad (A)$$

$$1.5Q_1^{1.852} + Q_3^{1.852} = \frac{0.35}{K} \qquad (B)$$

Also

$$Q_1 + Q_2 = Q_3 \qquad (C)$$

Solving for the three flow rates we get,

$$Q_1 = 16{,}677 \qquad Q_2 = 1000 \qquad \text{and} \qquad Q_3 = 17{,}677$$

1.12 Pumping Horsepower

In the previous sections we calculated the total pressure required at the beginning of the pipeline to transport a given volume of water over a certain distance. We will now calculate the pumping horsepower (HP) required to accomplish this.

Consider Example 1.18 in which we calculated the total pressure required to pump 11.5 Mgal/day of water from Corona to Red Mesa through a 500-mi-long, 20-in pipeline. We calculated the total pressure required to be 12,688 psi. Since the maximum allowable working pressure in the pipeline was limited to 1400 psi, we concluded that nine additional pump stations besides Corona were required. With a total of 10 pump stations, each pump station would be discharging at a pressure of approximately 1269 psi.

At the Corona pump station, water would enter the pump at some minimum pressure, say 50 psi and the pumps would boost the pressure to the required discharge pressure of 1269 psi. Effectively, the pumps would add the energy equivalent of 1269 − 50, or 1219 psi at a flow rate of 11.5 Mgal/day (7986.11 gal/min). The water horsepower (WHP) required is calculated as

$$\text{WHP} = \frac{(1219 \times 2.31) \times 7986.11 \times 1.0}{3960} = 5679 \text{ HP}$$

The general equation used to calculate WHP, also known as hydraulic horsepower (HHP), is as follows:

$$\text{WHP} = \frac{\text{ft of head} \times (\text{gal/min}) \times \text{specific gravity}}{3960} \qquad (1.63)$$

Assuming a pump efficiency of 80 percent, the pump brake horsepower (BHP) required is

$$\text{BHP} = \frac{5679}{0.8} = 7099 \text{ HP}$$

The general equation for calculating the BHP of a pump is

$$\text{BHP} = \frac{\text{ft of head} \times (\text{gal/min}) \times (\text{specific gravity})}{3960 \times \text{effy}} \qquad (1.64)$$

where effy is the pump efficiency expressed as a decimal value.

If the pump is driven by an electric motor with a motor efficiency of 95 percent, the drive motor HP required will be

$$\text{Motor HP} = \frac{7099}{0.95} = 7473 \text{ HP}$$

The nearest standard size motor of 8000 HP would be adequate for this application. Of course this assumes that the entire pumping requirement at the Corona pump station is handled by a single pump-motor unit. In reality, to provide for operational flexibility and maintenance two or more pumps will be configured in series or parallel configurations to provide the necessary pressure at the specified flow rate. Let us assume that two pumps are configured in parallel to provide the necessary head pressure of 1219 psi (2816 ft) at the Corona pump station. Each pump will be designed for one-half the total flow rate (7986.11) or 3993 gal/min and a head pressure of 2816 ft. If the pumps selected had an efficiency of 80 percent, we can calculate the BHP required for each pump as follows:

$$\text{BHP} = \frac{2816 \times 3993 \times 1.0}{3960 \times 0.80} \qquad \text{from Eq. (1.64)}$$

$$= 3550 \text{ HP}$$

Alternatively, if the pumps were configured in series instead of parallel, each pump will be designed for the full flow rate of 7986.11 gal/min but at half the total pressure required, or 1408 ft. The BHP required per pump will still be the same as determined by the preceding equation. Pumps are discussed in more detail in Sec. 1.13.

1.13 Pumps

Pumps are installed on water pipelines to provide the necessary pressure at the beginning of the pipeline to compensate for pipe friction and any elevation head and provide the necessary delivery pressure at the pipeline terminus. Pumps used on water pipelines are either positive displacement (PD) type or centrifugal pumps.

PD pumps generally have higher efficiency, higher maintenance cost, and a fixed volume flow rate at any pressure within allowable limits. Centrifugal pumps on the other hand are more flexible in terms of flow rates but have lower efficiency and lower operating and maintenance cost. The majority of liquid pipelines today are driven by centrifugal pumps.

Since pumps are designed to produce pressure at a given flow rate, an important characteristic of a pump is its performance curve. The performance curve is a graphic representation of how the pressure generated by a pump varies with its flow rate. Other parameters, such as efficiency and horsepower, are also considered as part of a pump performance curve.

1.13.1 Positive displacement pumps

Positive displacement (PD) pumps include piston pumps, gear pumps, and screw pumps. These are used generally in applications where a constant volume of liquid must be pumped against a fixed or variable pressure.

PD pumps can effectively generate any amount of pressure at the fixed flow rate, which depends on its geometry, as long as equipment pressure limits are not exceeded. Since a PD pump can generate any pressure required, we must ensure that proper pressure control devices are installed to prevent rupture of the piping on the discharge side of the PD pump. As indicated earlier, PD pumps have less flexibility with flow rates and higher maintenance cost. Because of these reasons, PD pumps are not popular in long-distance and distribution water pipelines. Centrifugal pumps are preferred due to their flexibility and low operating cost.

1.13.2 Centrifugal pumps

Centrifugal pumps consist of one or more rotating impellers contained in a casing. The centrifugal force of rotation generates the pressure in the liquid as it goes from the suction side to the discharge side of the pump. Centrifugal pumps have a wide range of operating flow rates with fairly good efficiency. The operating and maintenance cost of a centrifugal pump is lower than that of a PD pump. The performance

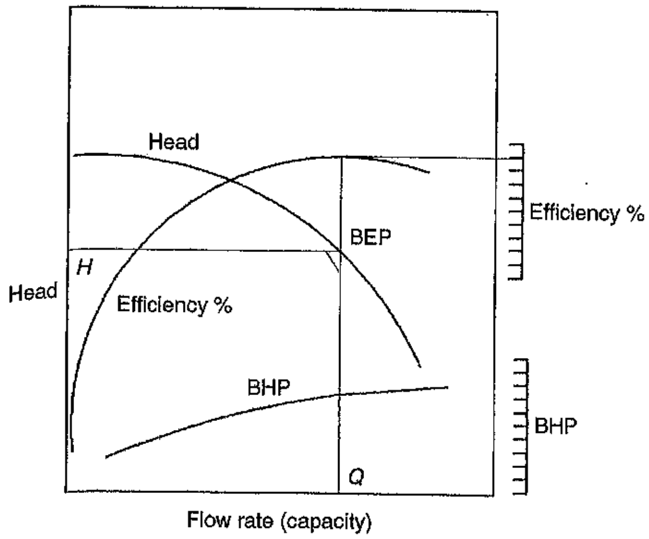


Figure 1.14 Performance curve for centrifugal pump.

curves of a centrifugal pump consist of head versus capacity, efficiency versus capacity, and BHP versus capacity. The term *capacity* is used synonymously with flow rate in connection with centrifugal pumps. Also the term *head* is used in preference to pressure when dealing with centrifugal pumps. Figure 1.14 shows a typical performance curve for a centrifugal pump.

Generally, the head-capacity curve of a centrifugal pump is a drooping curve. The highest head is generated at zero flow rate (shutoff head) and the head decreases with an increase in the flow rate as shown in Fig. 1.14. The efficiency increases with flow rate up to the best efficiency point (BEP) after which the efficiency drops off. The BHP calculated using Eq. (1.64) also generally increases with flow rate but may taper off or start decreasing at some point depending on the head-capacity curve.

The head generated by a centrifugal pump depends on the diameter of the pump impeller and the speed at which the impeller runs. The affinity laws of centrifugal pumps may be used to determine pump performance at different impeller diameters and pump speeds. These laws can bė mathematically stated as follows:
For impeller diameter change:

Flow rate: $$\frac{Q_1}{Q_2} = \frac{D_1}{D_2} \tag{1.65}$$

Head: $$\frac{H_1}{H_2} = \left(\frac{D_1}{D_2}\right)^2 \tag{1.66}$$

BHP: $$\frac{BHP_1}{BHP_2} = \left(\frac{D_1}{D_2}\right)^3 \tag{1.67}$$

For impeller speed change:

Flow rates: $$\frac{Q_1}{Q_2} = \frac{N_1}{N_2} \tag{1.68}$$

Heads: $$\frac{H_1}{H_2} = \left(\frac{N_1}{N_2}\right)^2 \tag{1.69}$$

BHP: $$\frac{BHP_1}{BHP_2} = \left(\frac{N_1}{N_2}\right)^3 \tag{1.70}$$

where subscript 1 refers to initial conditions and subscript 2 to final conditions. It must be noted that the affinity laws for impeller diameter change are accurate only for small changes in diameter. However, the affinity laws for impeller speed change are accurate for a wide range of impeller speeds.

Using the affinity laws if the performance of a centrifugal pump is known at a particular diameter, the corresponding performance at a slightly smaller diameter or slightly larger diameter can be calculated very easily. Similarly, if the pump performance for a 10-in impeller at 3500 revolutions per minute (r/min) impeller speed is known, we can easily calculate the performance of the same pump at 4000 r/min.

Example 1.21 The performance of a centrifugal pump with a 10-in impeller is as shown in the following table.

Capacity Q, gal/min	Head H, ft	Efficiency E, %
0	2355	0
1600	2340	57.5
2400	2280	72.0
3200	2115	79.0
3800	1920	80.0
4000	1845	79.8
4800	1545	76.0

(a) Determine the revised pump performance with a reduced impeller size of 9 in.

(b) If the given performance is based on an impeller speed of 3560 r/min, calculate the revised performance at an impeller speed of 3000 r/min.

Solution

(a) The ratio of impeller diameters is $\frac{9}{10} = 0.9$. Therefore, the Q values will be multiplied by 0.9 and the H values will be multiplied by $0.9 \times 0.9 = 0.81$.

Revised performance data are given in the following table.

Capacity Q, gal/min	Head H, ft	Efficiency E, %
0	1907	0
1440	1895	57.5
2160	1847	72.0
2880	1713	79.0
3420	1555	80.0
3600	1495	79.8
4320	1252	76.0

(b) When speed is changed from 3560 to 3000 r/min, the speed ratio = 3000/3560 = 0.8427. Therefore, Q values will be multiplied by 0.8427 and H values will be multiplied by $(0.8427)^2 = 0.7101$. Therefore, the revised pump performance is as shown in the following table.

Capacity Q, gal/min	Head H, ft	Efficiency E, %
0	1672	0
1348	1662	57.5
2022	1619	72.0
2697	1502	79.0
3202	1363	80.0
3371	1310	79.8
4045	1097	76.0

Example 1.22 For the same pump performance described in Example 1.21, calculate the impeller trim necessary to produce a head of 2000 ft at a flow rate of 3200 gal/min. If this pump had a variable-speed drive and the given performance was based on an impeller speed of 3560 r/min, what speed would be required to achieve the same design point of 2000 ft of head at a flow rate of 3200 gal/min?

Solution Using the affinity laws, the diameter required to produce 2000 ft of head at 3200 gal/min is as follows:

$$\left(\frac{D}{10}\right)^2 = \frac{2000}{2115}$$

$$D = 10 \times 0.9724 = 9.72 \text{ in}$$

The speed ratio can be calculated from

$$\left(\frac{N}{3560}\right)^2 = \frac{2000}{2115}$$

Solving for speed,

$$N = 3560 \times 0.9724 = 3462 \text{ r/min}$$

Strictly speaking, this approach is only approximate since the affinity laws have to be applied along iso-efficiency curves. We must create the new H-Q curves at the reduced impeller diameter (or speed) to ensure that at 3200 gal/min the head generated is 2000 ft. If not, adjustment must be made to the impeller diameter (or speed). This is left as an exercise for the reader.

Net positive suction head. An important parameter related to the operation of centrifugal pumps is the concept of net positive suction head (NPSH). This represents the absolute minimum pressure at the suction of the pump impeller at the specified flow rate to prevent pump cavitation. If the pressure falls below this value, the pump impeller may be damaged and render the pump useless.

The calculation of NPSH available for a particular pump and piping configuration requires knowledge of the pipe size on the suction side of the pump, the elevation of the water source, and the elevation of the pump impeller along with the atmospheric pressure and vapor pressure of water at the pumping temperature. The pump vendor may specify that a particular model of pump requires a certain amount of NPSH (known as NPSH required or NPSH_R) at a particular flow rate. Based on the actual piping configuration, elevations, etc., the calculated NPSH (known as NPSH available or NPSH_A) must exceed the required NPSH at the specified flow rate. Therefore,

$$\text{NPSH}_A > \text{NPSH}_R$$

If the NPSH_R is 25 ft at a 2000 gal/min pump flow rate, then NPSH_A must be 35 ft or more, giving a 10-ft cushion. Also, typically, as the flow rate increases, NPSH_R increases fairly rapidly as can be seen from the typical centrifugal pump curve in Fig. 1.14. Therefore, it is important that the engineer perform calculations at the expected range of flow rates to ensure that the NPSH available is always more than the required NPSH, per the vendor's pump performance data. As indicated earlier, insufficient NPSH available tends to cavitate or starve the pump and eventually causes damage to the pump impeller. The damaged impeller will not be able to provide the necessary head pressure as indicated on the pump performance curve. NPSH calculation will be illustrated using an example next.

Figure 1.15 shows a centrifugal pump installation where water is pumped out of a storage tank that is located at a certain elevation above that of the centerline of the pump. The piping from the storage tank to the pump suction consists of straight pipe, valves, and fittings. The NPSH available is calculated as follows:

$$\text{NPSH} = (P_a - P_v)\frac{2.31}{\text{Sg}} + H + E_1 - E_2 - h_f \qquad (1.71)$$

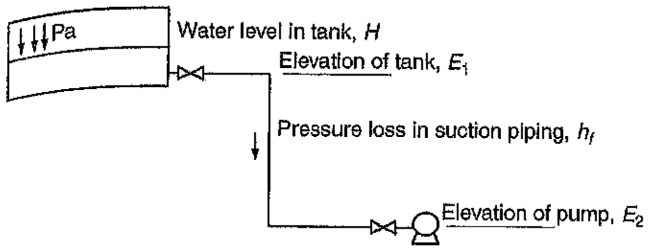


Figure 1.15 NPSH calculations.

where P_a = atmospheric pressure, psi
P_v = liquid vapor pressure at flowing temperature, psia
Sg = liquid specific gravity
H = liquid head in tank, ft
E_1 = elevation of tank bottom, ft
E_2 = elevation of pump suction, ft
h_f = friction loss in suction piping from tank to pump suction, ft

All terms in Eq. (1.71) are known except the head loss h_f. This item must be calculated considering the flow rate, pipe size, and liquid properties. We will use the Hazen-Williams equation with $C = 120$ for calculating the head loss in the suction piping. We get

$$P_m = 23{,}909\left(\frac{3000}{120}\right)^{1.852}\frac{1}{13.5^{4.87}} = 29.03 \text{ psi/mi}$$

The pressure loss in the piping from the tank to the pump $= \frac{29.03\times 500}{5280} =$ 2.75 psi. Substituting the given values in Eq. (1.71) assuming the vapor pressure of water is 0.5 psia at the pumping temperature,

$$\text{NPSH} = (14.7 - 0.5) \times 2.31 + 10 + 102 - 95 - 2.75 = 47.05 \text{ ft}$$

The required NPSH for the pump must be less than this value. If the flow rate increases to 5000 gal/min and the liquid level in turn drops to 1 ft, the revised NPSH available is calculated as follows.

With the flow rate increasing from 3200 to 5000 gal/min, the pressure loss due to friction P_m is approximately,

$$P_m = \left(\frac{5000}{3200}\right)^{1.852} \times 29.03 = 66.34 \text{ psi/mi}$$

$$\text{Head loss in 500 ft of pipe} = 66.34 \times \frac{500}{5280} = 6.3 \text{ psi}$$

Therefore,

$$\text{NPSH} = (14.7 - 0.5) \times 2.31 + 1 + 102 - 95 - 6.3 = 34.5 \text{ ft}$$

It can be seen that the NPSH available dropped off considerably with the reduction in liquid level in the tank and the increased friction loss in the suction piping at the higher flow rate.

The required NPSH for the pump (based on vendor data) must be lower than the preceding available NPSH calculations. If the pump data shows 38 ft NPSH required at 5000 gal/min, the preceding calculation indicates that the pump will cavitate since NPSH available is only 34.5 ft.

Specific speed. An important parameter related to centrifugal pumps is the specific speed. The specific speed of a centrifugal pump is defined as the speed at which a geometrically similar pump must be run such that it will produce a head of 1 ft at a flow rate of 1 gal/min. Mathematically, the specific speed is defined as follows

$$N_S = \frac{NQ^{1/2}}{H^{3/4}} \tag{1.72}$$

where N_S = specific speed
N = impeller speed, r/min
Q = flow rate, gal/min
H = head, ft

It must be noted that in Eq. (1.72) for specific speed, the capacity Q and head H must be measured at the best efficiency point (BEP) for the maximum impeller diameter of the pump. For a multistage pump the value of the head H must be calculated per stage. It can be seen from Eq. (1.72) that low specific speed is attributed to high head pumps and high specific speed for pumps with low head.

Similar to the specific speed another term known as *suction specific speed* is also applied to centrifugal pumps. It is defined as follows:

$$N_{SS} = \frac{NQ^{1/2}}{(\text{NPSH}_R)^{3/4}} \tag{1.73}$$

where N_{SS} = suction specific speed
N = impeller speed, r/min
Q = flow rate, gal/min
NPSH_R = NPSH required at the BEP

With single or double suction pumps the full capacity Q is used in Eq. (1.73) for specific speed. For double suction pumps one-half the value of Q is used in calculating the suction specific speed.

Example 1.23 Calculate the specific speed of a four-stage double suction centrifugal pump with a 12-in-diameter impeller that runs at 3500 r/min and generates a head of 2300 ft at a flow rate of 3500 gal/min at the BEP. Calculate the suction specific speed of this pump, if the NPSH required is 23 ft.

Solution From Eq. (1.72), the specific speed is

$$N_S = \frac{NQ^{1/2}}{H^{3/4}}$$

$$= \frac{3500(3500)^{1/2}}{(2300/4)^{3/4}} = 1763$$

The suction specific speed is calculated using Eq. (1.73):

$$N_{SS} = \frac{NQ^{1/2}}{\text{NPSH}_R{}^{3/4}}$$

$$= \frac{3500(3500/2)^{1/2}}{(23)^{3/4}} = 13{,}941$$

1.13.3 Pumps in series and parallel

In the discussions so far we considered the performance of a single centrifugal pump. Sometimes, because of head limitations of a single pump or flow rate limits, we may have to use two or more pumps together at a pump station to provide the necessary head and flow rate. When more than one pump is used, they may be operated in series or parallel configurations. Series pumps are so arranged that each pump delivers the same volume of water, but the total pressure generated by the combination is the sum of the individual pump heads. Parallel pumps are configured such that the total flow delivered is the sum of the flow rates through all pumps, while each pump delivers a common head pressure. For higher pressures, pumps are operated in series, and when larger flow is required they are operated in parallel.

In Example 1.18 we found that the Corona pump station required pumps that would provide a pressure of 1219 psi at a flow rate of 7986.11 gal/min. Therefore we are looking for a pump or a combination of pumps at Corona that would provide the following:

$$\text{Flow rate} = 7986.11 \text{ gal/min} \qquad \text{and} \qquad \text{Head} = 1219 \times 2.31 = 2816 \text{ ft}$$

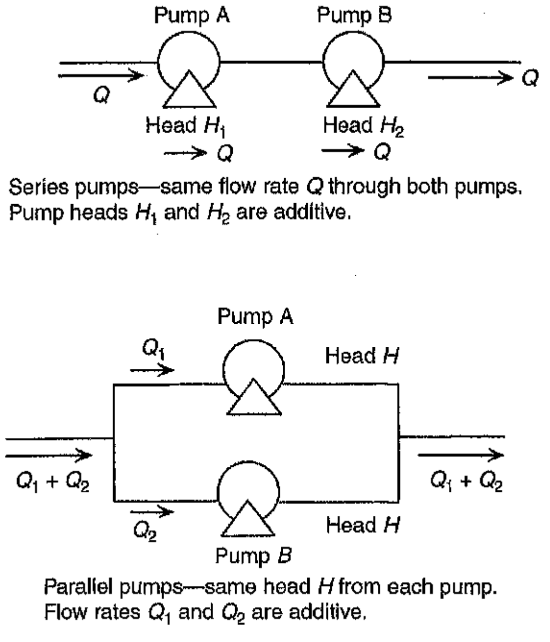


Figure 1.16 Pumps in series and parallel.

From a pump manufacturer's catalog, we can select a single pump that can match this performance. We could also select two smaller pumps that can generate 2816 ft of head at 3993 gal/min. We would operate these two pumps in parallel to achieve the desired flow rate and pressure. Alternatively, if we chose two other pumps that would each provide 1408 ft of head at the full flow rate of 7986.11 gal/min, we would operate these pumps in series. Example of pumps in series and parallel are shown in Fig. 1.16.

In some instances, pumps must be configured in parallel, while other situations might require pumps be operated in series. An example of where parallel pumps are needed would be in pipelines that have a large elevation difference between pump stations. In such cases, if one pump unit fails, the other pump will still be able to handle the head at a reduced flow rate. If the pumps were in series, the failure of one pump would cause the entire pump station to be shut down, since the single pump will not be able to generate enough head on its own to overcome the static elevation head between the pump stations. Figure 1.17 shows how the performance of a single pump compares with two identical pumps in series and parallel configurations.

Example 1.24 Two pumps with the head-capacity characteristics defined as follows are operated in series.

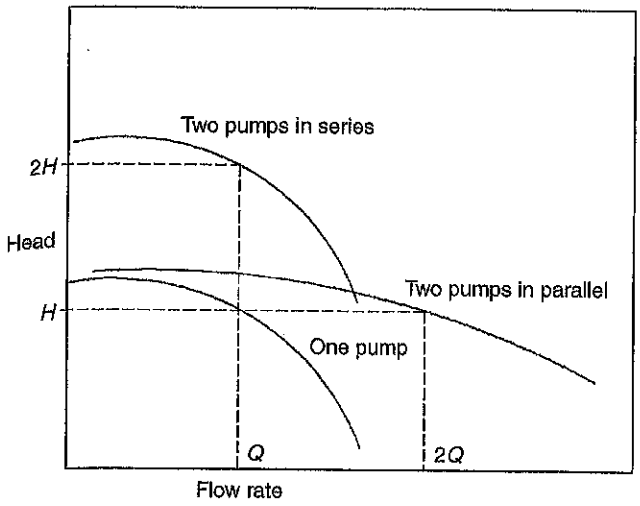


Figure 1.17 Pump performance—series and parallel.

Pump A:

Q, gal/min	0	600	1400	2200	3200
H, ft	2400	2350	2100	1720	1200

Pump B:

Q, gal/min	0	600	1400	2200	3200
H, ft	800	780	700	520	410

(a) Calculate the combined performance of the two operated in series.

(b) When operated in series, what impeller trims must be made to either pump, to meet the requirement of 2080 ft of head at 2200 gal/min?

(c) Can these pumps be operated in parallel configuration?

Solution

(a) Pumps in series cause the heads to be additive at the same flow rate. Therefore, at each flow rate, we add the corresponding heads to create the new H-Q curve for the combined pumps in series.

The combined performance of pump A and pump B in series is as follows:

Q, gal/min	0	600	1400	2200	3200
H, ft	3200	3130	2800	2240	1610

(b) Reviewing the combined pump curve, we see that the head generated at 2200 gal/min is 2240 ft. Since our requirement is 2080 ft of head at 2200 gal/min, clearly we must trim one of the pump impellers. We will leave the smaller pump B alone and trim the impeller of the larger pump A to achieve the total head of 2080 ft.

$$\text{Pump A head trim required} = 2240 - 2080 = 160 \text{ ft}$$

At the desired flow rate of 2200 gal/min, pump A produces 1720 ft. We must reduce this head by 160 ft, by trimming the impeller, or the head must become 1720 − 160 = 1560 ft. Using the affinity laws, the pump trim required is

$$\left(\frac{1560}{1720}\right)^{1/2} = 0.9524 \quad \text{or} \quad 95.24 \text{ percent trim}$$

It must be noted that this calculation is only approximate. We must create the new pump performance curve at 95.24 percent trim and verify that the trimmed pump will generate the desired head of 1560 ft at a flow rate of 2200 gal/min. This is left as an exercise for the reader.

(c) For parallel pumps, since flow is split between the pumps at the common head, the individual pump curves should each have approximately the same head at each flow rate, for satisfactory operation. Reviewing the individual curves for pumps A and B, we see that the pumps are mismatched. Therefore, these pumps are not suitable for parallel operation, since they do not have a common head range.

Example 1.25 Two identical pumps with the head-capacity characteristic defined as follows are operated in parallel. Calculate the resultant pump performance.

Q, gal/min	0	600	1400	2200	3200
H, ft	2400	2350	2100	1720	1200

Solution Since the pumps operated in parallel will have common heads at the combined flow rates, we can generate the combined pump curve by adding the flow rates corresponding to each head value. The resulting combined performance curve is as follows:

Q, gal/min	0	1200	2800	4400	6400
H, ft	2400	2350	2100	1720	1200

1.13.4 System head curve

A *system head curve*, or a system head characteristic curve, for a pipeline is a graphic representation of how the pressure needed to pump water through the pipeline varies with the flow rate. If the pressures required at 1000, 2000, up to 10,000 gal/min are plotted on the vertical axis, with

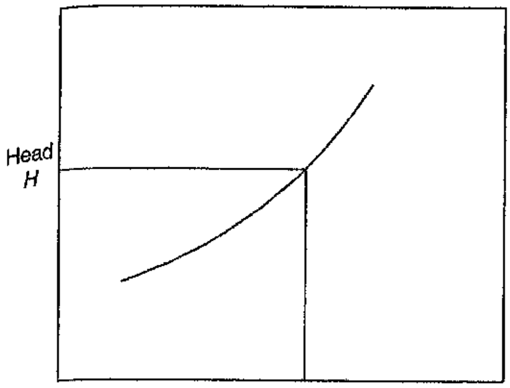


Figure 1.18 System head curve.

the flow rates on the horizontal axis, we get the system head curve as shown in Fig. 1.18.

It can be seen that the system curve is not linear. This is because the pressure drop due to friction varies approximately as the square of the flow rate, and hence the additional pressure required when the flow is increased 2000 to 3000 gal/min is more than that required when the flow rate increases from 1000 to 2000 gal/min.

Consider a pipeline used to transport water from point A to point B. The pipe inside diameter is D and the length is L. By knowing the elevation along the pipeline we can calculate the total pressure required at any flow rate using the techniques discussed earlier. At each flow rate we would calculate the pressure drop due to friction and multiply by the pipe length to get the total pressure drop. Next we will add the equivalent of the static head difference between A and B converted to psi. Finally, the delivery pressure required at B would be added to come up with the total pressure required similar to Eq. (1.29). The process would be repeated for multiple flow rates so that a system head curve can be constructed as shown in Fig. 1.18. If we plotted the feet of head instead of pressure on the vertical axis, we could use the system curve in conjunction with the pump curve for the pump at A. By plotting both the pump H-Q curve and the system head curve on the same graph, we can determine the point of operation for this pipeline with the specified pump curve. This is shown in Fig. 1.19.

When there is no elevation difference between points A and B, the system head curve will start at the point where the flow rate and head are both zero. If the elevation difference were 100 ft, B being higher than A, the system head curve will start at $H = 100$ ft and flow $Q = 0$.

This means at zero flow rate the pressure required is not zero. This simply means that even at zero flow rate, a minimum pressure must be present at A to overcome the static elevation difference between A and B.

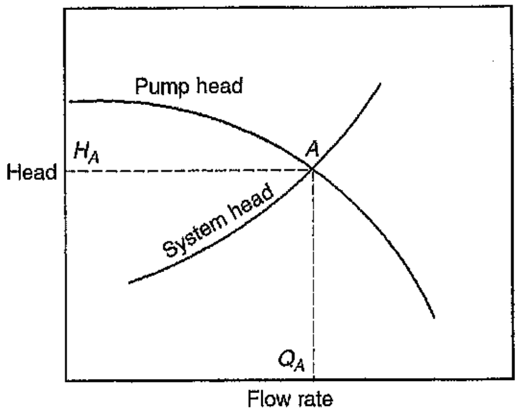


Figure 1.19 Pump head curve and system head curve.

1.13.5 Pump curve versus system head curve

The system head curve for a pipeline is a graphic representation of the head required to pump water through the pipeline at various flow rates and is an increasing curve, indicating that more pressure is required for a higher flow rate. On the other hand, the pump performance (head versus capacity) curve shows the head the pump generates at various flow rates, generally a drooping curve. When the required head per the system head curve equals the available pump head, we have a match of the required head versus the available head. This point of intersection of the system head curve and the pump head curve is the operating point for this particular pump and pipeline system. This is illustrated in Fig. 1.19.

It is possible that in some cases there may not be a point of intersection between a system head curve and a pump curve. This may be because the pump is too small and therefore the system head curve starts off at a point above the shutoff head of the curve and it diverges from the pump curve. Such a situation is shown in Fig. 1.20. It can be seen from this figure that even though there is no operating point between the system head curve and the single pump curve, by adding a second pump in series, we are able to get a satisfactory operating point on the system head curve.

When we use multiple pumps in series or parallel, a combined pump curve is generated and superimposed on the system head curve to get the operating point. Figure 1.21 shows how for a given pipeline system head curve, the operating point changes when we switch from a series pump configuration to a parallel pump configuration.

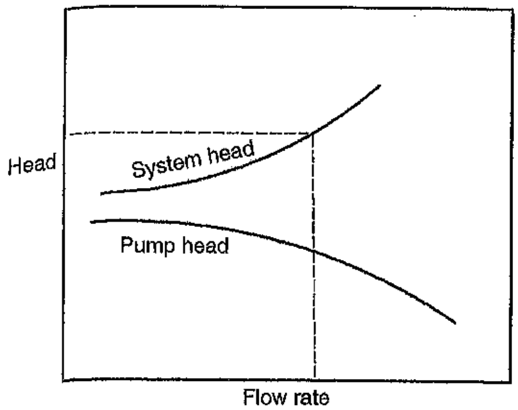


Figure 1.20 Diverging pump head curve and system head curve.

In Fig. 1.21, the pipeline system head curve is plotted along with the pump curves. Also shown are the combined pump curves for both series and parallel operation of two identical pumps. It can be seen that A represents the operating point with one pump, C the operating point for two pumps in series, and finally B the operating point with the two pumps in parallel. Corresponding to these points, the pipeline (and pump) flow rates are Q_A, Q_C, and Q_B, respectively.

The relative magnitudes of these flow rates would depend upon the nature of the system head curve. A steep system head curve will produce a higher flow rate with pumps in series, whereas a flat system head curve will produce a higher flow rate with parallel pumps.

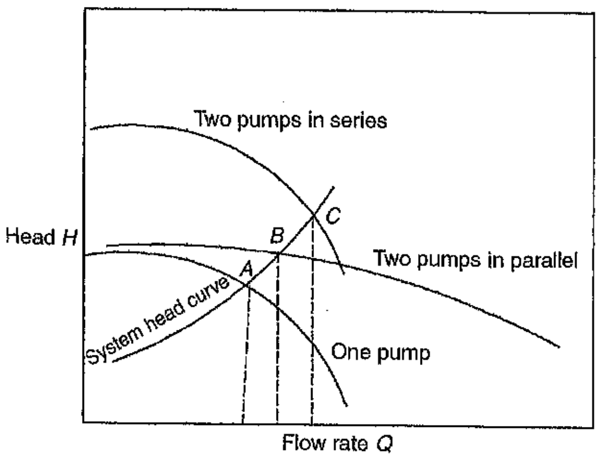


Figure 1.21 Multiple pumps with system head curve.

1.14 Flow Injections and Deliveries

So far we have discussed water pipelines with flow entering the pipeline at the beginning and exiting at the end of the pipeline. There was no flow injection or flow delivery along the pipeline between the entrance and exit. In many instances a certain volume of water would be pumped out of a storage tank and on its way to the destination several intermediate deliveries may be made at various points as shown in Fig. 1.22.

In Fig. 1.22 we see a pipeline that carries 10,000 gal/min from point *A* and at two intermediate points *C* and *D* delivers 2000 and 5000 gal/min, respectively, ultimately carrying the remainder of 3000 gal/min to the termination point *B*. Such a water pipeline would be typical of a small distribution system that serves three communities along the path of the pipeline. The hydraulic analysis of such a pipeline must take into account the different flow rates and hence the pressure drops in each segment. The pressure drop calculation for the section of pipe between *A* and *C* will be based on a flow rate of 10,000 gal/min. The pressure drop in the last section between *D* and *B* would be based on 3000 gal/min. The pressure drop in the intermediate pipe segment *CD* will be based on 8000 gal/min. The total pressure required for pumping at *A* will be the sum of the pressure drops in the three segments *AC*, *CD*, and *DB* along with adjustment for any elevation differences plus the delivery pressure required at *B*. For example, if the pressure drops in the three segments are 500, 300, and 150 psi, respectively, and the delivery pressure required at *B* is 50 psi and the pipeline is on a flat terrain, the total pressure required at *A* will be

$$500 + 300 + 150 + 50 = 1000 \text{ psi}$$

In comparison if there were no intermediate deliveries at *C* and *D*, the entire flow rate of 10,000 gal/min would be delivered at *B* necessitating a much higher pressure at *A* than the 1000 psi calculated.

Similar to intermediate deliveries previously discussed, water may be injected into the pipeline at some locations in between, causing additional volumes to be transported through the pipeline to the terminus *B*. These injection volumes may be from other storage facilities or

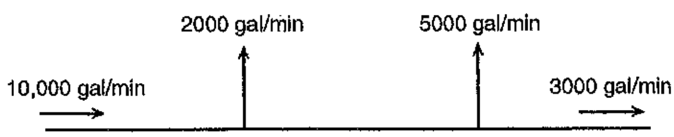


Figure 1.22 Water pipeline with multiple deliveries.

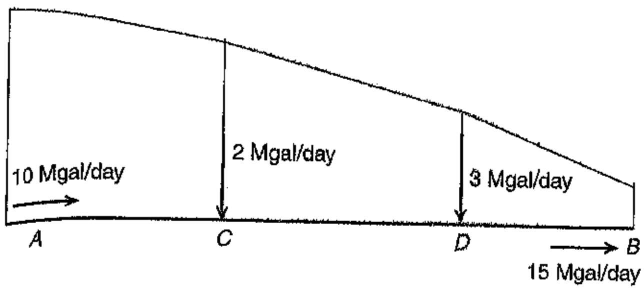


Figure 1.23 Hydraulic gradient with injections and deliveries.

water wells. The impact of the injections and deliveries on the hydraulic pressure gradient is illustrated in Fig. 1.23.

Because of the varying flow rates in the three pipe sections, the slope of the hydraulic gradient, which represents the pressure loss per mile, will be different for each section. Hence the hydraulic gradient appears as a series of broken lines. If the flow through the entire pipeline were a constant value as in previous examples, the hydraulic gradient will be one continuous line with a constant slope equal to the head loss per mile. We will illustrate injection and delivery in a water pipeline system using an example.

Example 1.26 An NPS 30 water pipeline (0.5-in wall thickness) 106 mi long from A to B is used to transport 10,000 gal/min with intermediate deliveries at C and D of 2000 and 3000 gal/min, respectively, as shown in Fig. 1.24. At E, 4000 gal of water is injected into the pipeline so that a total of 9000 gal/min is delivered to the terminus at B at 50 psi. Calculate the total pressure and pumping HP required at A based on 80 percent pump efficiency. Use the Hazen-Williams equation with $C = 120$. The elevations of points A through E are as follows:

$$A = 100 \text{ ft} \quad B = 340 \text{ ft} \quad C = 180 \text{ ft} \quad D = 150 \text{ ft} \quad \text{and} \quad E = 280 \text{ ft}$$

Solution Section AC has a flow rate of 10,000 gal/min and is 23 mi long. Using the Hazen-Williams equation (1.33), we calculate the pressure drop in

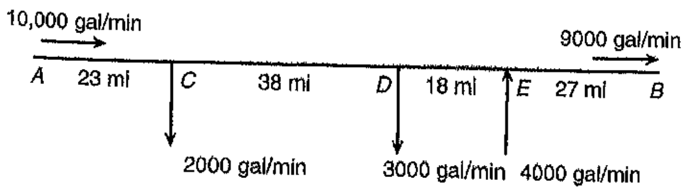


Figure 1.24 Example of water pipeline with injections and deliveries.

this section of pipe to be

$$P_m = 23{,}909\left(\frac{10{,}000}{120}\right)^{1.852}\left(\frac{1}{29.0}\right)^{4.87}$$

$$= 6.5169 \text{ psi/mi}$$

$$\text{Total pressure drop in } AC = 6.52 \times 23 = 149.96 \text{ psi}$$

$$\text{Elevation head for } AC = \frac{180 - 100}{2.31} = 34.63 \text{ psi}$$

Section CD has a flow rate of 8000 gal/min and is 38 mi long. Therefore, the pressure drop is

$$P_m = \left(\frac{8000}{10{,}000}\right)^{1.852} \times 6.5169 = 4.3108 \text{ psi/mi}$$

$$\text{Total pressure drop in } CD = 4.3108 \times 38 = 163.81 \text{ psi}$$

$$\text{Elevation head for } CD = \frac{150 - 180}{2.31} = -12.99 \text{ psi}$$

Section DE flows 5000 gal/min and is 18 mi long. We calculate the pressure drop in this section of pipe to be

$$P_m = \left(\frac{5000}{10{,}000}\right)^{1.852} \times 6.5169 \qquad \text{using proportions}$$

$$= 1.8052 \text{ psi/mi}$$

$$\text{Total pressure drop in } DE = 1.8052 \times 18 = 32.49 \text{ psi}$$

$$\text{Elevation head for } DE = \frac{280 - 150}{2.31} = 56.28 \text{ psi}$$

Section EB flows 9000 gal/min and is 27 mi long. We calculate the pressure drop in this section of pipe to be

$$P_m = \left(\frac{9000}{10{,}000}\right)^{1.852} \times 6.5169 = 5.3616 \text{ psi/mi}$$

$$\Delta P_{EB} = 5.3616 \times 27 = 144.76 \text{ psi}$$

$$\text{Elevation head for } EB = \frac{340 - 280}{2.31} = 25.97 \text{ psi}$$

Adding all the pressure drops and adjusting for elevation difference we get the total pressure required at A including the delivery pressure of 50 psi at B as follows:

$$P_A = (149.96 + 34.63) + (163.81 - 12.99) + (32.49 + 56.28)$$
$$+ (144.76 + 25.97) + 50$$

Therefore, $P_A = 644.91$ psi.

Approximately 645 psi is therefore required at the beginning of pipeline A to pump the given volumes through the pipeline system. The pump HP required at A is calculated next. Assuming a pump suction pressure of 50 psi

$$\text{Pump head} = (645 - 50) \times 2.31 = 1375 \text{ ft}$$

Therefore, the BHP required using Eq. (1.64) is

$$\text{BHP} = 1375 \times 10{,}000 \times \frac{1}{3960 \times 0.8} = 4341$$

Therefore, a 5000-HP motor-driven pump will be required at A.

1.15 Valves and Fittings

Water pipelines include several appurtenances as part of the pipeline system. Valves, fittings, and other devices are used in a pipeline system to accomplish certain features of pipeline operations. Valves may be used to communicate between the pipeline and storage facilities as well as between pumping equipment and storage tanks. There are many different types of valves, each performing a specific function. Gate valves and ball valves are used in the main pipeline as well as within pump stations and tank farms. Pressure relief valves are used to protect piping systems and facilities from overpressure due to upsets in operational conditions. Pressure regulators and control valves are used to reduce pressures in certain sections of piping systems as well as when delivering water to third-party pipelines which may be designed for lower operating pressures. Check valves are found in pump stations and tank farms to prevent backflow as well as separating the suction piping from the discharge side of a pump installation. On long-distance pipelines with multiple pump stations, the pigging process necessitates a complex series of piping and valves to ensure that the pig passes through the pump station piping without getting stuck.

All valves and fittings such as elbows and tees contribute to the frictional pressure loss in a pipeline system. Earlier we referred to some of these head losses as minor losses. As described earlier, each valve and fitting is converted to an equivalent length of straight pipe for the purpose of calculating the head loss in the pipeline system.

A control valve functions as a pressure reducing device and is designed to maintain a specified pressure at the downstream side as shown in Fig. 1.25.

If P_1 is the upstream pressure and P_2 is the downstream pressure, the control valve is designed to handle a given flow rate Q at these pressures. A coefficient of discharge C_v is typical of the control valve design and is related to the pressures and flow rates by the following equation:

$$Q = C_v A (P_1 - P_2)^{1/2} \tag{1.74}$$

where A is a constant.

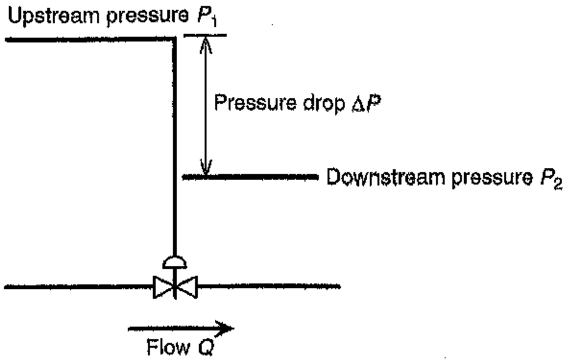


Figure 1.25 Control valve.

Generally, the control valve is selected for a specific application based on P_1, P_2, and Q. For example, a particular situation may require 800 psi upstream pressure, 400 psi downstream pressure, and a flow rate of 3000 gal/min. Based on these numbers, we may calculate a $C_v = 550$. We would then select the correct size of a particular vendor's control valve that can provide this C_v value at a specified flow rate and pressures. For example, a 10-in valve from vendor A may have a C_v of 400, while a 12-in valve may have a $C_v = 600$. Therefore, in this case we would choose a 12-in valve to satisfy our requirement of $C_v = 550$.

1.16 Pipe Stress Analysis

In this section we will discuss how a pipe size is selected based on the internal pressure necessary to transport water through the pipeline. If 1000 psi pressure is required at the beginning of a pipeline to transport a given volume of water a certain distance, we must ensure that the pipe has adequate wall thickness to withstand this pressure. In addition to being able to withstand the internal pressure, the pipeline also must be designed not to collapse under external loads such as soil loading and vehicles in case of a buried pipeline.

Since pipe may be constructed of different materials such as reinforced concrete, steel, wrought iron, plastic, or fiberglass, the necessary wall thickness will vary with the strength of the pipe material. The majority of pipelines are constructed of some form of material conforming to the American National Standards Institute (ANSI), American Society for Testing and Materials (ASTM), American Petroleum Institute (API), American Water Works Association (AWWA), Plastic Pipe Institute (PPI), or Federal Specification.

Barlow's equation is used to calculate the amount of internal pressure that a pipe can withstand, based on the pipe diameter, wall thickness, and the yield strength of the pipe material. Once we calculate this allowable internal operating pressure of the pipeline, we can then determine a hydrostatic test pressure, to ensure safe operation. The hydrostatic test pressure is generally 125 percent of the safe working pressure. The pipeline will be pressurized to this hydrostatic test pressure and the pressure held for a specified period of time to ensure no leaks and no pipe rupture. Generally, aboveground pipelines are hydrotested to 4 h minimum and underground pipelines for 8 h. Various local, city, state, and federal government codes may dictate more rigorous requirements for hydrotesting water pipelines.

Barlow's equation. Consider a circular pipe of outside diameter D and wall thickness T. Depending on the D/T ratio, the pipe may be classified as thin walled or thick walled. Most water pipelines constructed of steel are thin-walled pipes. If the pipe is constructed of some material (with a yield strength S psi) an internal pressure of P psi will generate stresses in the pipe material. At any point within the pipe material two stresses are present. The hoop stress S_h acts along the circumferential direction at a pipe cross section. The longitudinal or axial stress S_a acts along the length or axis of the pipe and therefore normal to the pipe cross section. It can be proved that the hoop stress S_h is twice the axial stress S_a. Therefore, the hoop stress becomes the controlling stress that determines the pipe wall thickness required. As the internal pressure P is increased, both S_h and S_a increase, but S_h will reach the yield stress of the material first. Therefore, the wall thickness necessary to withstand the internal pressure P will be governed by the hoop stress S_h generated in the pipe of diameter D and yield strength S.

Barlow's equation is as follows

$$S_h = \frac{PD}{2T} \tag{1.75}$$

The corresponding formula for the axial (or longitudinal) stress S_a is

$$S_a = \frac{PD}{4T} \tag{1.76}$$

Equation (1.75) for hoop stress is modified slightly by applying a design factor to limit the stress and a seam joint factor to account for the method of manufacture of pipe. The modified equation for calculating the internal design pressure in a pipe in U.S. Customary units is as follows:

$$P = \frac{2TSEF}{D} \tag{1.77}$$

where P = internal pipe design pressure, psi
D = pipe outside diameter, in
T = nominal pipe wall thickness, in
S = specified minimum yield strength (SMYS) of pipe material, psig
E = seam joint factor, 1.0 for seamless and submerged arc welded (SAW) pipes (see Table 1.7)
F = design factor, usually 0.72 for water and petroleum pipelines

The design factor is sometimes reduced from the 0.72 value in the case of offshore platform piping or when certain city regulations require buried pipelines to be operated at a lower pressure. Equation (1.77) for calculating the internal design pressure is found in the Code of Federal Regulations, Title 49, Part 195, published by the U.S. Department of Transportation (DOT). You will also find reference to this equation in ASME standard B31.4 for design and transportation of liquid pipelines.

TABLE 1.7 Pipe Design Joint Factors

Pipe specification	Pipe category	Joint factor E
ASTM A53	Seamless	1.00
	Electric resistance welded	1.00
	Furnace lap welded	0.80
	Furnace butt welded	0.60
ASTM A106	Seamless	1.00
ASTM A134	Electric fusion arc welded	0.80
ASTM A135	Electric Resistance Welded	1.00
ASTM A139	Electric fusion welded	0.80
ASTM A211	Spiral welded pipe	0.80
ASTM A333	Seamless	1.00
ASTM A333	Welded	1.00
ASTM A381	Double submerged arc welded	1.00
ASTM A671	Electric fusion welded	1.00
ASTM A672	Electric fusion welded	1.00
ASTM A691	Electric fusion welded	1.00
API 5L	Seamless	1.00
	Electric resistance welded	1.00
	Electric flash welded	1.00
	Submerged arc welded	1.00
	Furnace lap welded	0.80
	Furnace butt welded	0.60
API 5LX	Seamless	1.00
	Electric resistance welded	1.00
	Electric flash welded	1.00
	Submerged arc welded	1.00
API 5LS	Electric resistance welded	1.00
	Submerged arc welded	1.00

In SI units, the internal design pressure equation is the same as shown in Eq. (1.77), except the pipe diameter and wall thickness are in millimeters and the SMYS of pipe material and the internal design pressures are both expressed in kilopascals.

For a particular application the minimum wall thickness required for a water pipeline can be calculated using Eq. (1.77). However, this wall thickness may have to be increased to account for corrosion effects, if any, and for preventing pipe collapse under external loading conditions. For example, if corrosive water is being transported through a pipeline and it is estimated that the annual corrosion allowance of 0.01 in must be added, for a pipeline life of 20 years we must add $0.01 \times 20 = 0.20$ in to the minimum calculated wall thickness based on internal pressure. If such a pipeline were to be designed to handle 1000 psi internal pressure and the pipeline is constructed of NPS 16, SAW steel pipe with 52,000 psi SMYS, then based on Eq. (1.77) the minimum wall thickness for 1000 psi internal pressure is

$$T = 1000 \times \frac{16}{2 \times 52{,}000 \times 1.0 \times 0.72} = 0.2137 \text{ in}$$

Adding $0.01 \times 20 = 0.2$ in for corrosion allowance for 20-year life, the revised wall thickness is

$$T = 0.2137 + 0.20 = 0.4137 \text{ in}$$

Therefore, we would use the nearest standard wall thickness of 0.500 in.

Example 1.27 What is the internal design pressure for an NPS 20 water pipeline (0.375-in wall thickness) if it is constructed of SAW steel with a yield strength of 42,000 psi? Assume a design factor of 0.66. What would be the required hydrotest pressure range for this pipe?

Solution Using Eq. (1.77),

$$P = 2 \times 0.375 \times 42{,}000 \times 1.0 \times \frac{0.66}{20} = 1039.5$$

$$\text{Hydrotest pressure} = 1.25 \times 1039.5 = 1299.38 \text{ psi}$$

The internal pressure that will cause the hoop stress to reach the yield stress of 42,000 psi will correspond to $1039.5/0.66 = 1575$ psi. Therefore, the hydrotest pressure range is 1300 to 1575 psi.

1.17 Pipeline Economics

In pipeline economics we are concerned with the objective of determining the optimum pipe size and material to be used for transporting a given volume of water from a source to a destination. The criterion would be to minimize the capital investment as well as annual operating and maintenance cost. In addition to selecting the pipe itself to handle the flow rate we must also evaluate the optimum size of pumping equipment required. By installing a smaller-diameter pipe we may reduce the pipe material cost and installation cost. However, the smaller pipe size would result in a larger pressure drop due to friction and hence higher horsepower, which would require larger more costly pumping equipment. On the other hand, selecting a larger pipe size would increase the capital cost of the pipeline itself but would reduce the capital cost of pumping equipment. Larger pumps and motors will also result in increased annual operating and maintenance cost. Therefore, we need to determine the optimum pipe size and pumping power required based on some approach that will minimize both capital investment as well as annual operating costs. The least present value approach, which considers the total capital investment, the annual operating costs over the life of the pipeline, time value of money, borrowing cost, and income tax rate, seems to be an appropriate method in this regard.

In determining the optimum pipe size for a given pipeline project, we would compare three or four different pipe diameters based on the capital cost of pipeline and pump stations, annual operating costs (pump station costs, electricity costs, demand charges, etc.), and so forth. Taking into consideration the project life, depreciation of capital assets, and tax rate, along with the interest rate on borrowed money, we would be able to annualize all costs. If the annualized cost is plotted against the different pipe diameters, we will get a set of curves as shown in Fig. 1.26. The pipe diameter that results in the least annual cost would be considered the optimum size for this pipeline.

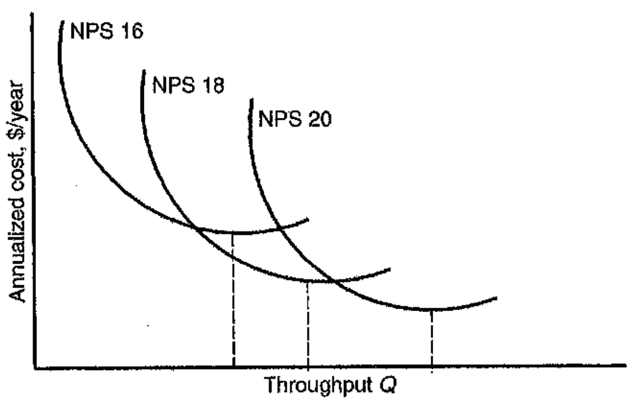


Figure 1.26 Pipeline costs versus pipe diameter.

Example 1.28 A 25-mi-long water pipeline is used to transport 15 Mgal/day of water from a pumping station at Parker to a storage tank at Danby. Determine the optimum pipe size for this application based on the minimum initial cost. Consider three different pipe sizes: NPS 20, NPS 24, and NPS 30. Use the Hazen-Williams equation with $C = 120$ for all pipes. Assume the pipeline is on fairly flat terrain. Use 85 percent pump efficiency. Use \$700 per ton for pipe material cost and \$1500 per HP for pump station installation cost. Labor costs for installing the three pipe sizes are \$100, \$120, and \$130 per ft, respectively. The pipeline will be designed for an operating pressure of 1400 psi. Assume the following wall thickness for the pipes:

NPS 20 pipe: 0.312 in

NPS 24 pipe: 0.375 in

NPS 30 pipe: 0.500 in

Solution First we determine the flow in gal/min:

$$15 \text{ Mgal/day} = \frac{15 \times 10^6}{(24 \times 60)} = 10,416.7 \text{ gal/min}$$

For the NPS 20 pipe we will first calculate the pressure and pumping HP required. The pressure drop per mile from the Hazen-Williams equation (1.33) is

$$P_m = 23{,}909 \left(\frac{10{,}416.7}{120}\right)^{1.852} \frac{1}{19.376^{4.87}}$$
$$= 50.09 \text{ psi/mi}$$

$$\text{Total pressure drop in 25 mi} = 25 \times 50.09 = 1252.25 \text{ psi}$$

Assuming a 50-psi delivery pressure at Danby and a 50-psi pump suction pressure, we obtain

$$\text{Pump head required at Parker} = 1252.25 \times 2.31 = 2893 \text{ ft}$$

$$\text{Pump flow rate} = 10{,}416.7 \text{ gal/min}$$

$$\text{Pump HP required at Parker} = 2893 \times 10{,}416.7 \times \frac{1}{3960 \times 0.85}$$
$$= 8953 \text{ HP}$$

Therefore, a 9000-HP pump unit will be required.

Next we will calculate the total pipe required. The total tonnage of NPS 20 pipe is calculated as follows:

$$\text{Pipe weight per ft} = 10.68 \times 0.312(20 - 0.312) = 65.60 \text{ lb/ft}$$

$$\text{Total pipe tonnage for 25 mi} = 25 \times 65.6 \times \frac{5280}{2000} = 4330 \text{ tons}$$

Increasing this by 5 percent for contingency and considering $700 per ton material cost, we get

$$\text{Total pipe material cost} = 700 \times 4330 \times 1.05 = \$3.18 \text{ million}$$

$$\text{Labor cost for installing NPS 20 pipeline} = 100 \times 25 \times 5280 = \$13.2 \text{ million}$$

$$\text{Pump station cost} = 1500 \times 9000 = \$13.5 \text{ million}$$

Therefore, the total capital cost of NPS 20 pipeline = $3.18 + $13.2 + $13.5 = $29.88 million.

Next we calculate the pressure and HP required for the NPS 24 pipeline. The pressure drop per mile from the Hazen-Williams equation is

$$P_m = 23{,}909 \left(\frac{10{,}416.7}{120}\right)^{1.852} \frac{1}{23.25^{4.87}}$$

$$= 20.62 \text{ psi/mi}$$

$$\text{Total pressure drop in 25 mi} = 25 \times 20.62 = 515.5 \text{ psi}$$

Assuming a 50-psi delivery pressure at Danby and a 50-psi pump suction pressure, we obtain

$$\text{Pump head required at Parker} = 515.5 \times 2.31 = 1191 \text{ ft}$$

$$\text{Pump flow rate} = 10{,}416.7 \text{ gal/min}$$

$$\text{Pump HP required at Parker} = 1191 \times 10{,}416.7 \times \frac{1}{3960 \times 0.85}$$

$$= 3686 \text{ HP}$$

Therefore a 4000-HP pump unit will be required.

Next we will calculate the total pipe required. The total tonnage of NPS 24 pipe is calculated as follows:

$$\text{Pipe weight per ft} = 10.68 \times 0.375\,(24 - 0.375) = 94.62 \text{ lb/ft}$$

$$\text{Total pipe tonnage for 25 mi} = 25 \times 94.62 \times \frac{5280}{2000} = 6245 \text{ tons}$$

Increasing this by 5 percent for contingency and considering $700 per ton material cost, we obtain

$$\text{Total pipe material cost} = 700 \times 6245 \times 1.05 = \$4.59 \text{ million}$$

$$\text{Labor cost for installing NPS 24 pipeline} = 120 \times 25 \times 5280 = \$15.84 \text{ million}$$

$$\text{Pump station cost} = 1500 \times 4000 = \$6.0 \text{ million}$$

Therefore, the total capital cost of NPS 24 pipeline = $4.59 + $15.84 + $6.0 = $26.43 million.

Next we calculate the pressure and HP required for the NPS 30 pipeline. The pressure drop per mile from the Hazen-Williams equation is

$$P_m = 23{,}909 \left(\frac{10{,}416.7}{120}\right)^{1.852} \frac{1}{29.0^{4.87}}$$

$$= 7.03 \text{ psi/mi}$$

Total pressure drop in 25 mi = 25 × 7.03 = 175.75 psi

Assuming a 50-psi delivery pressure at Danby and a 50-psi pump suction pressure, we obtain

Pump head required at Parker = 175.75 × 2.31 = 406 ft

Pump flow rate = 10,416.7 gal/min

$$\text{Pump HP required at Parker} = 406 \times 10{,}416.7 \times \frac{1}{3960 \times 0.85} = 1257 \text{ HP}$$

Therefore a 1500-HP pump unit will be required.

Next we will calculate the total pipe required. The total tonnage of NPS 30 pipe is calculated as follows:

$$\text{Pipe weight per ft} = 10.68 \times 0.500(30 - 0.500) = 157.53 \text{ lb/ft}$$

$$\text{Total pipe tonnage for 25 mi} = 25 \times 157.53 \times \frac{5280}{2000} = 10{,}397 \text{ tons}$$

Increasing this by 5 percent for contingency and considering $700 per ton material cost, we obtain

Total pipe material cost = 700 × 10,397 × 1.05 = $7.64 million

Labor cost for installing
NPS 30 pipeline = 130 × 25 × 5280 = $17.16 million

Pump station cost = 1500 × 1500 = $2.25 million

Therefore, the total capital cost of NPS 30 pipeline = $7.64 + $17.16 + $2.25 = $27.05 million.

In summary, the total capital cost of the NPS 20, NPS 24, and NPS 30 pipelines are

NPS 20 capital cost = $29.88 million

NPS 24 capital cost = $26.43 million

NPS 30 capital cost = $27.05 million

Based on initial cost alone, it appears that NPS 24 is the preferred pipe size.

Example 1.29 A 70-mi-long water pipeline is constructed of 30-in (0.375-in wall thickness) pipe for transporting 15 Mgal/day from Hampton pump station to a delivery tank at Derry. The delivery pressure required at Derry is 20 psi. The elevation at Hampton is 150 ft and at Derry it is 250 ft. Calculate the pumping horsepower required at 85 percent pump efficiency.

This pipeline system needs to be expanded to handle increased capacity from 15 Mgal/day to 25 Mgal/day. The maximum pipeline pressure is 800 psi. One option would be to install a parallel 30-in-diameter pipeline (0.375 wall thickness) and provide upgraded pumps at Hampton. Another option would require expanding the capacity of the existing pipeline by installing an intermediate booster pump station. Determine the more economical alternative for the expansion. Use the Hazen-Williams equation for pressure drop with $C = 120$.

Solution At 15 Mgal/day flow rate,

$$Q = \frac{15 \times 10^6}{24 \times 60} = 10,416.7 \text{ gal/min}$$

Using the Hazen-Williams equation,

$$P_m = 23{,}909 \left(\frac{10{,}416.7}{120}\right)^{1.852} \frac{1}{29.25^{4.87}} = 6.74 \text{ psi/mi}$$

The total pressure required at Hampton is

$$P_t = P_f + P_{\text{elev}} + P_{\text{del}} \qquad \text{from Eq. (1.29)}$$

$$= (6.74 \times 70) + \frac{250 - 150}{2.31} + 20 = 535.1 \text{ psi}$$

Therefore the Hampton pump head required is $(535.1 - 50) \times 2.31 = 1121$ ft, assuming a 50-psi suction pressure at Hampton.

The pump HP required at Hampton [using Eq. (1.64)] is

$$\text{HP} = 1121 \times 10{,}416.7 \frac{1}{3960 \times 0.85} = 3470 \text{ HP}, \quad \text{say 4000 HP installed}$$

For expansion to 25 Mgal/day, the pressure drop will be calculated using proportions:

$$25 \text{ Mgal/day} = \frac{25 \times 10^6}{24 \times 60} = 17{,}361.11 \text{ gal/min}$$

$$P_m = 6.74 \times \left(\frac{25}{15}\right)^{1.852} = 17.36 \text{ psi/mi}$$

The total pressure required is

$$P_t = (17.36 \times 70) + \frac{250 - 150}{2.31} + 20 = 1279 \text{ psi}$$

Since the maximum pipeline pressure is 800 psi, the number of pump stations required

$$= 1279/800 = 1.6, \qquad \text{or 2 pump stations}$$

With two pump stations, the discharge pressure at each pump station = 1279/2 = 640 psi. Therefore, the pump head required at each pump station = (640 − 50) × 2.31 = 1363 ft, assuming a 50-psi suction pressure at each pump station.

The pump HP required [using Eq. (1.64)] is

$$\text{HP} = 1363 \times 17{,}361.11 \frac{1}{3960 \times 0.85}$$

$$= 7030 \text{ HP, say 8000 HP installed}$$

$$\text{Increase in HP for expansion} = 2 \times 8000 - 4000 = 12{,}000 \text{ HP}$$

Incremental pump station cost based on \$1500 per HP = 1500 × 12,000 = \$18 million

This cost will be compared to looping a section of the pipeline with a 30-in pipe. If a certain length of the 70-mi pipeline is looped with 30-in pipe, we could reduce the total pressure required for the expansion from 1279 psi to the maximum pipeline pressure of 800 psi. The equivalent diameter of two 30-in pipes is

$$D_e = 29.25 \left(\frac{2}{1}\right)^{0.3803} = 38.07 \text{ in}$$

The pressure drop in the 30-in pipe at 25 Mgal/day was calculated earlier as 17.36 psi/mi. Hence,

$$P_m \text{ for the 38.07-in pipe} = 17.36 \times (29.25/38.07)^{4.87} = 4.81 \text{ psi/mi}$$

If we loop x miles of pipe, we will have x miles of pipe at $P_m = 4.81$ psi/mi and $(70 - x)$ mi of pipe at 17.36 psi/mi. Therefore, since the total pressure cannot exceed 800 psi, we can write

$$4.81x + 17.36(70 - x) + 43.3 + 20 \leq 800$$

Solving for x we get,

$$x \geq 38.13$$

Therefore we must loop about 39 mi of pipe to be within the 800-psi pressure limit.

If we loop loop 39 mi of pipe, the pressure required at the 25 Mgal/day flow rate is

$$(39 \times 4.81) + (31 \times 17.36) + 43.3 + 20 = 789.1 \text{ psi}$$

The cost of this pipe loop will be calculated based on a pipe material cost of \$700 per ton and an installation cost of \$120 per ft.

$$\text{Pipe weight per foot} = 10.68 \times 0.375 \times (30 - 0.375)$$

$$= 118.65 \text{ lb/ft}$$

$$\text{Material cost of 39 mi of 30-in loop} = \$700 \times 118.65 \times 5280 \times 39$$

$$= \$17.1 \text{ million}$$

$$\text{Pipe labor cost for installing 39 mi of 30-in loop} = \$120 \times 5280 \times 39 = \$24.7 \text{ million}$$

$$\text{Total cost of pipe loop} = \$17.1 + \$24.7 = \$41.8 \text{ million}$$

compared to

$$\text{Incremental pump station cost based on adding a booster pump station} = \$18 \text{ million}$$

Therefore, based on the minimum initial cost alone, looping is not the economical option.

In conclusion, at the expanded flow rate of 25 Mgal/day, it is more cost effective to add HP at Hampton and build the second pump station to limit pipe pressure to 800 psi.